Lecture 2 More examples (1.1, 1.2) MA 266 Steve Bell

MyLab : HW01, 02, 03 due Wed 1/21

HW04 due Fri 1/23

Gradescope : Written HW02, 03, 04 due Fri 1/23

Last time Easiest ODE : $\frac{dy}{dx} = x^2$

$$y = \int x^2 dx + C$$

$$y = \frac{1}{3}x^3 + C$$

Pin down C via an intitial condition : $y(x_0) = y_0$

Today $\frac{dy}{dx} = Ky$ ← RHS is a fcn of y

$K > 0$. y = rabbits in Aus., x = time

$K < 0$ Radioactive decay

$$\underbrace{\frac{1}{y}\frac{dy}{dx}} = K$$

Chain rule

$$\frac{d}{dx}(\sin 3x)^5 = 5(\sin 3x)^4 3\cos 3x$$

$$\frac{d}{dx}(y^5) = 5y^4\frac{dy}{dx}$$

$$\frac{d}{dx}(\ln|y|) = \frac{1}{y}\frac{dy}{dx}$$

Aha! $\frac{d}{dx}[\ln|y|] = K$

$\ln|y|$ = antiderivative for K

$$= \int K\,dx + C$$

$$\ln|y| = Kx + C$$

$$e^{\ln|y|} = e^{kx + c}$$

$$|y| = e^{kx + c} = e^{kx} e^{c}$$

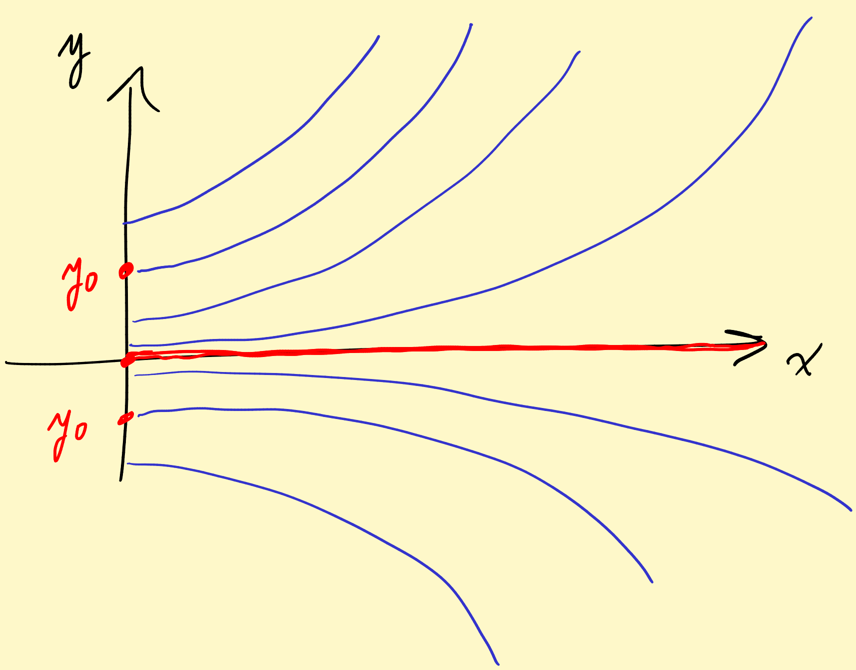
$$y = \underbrace{\left(\pm e^{c}\right)}_{\tilde{c} \,=\, \text{any constant } \underline{\text{except}}\ 0} e^{kx}$$

Change name $\tilde{c}$ to c.

Gen^l Sol^n: $\quad y = ce^{kx}$

Hmmm. $y \equiv 0$ is a sol^n. Method missed it!

Life in ODE!



$y = ce^{kx}, \quad y(0) = y_0$

$$y(0) = c\underbrace{e^{k \cdot 0}}_{1} = c$$

$$y_0 = c$$

$$y = y_0 e^{kx}$$

$\frac{dy}{dx} = ky$ is a <u>linear</u> ODE because $\frac{dy}{dx}$, y appear as linear terms.

Nice thing: Sol^{n}s are defined for <u>all</u> x.

Next: $$\frac{dy}{dx} = y^2$$ ← non-linear

$$\underbrace{\frac{1}{y^2}\frac{dy}{dx}}_{} = 1$$

Aha! $\frac{d}{dx}\left[-\frac{1}{y}\right]$

$$-\frac{1}{y} = \int 1\, dx = x + C$$

$$\boxed{y = \frac{-1}{x+C}}$$

<u>Chain rule</u>

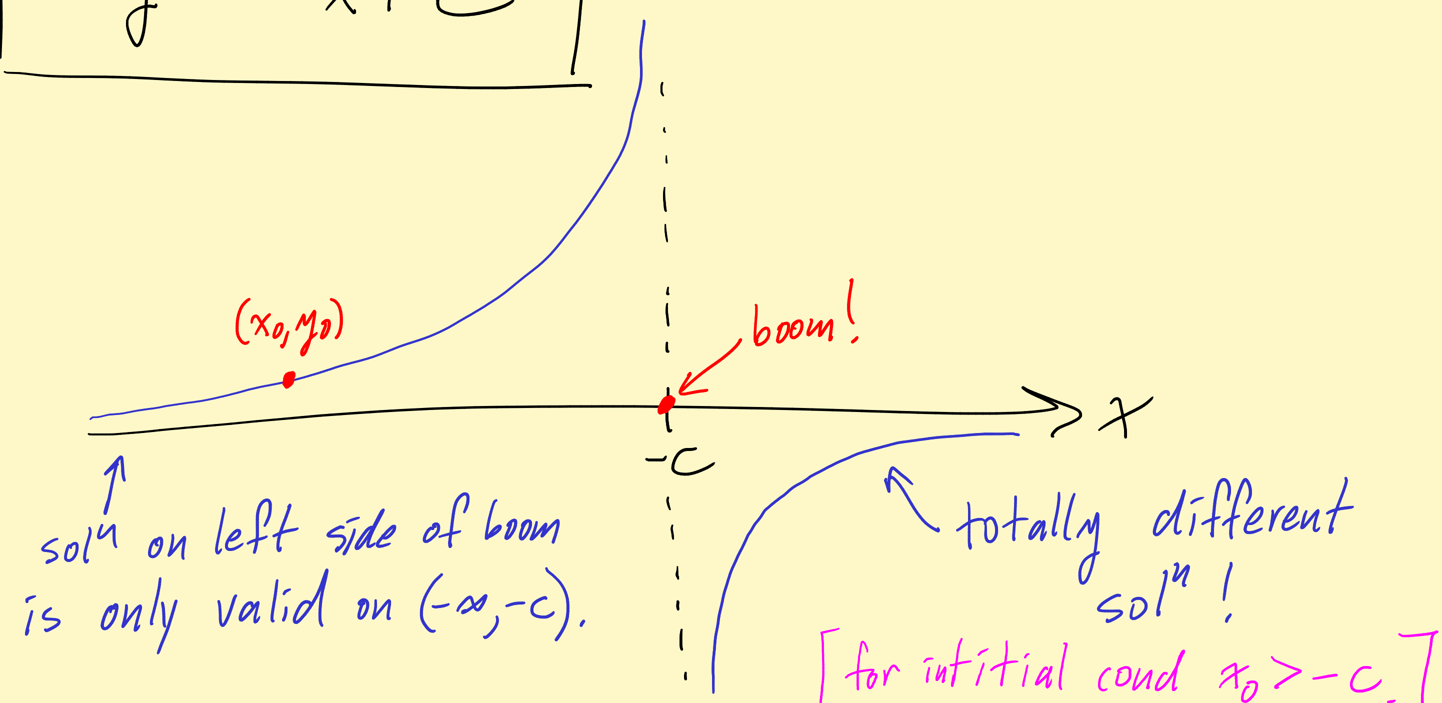
$$\frac{d}{dx}\left(y^{-N}\right) = (-N)\, y^{(-N-1)}\frac{dy}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{y^N}\right) = \frac{-N}{y^{N+1}}\frac{dy}{dx}$$

want $N+1 = 2$

$N = 1$

$$\frac{d}{dx}\left(\frac{1}{y}\right) = \frac{-1}{y^2}\frac{dy}{dx}$$



Hmmm. $y \equiv 0$ is a sol^n. Method missed it again!

Think: $\lim_{c \to \infty} \frac{-1}{x+c} = 0$

$$\left[\lim_{c \to -\infty} (\pm e^c) = 0 \qquad \frac{dy}{dx} = ky \right]$$

EX Rabbits in Aus

$$\frac{dp}{dt} = \underbrace{K_1 P}_{\text{birth rate}} - \underbrace{K_2}_{\text{death rate}}$$

EX: $\frac{dp}{dt} = 2p - 3$

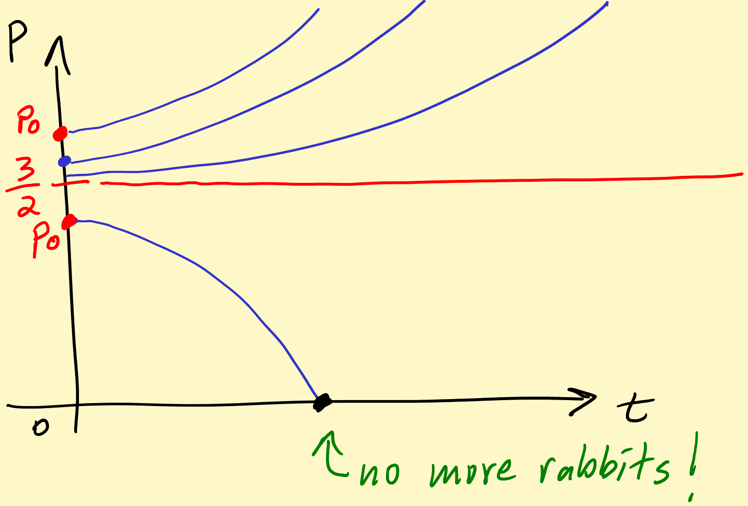
Do chain rule backwards: Get $\underbrace{p = Ce^{2t} + \frac{3}{2}}_{\text{Gen}^l\ \text{Sol}^n}$

$p(0) = p_0 \leftarrow$ initial pop (in millions)

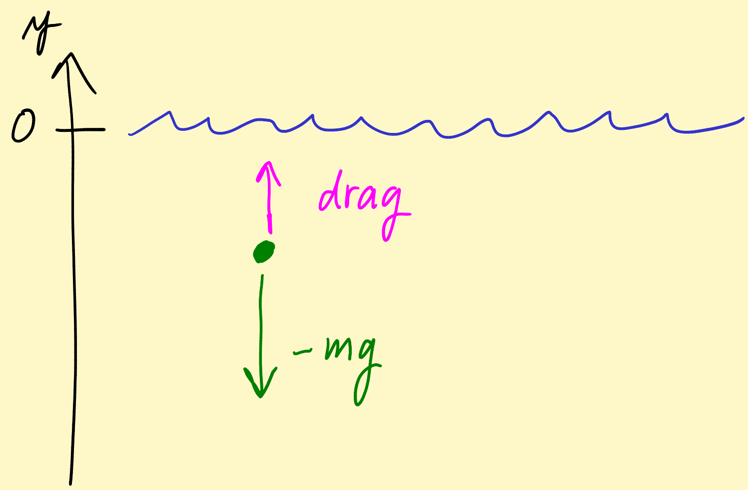
$p(0) = C\underbrace{e^{2 \cdot 0}}_{1} + \frac{3}{2} \overset{\text{want}}{=} p_0$

$$\boxed{C = p_0 - \frac{3}{2}}$$

Sol^n: $p = \left(p_0 - \frac{3}{2}\right)e^{2t} + \frac{3}{2}$



Drop a rock in ocean



Stokes' law

$$F_{drag} = -cV$$

$$\underset{\uparrow \text{ net force}}{F} = ma$$

$$-mg - cV = m\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = -\frac{c}{m}V - g}$$

Same problem!