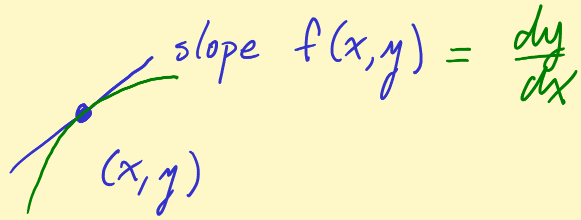
Lecture 3 Direction fields (1.3)

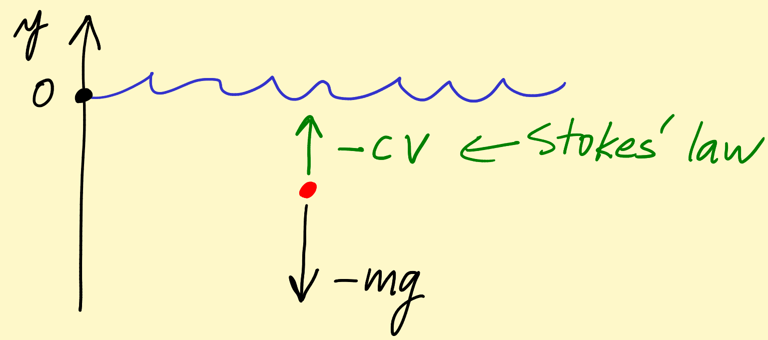
ODE: $\frac{dy}{dx} = f(x, y)$ IVP: $y(x_0) = y_0$



Sol^n: $y(x)$: $y'(x) = f(x, y(x))$

goes through (x_0, y_0)

Drop rock in ocean



$v = \frac{dy}{dt} < 0$ (down)

$$F = ma$$

$$-mg - cv = m\frac{dv}{dt}$$

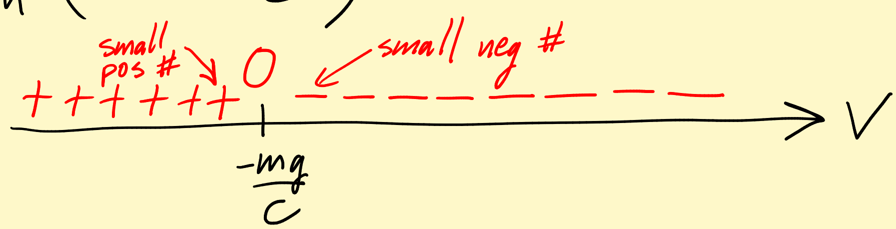
$$\boxed{\frac{dv}{dt} = -\frac{c}{m}v - g}$$

Drop from rest at surface $y=0$.

$$v(0) = 0$$

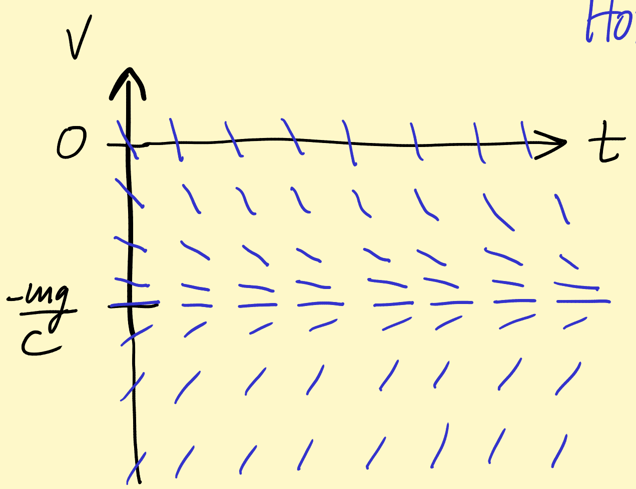
$$\frac{dv}{dt} = -\frac{c}{m}\left(v + \frac{mg}{c}\right)$$

Sign of $\frac{dv}{dt}$

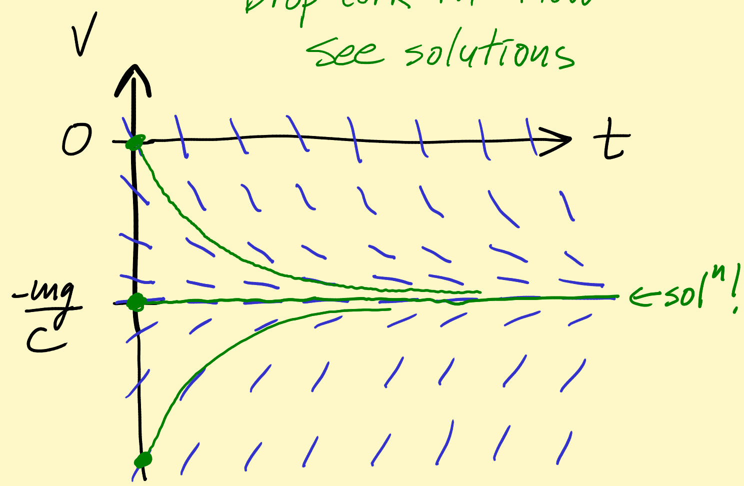


Isoclines : $-\frac{c}{m}\left(v+\frac{mg}{c}\right) = s$ ⟵ slope (constant)

Horizontal lines

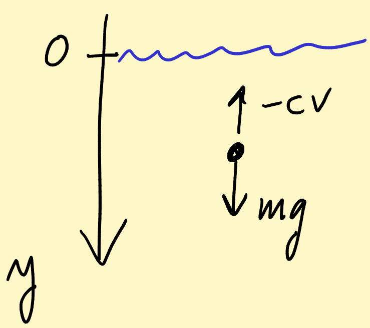


"Drop cork in flow"
see solutions



Terminal velocity $= \frac{-mg}{c}$

Book :

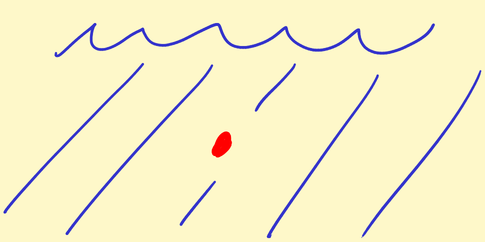


$$\frac{dv}{dt} = mg - cv$$

Get same conclusions.

Hot rock in ocean

Newton's law of cooling



T = temp of rock at time t

A = temp of ocean (ambient temp)

$T(0) = T_0$

$$\frac{dT}{dt} = -k(T - A)$$

hot rock cools. So const < 0

HW: Analyze via D. Field.

Sol^n: $$\underbrace{\frac{1}{T-A}\frac{dT}{dt}}_{\frac{d}{dt}\left[\ln|T-A|\right]} = -k$$

$\ln|T-A|$ = antiderive for $-k$

$$\ln|T-A| = -kt + C$$

$e^{[\ \ ]}$: $$|T-A| = e^{-kt+C} = e^{-kt}e^{C}$$

$$T-A = \underbrace{\left(\pm e^{C}\right)}_{\text{small } c} e^{-kt}$$

$$T = A + ce^{-kt} \quad \leftarrow \text{Gen}^l \text{ Sol}^n$$

IVP $$T(0) = A + c\underbrace{e^{0}}_{1}$$

$$T_0 = A + c \qquad \boxed{c = T_0 - A}$$

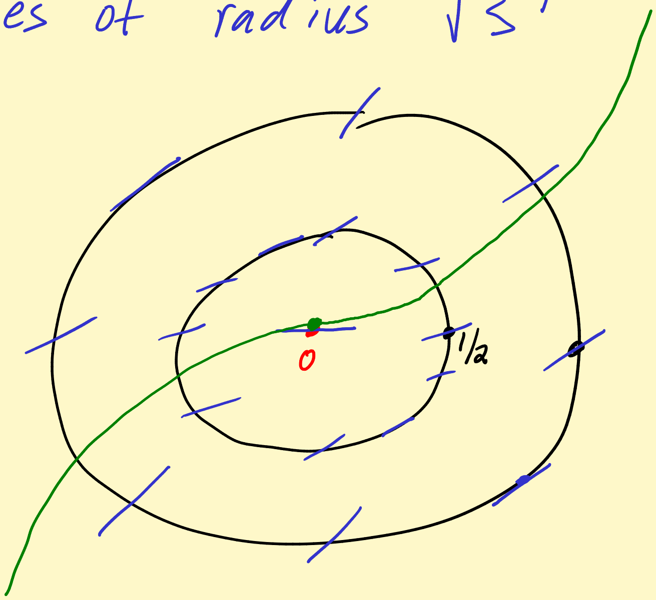
Sol^n to IVP $\boxed{T = A + (T_0 - A)e^{-kt}}$

D. Fields

EX $\dfrac{dy}{dx} = x^2 + y^2$

Isoclines : $x^2 + y^2 = s \leftarrow$ slope (constant)

Circles of radius $\sqrt{s}$ Note: $s \geq 0$



$r = \underbrace{\sqrt{\tfrac{1}{4}}}_{\frac{1}{2}}$ $s = \frac{1}{4}$

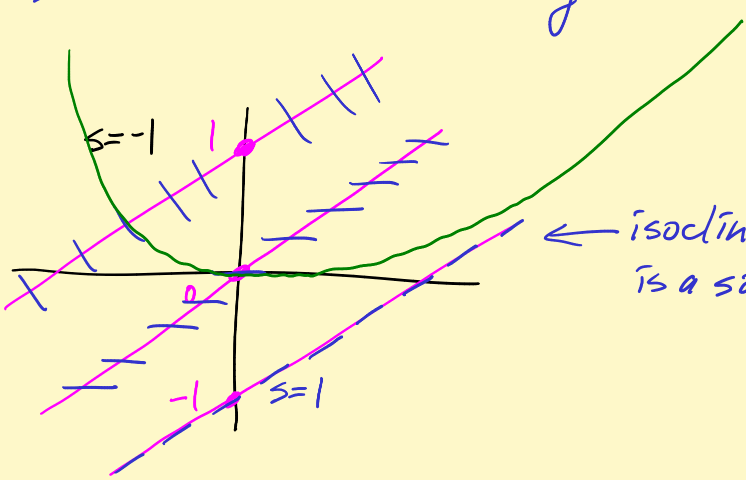
$r = 1 = \sqrt{1}$. $s = 1$

EX $\dfrac{dy}{dx} = x - y$

Isoclines : $x - y = s$ $y = x - s$

lines : slope 1

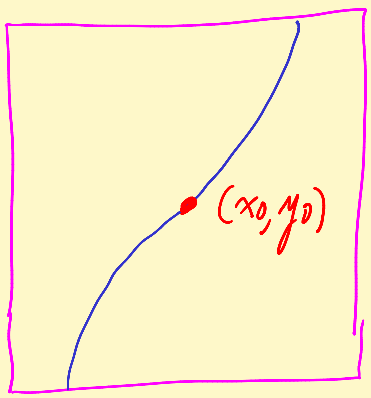
y intercept $-s$



$\leftarrow$ isocline is a sol^n!

Existence & Uniqueness thm

$$\frac{dy}{dx} = f(x,y) \qquad y(x_0) = y_0$$



R = rectangle

sol^n $y(x)$

If 1) $f(x,y)$ is continuous on R

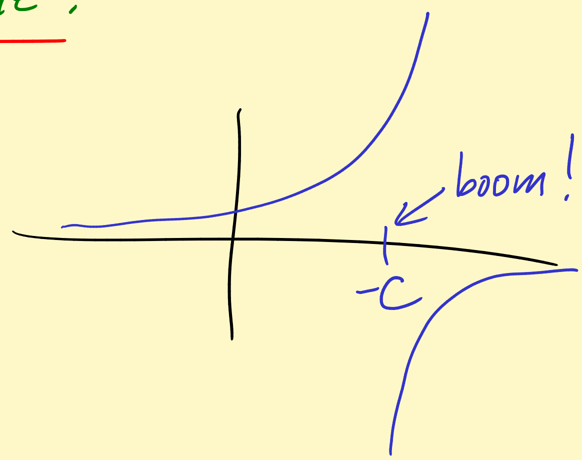
and

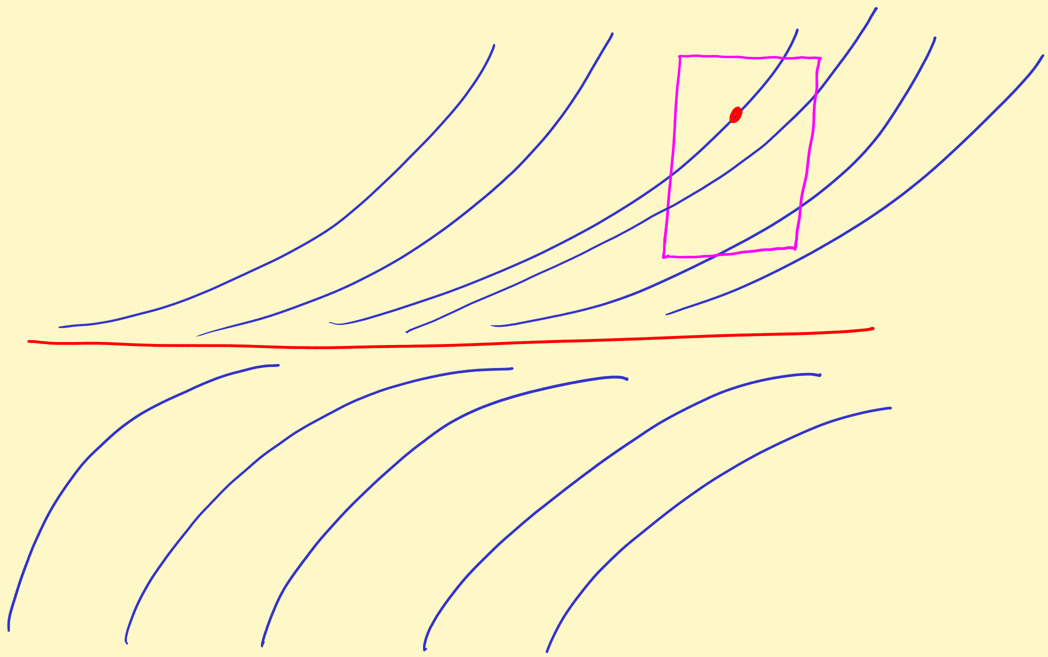
2) $\frac{\partial f}{\partial y}$ is also continuous on R,

then a sol^n to ODE with $y(x_0) = y_0$ exists

and it is unique.

EX: $\frac{dy}{dx} = y^2$





Sol^{n}s exist on (x_0-h, x_0+k) for some $h,k>0$.