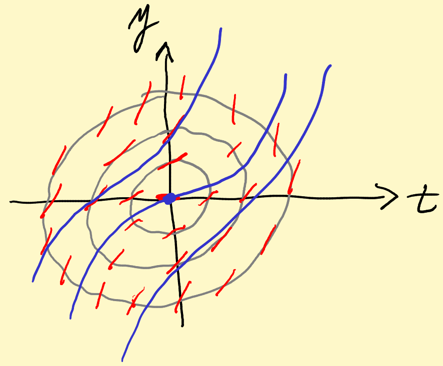
Lecture 10 Autonomous first order ODE (2.2)

MyLab HW 09
GS written HW 08, 09

EX $\dfrac{dy}{dt} = t^2 + y^2 \leftarrow$ t on RHS. Not autonomous



Isoclines: $t^2 + y^2 = s$
$\uparrow$ slope s

Circles

EX $\dfrac{dy}{dt} = f(y) \leftarrow$ no t on RHS. Autonomous

Isoclines $f(y) = s$ $\quad y = f^{-1}(s) \leftarrow$ horiz lines

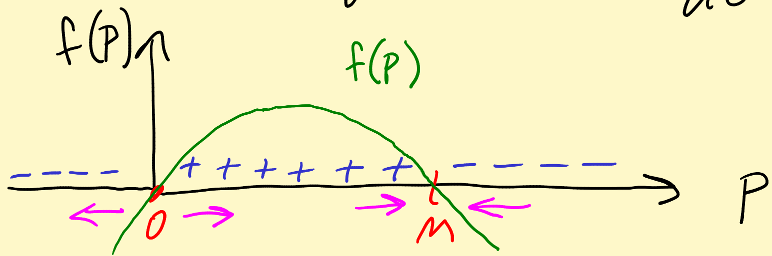
Def^{n} $\dfrac{dy}{dt} = f(y)$ Let z_0 be a zero of f.

Then $y \equiv z_0$ is a solution!

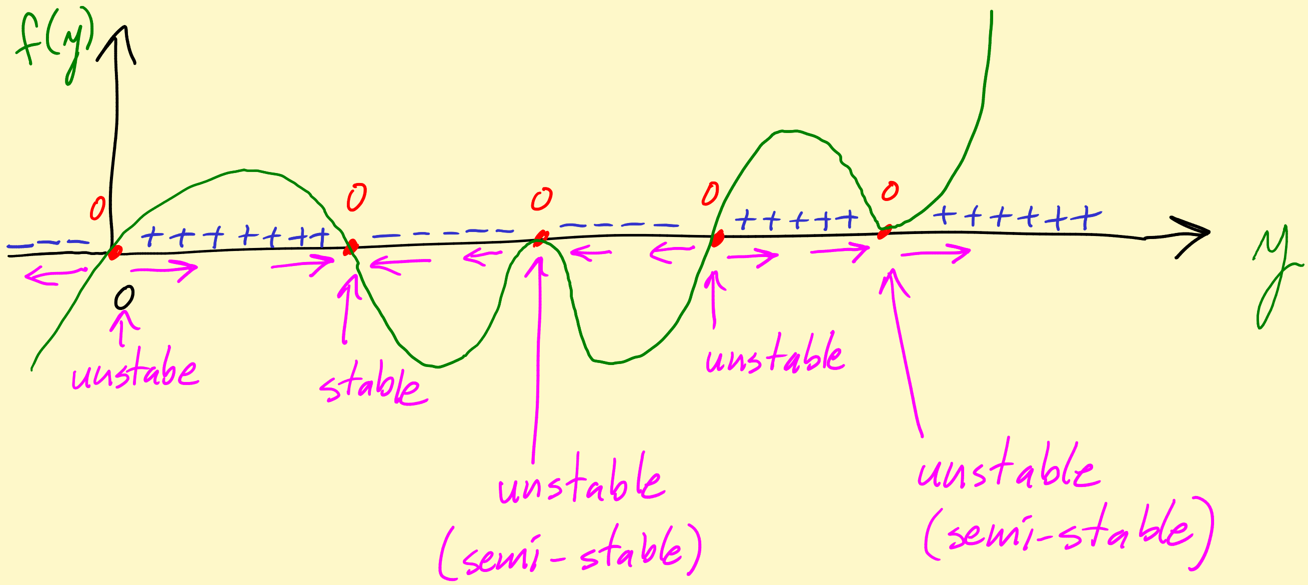
$$\underbrace{\frac{d}{dt}[z_0]}_{0} = \underbrace{f(z_0)}_{0} \quad \checkmark$$

$y \equiv z_0$ is called an equillibrium solution.

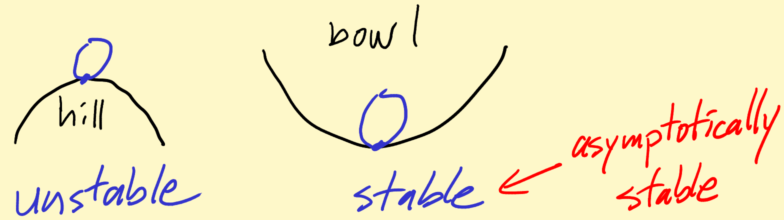
Important diagram: $\dfrac{dP}{dt} = K(M-P)P$ (Logistics eqn)

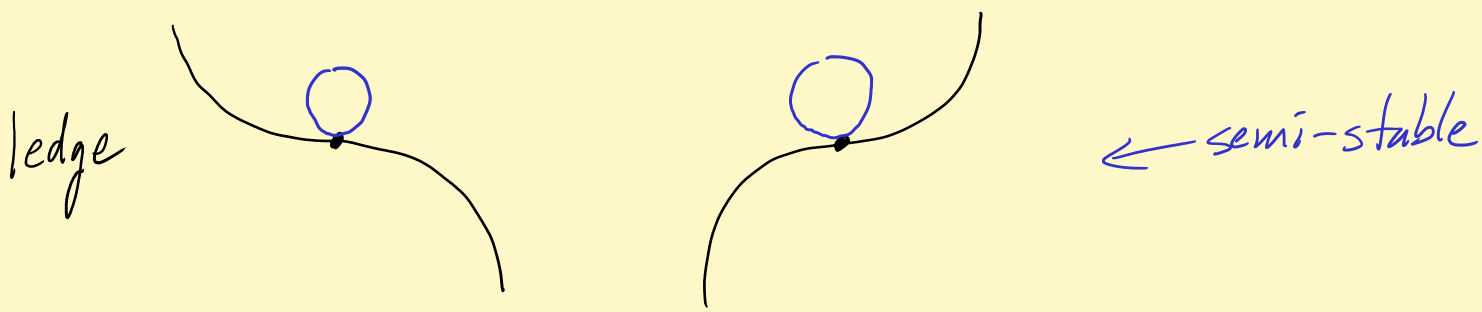


General case $\quad \dfrac{dy}{dt} = f(y)$



Feeling: Sticky ping pong ball

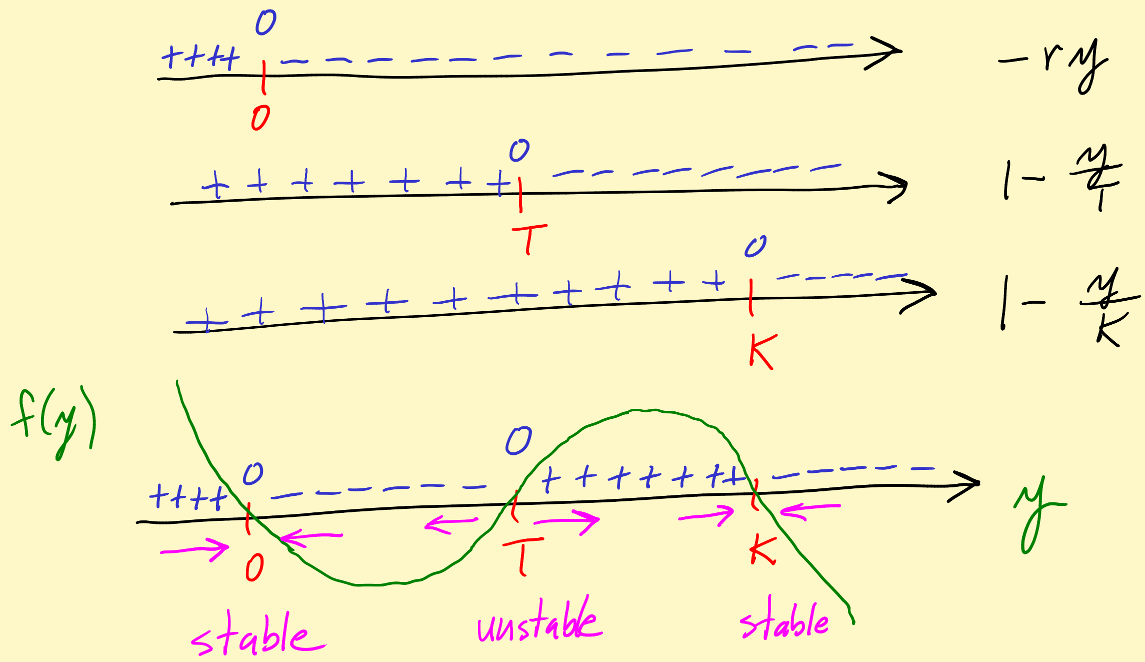


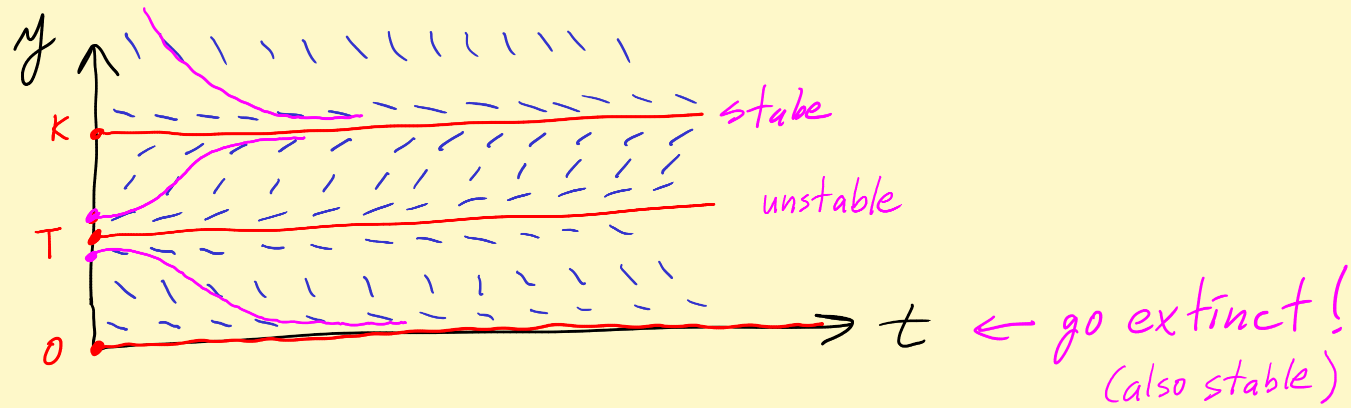


Something that's stable, but not asymptotically stable:

The solar system! Add a rock to the moon. The system shifts a little, but it stays close to what it was. It doesn't rattle apart!

EX Carrier pigeons $\quad \dfrac{dy}{dt} = -ry\left(1-\dfrac{y}{T}\right)\left(1-\dfrac{y}{K}\right)$





Finding inflection pts. $\qquad \dfrac{dy}{dt} = f(y)$

$$\frac{d^2y}{dt^2} = f'(y)\underbrace{\frac{dy}{dt}}_{\frac{dy}{dt}=f(y)}$$

$$\boxed{\frac{d^2y}{dt^2} = f'(y)\,f(y)}$$

Get zeroes of $f'(y)$ and $f(y)$, and sign of product between zeroes, check where $f'(y)f(y)$ <u>changes</u> sign. Say at z_0:

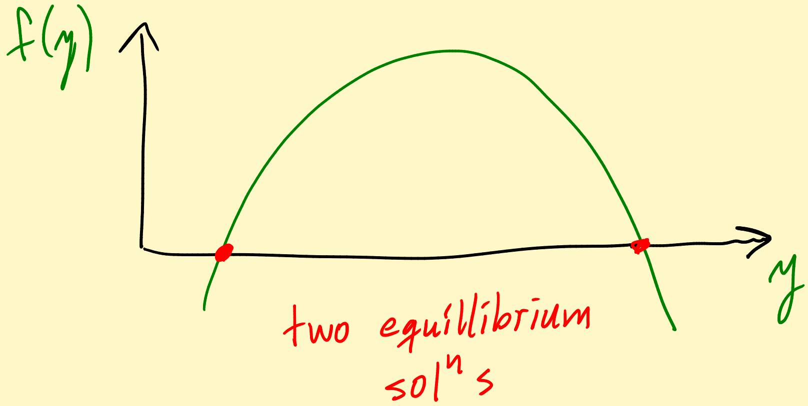
Sol^{n}s change concavity when they pass throug $y \equiv z_0$.

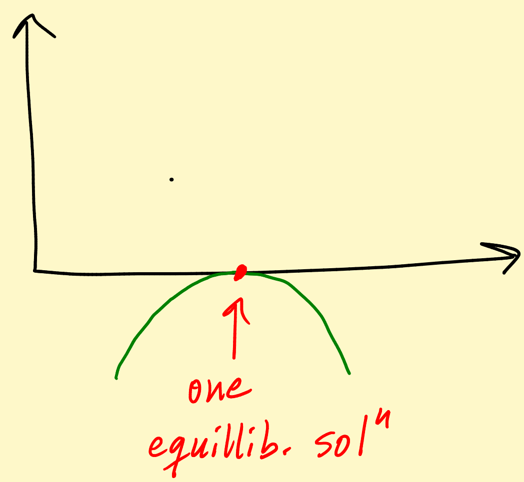
Bifurcation pts

EX $\qquad \dfrac{dy}{dt} = \underbrace{y(4-y) - h}_{f(y)} \qquad$ parameter h

Hmmm.

$f(y)$ is quadractic!

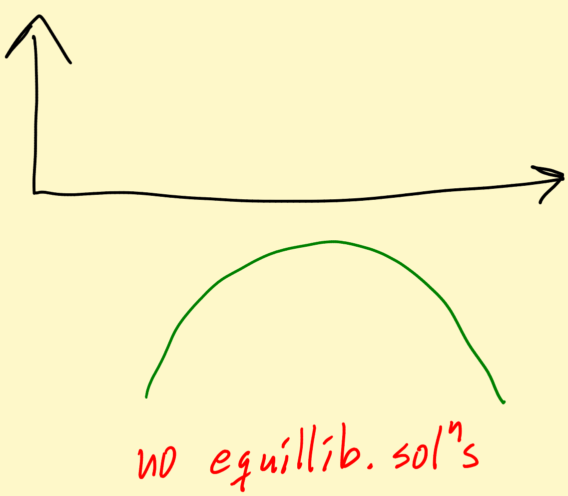




Delicate!

Magic h where this change occurs.

This h is the "bifurcation pt."



See p. 92.

Also, see last page (p. 7) of Lecture 9 for corrected and improved discussion of inflection pts. for sol^n of the logistic eqn.

Here is a copy of p.7 of Lecture 9 :

Fun thing: Where are inflection pts in $0 < P < M$ of the logistics equ: Pts where $\frac{d^2P}{dt^2}$ changes sign

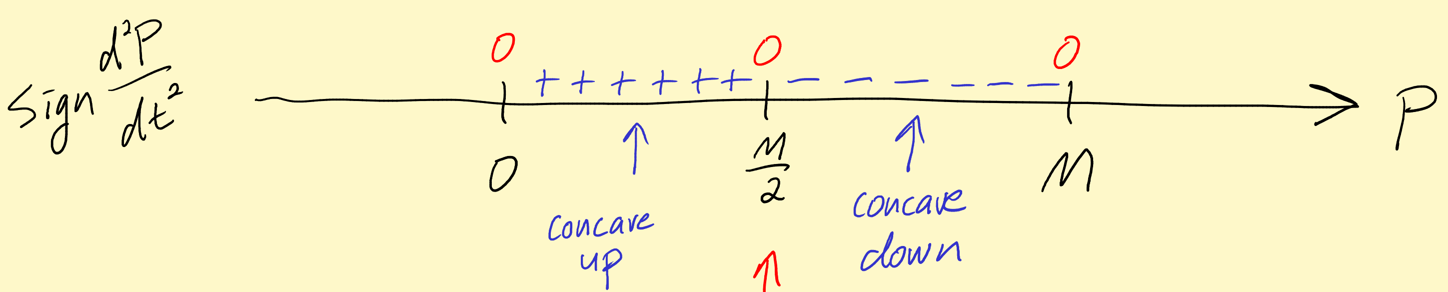
$$\frac{dP}{dt} = k(M-P)P = kMP - kP^2$$

$$\frac{d^2P}{dt^2} = kM\frac{dP}{dt} - k2P\frac{dP}{dt}$$

$$\frac{dP}{dt} = k(M-P)P$$

$$= \left(kM - k2P\right)\frac{dP}{dt}$$

$$= k(M-2P)\,k(M-P)P$$



Inflection when sol'ns pass through $P = \frac{M}{2}$