Lecture 17 Vibrations (3.4)

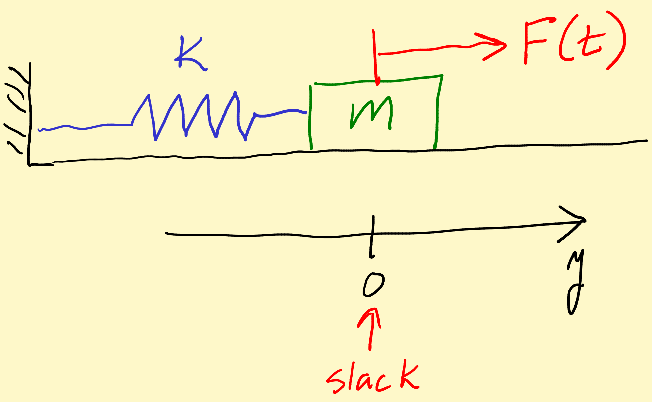
MyLab HW 16

Exam 1 : Tuesday, Feb 24 6:30 pm Elliott Hall of Music

Exam covers up to Lesson 16 on 3.3. Ten problems:
1-7 multiple choice (show work, circle letter A. of correct answer – no scantron), 8, 9, 10 written answer problems. Bring pencils & erasers. Write on problem side of paper only – will be scanned to Gradescope. (Can use back side for scratch – won't be seen.) Seating chart, old exam archive, and practice problems at www.math.purdue.edu/MA266

Probs 1-16 of practice problems a good review.

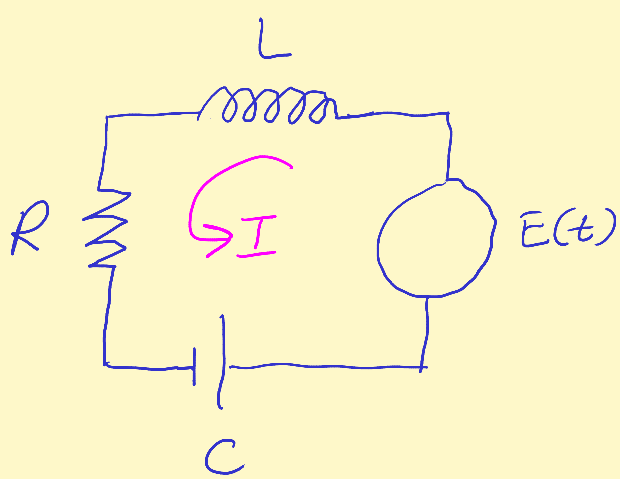
Monday : Review lecture



$$F_{net} = ma$$

$$F(t) - Ky - cV = ma$$

external force   Hooke's law   friction



Kirchof's law

$$E(t) = L\frac{dI}{dt} + RI + \frac{1}{C}Q$$

$$E'(t) = L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I$$

$$m\frac{d^2y}{dt} + c\frac{dy}{dt} + ky = F(t)$$

Case No friction, no external force F

$$my'' + ky = 0$$

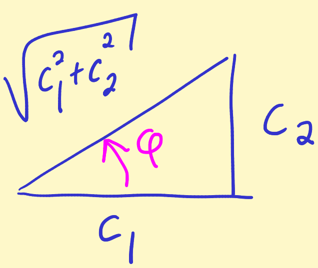
$$mr^2 + k = 0$$

$$r = \sqrt{\frac{k}{m}}\, i \qquad \leftarrow \omega_0 = \sqrt{\frac{k}{m}}$$

Gen^l Sol^n $\quad y = c_1 \cos\sqrt{\frac{k}{m}}\,t + c_2 \sin\sqrt{\frac{k}{m}}\,t$

Classic important trick:

$$y = \underbrace{\sqrt{c_1+c_2^2}}_{A}\left(\underbrace{\frac{c_1}{\sqrt{c_1^2+c_2^2}}}_{\cos\varphi} \cos\omega_0 t + \underbrace{\frac{c_2}{\sqrt{c_1^2+c_2^2}}}_{\sin\varphi} \sin\omega_0 t\right)$$



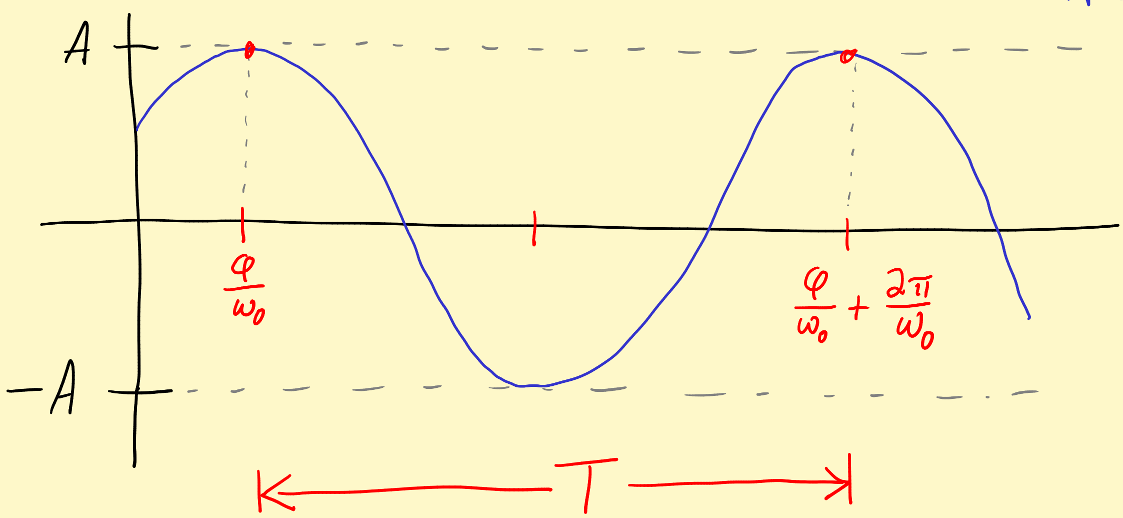
$$\boxed{y = A\cos(\omega_0 t - \varphi)}$$

$\leftarrow$ easy to graph!

$\varphi = \mathrm{Tan}^{-1}\frac{c_2}{c_1} \longleftarrow$ "phase shif"

$A = \sqrt{c_1^2 + c_2^2} \longleftarrow$ "amplitude" $A > 0.$

$A\cos(\omega_0 t - \varphi)$



$\omega_0 t - \varphi = 2\pi$

$T =$ "period" $= \frac{2\pi}{\omega_0}$ Frequency: $f = \frac{1}{T} = \frac{\omega_0}{2\pi}$

Trig identities: $e^{(\alpha+\beta)i} = e^{\alpha i} e^{\beta i} = [\cos\alpha + i\sin\alpha]\cdot[\cos\beta + i\sin\beta]$

$\underline{\cos(\alpha+\beta)} + i\,\underline{\sin(\alpha+\beta)} = \left(\underline{\cos\alpha\cos\beta - \sin\alpha\sin\beta}\right) + i\left(\underline{\cos\alpha\sin\beta + \sin\alpha\cos\beta}\right)$

$\beta = -\varphi$

$\sin\beta = -\sin\varphi$

<u>Damping</u> $m\ddot{y} + c\dot{y} + ky = 0$

$mr^2 + cr + k = 0$

Roots $r_1, r_2 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$

Case r_1, r_2 real and unequal — Overdamped

$$r_1 = -\frac{c}{2m} - \frac{\sqrt{c^2-4mk}}{2m} \longleftarrow \text{negative } \checkmark$$

$$r_2 = -\frac{c}{2m} + \frac{\sqrt{c^2-4mk}}{2m} \longleftarrow \text{also negative!}$$

Hmmm. $\quad c^2 > c^2 - 4mk > 0$

know

Take square root: $\quad c > \sqrt{c^2-4mk} > 0$

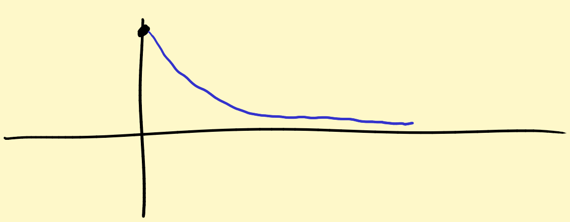
Divide by $2m$: $\quad \frac{c}{2m} > \frac{\sqrt{c^2-4mk}}{2m} > 0$

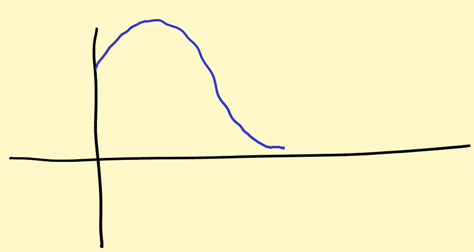
Subtract $\frac{c}{2m}$: $\quad 0 > -\frac{c}{2m} + \frac{\sqrt{c^2-4mk}}{2m} > -\frac{c}{2m}$ $\checkmark$

Conclude $\quad y = c_1 e^{r_1 t} + c_2 e^{r_2 t} \xrightarrow[t\to\infty]{} 0$ fast

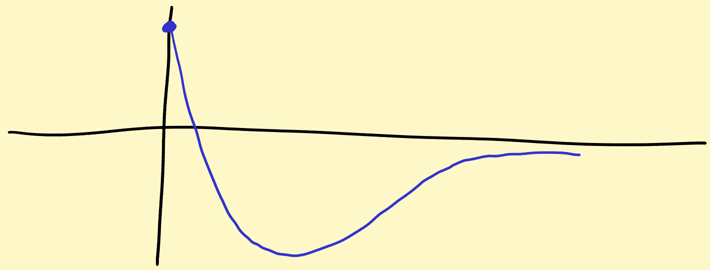
because r_1, r_2 both < 0.

Cool fact: It "wiggles" at most once.





"At most one hump." "Crosses $y=0$ at most once."



Critically damped   Two real roots, the same.

$$c^2 - 4km = 0 \qquad r_1, r_1 = \frac{-c}{2m}$$

$$y = c_1 e^{-\frac{c}{2m}t} + c_2 t e^{-\frac{c}{2m}t}$$

also $\to 0$ fast as $t \to \infty$.

Same behavior: At most one hump and crossing of $y=0$.

Delicate! One speck of dust added to m: Get complex roots — wiggling.

A little Lighter: Overdamped. No wiggling.

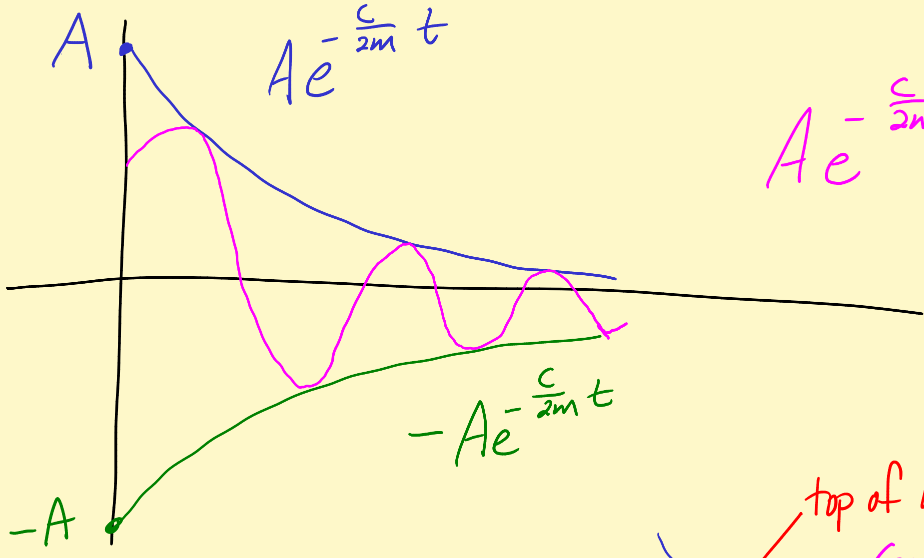
Underdamped   Complex roots: $c^2 - 4km < 0$

$$r_1, r_2 = \frac{-c}{2m} \pm \underbrace{\frac{\sqrt{|c^2-4km|}}{2m}}_{\omega} i$$

$$y = e^{-\frac{c}{2m}t}\left(c_1 \cos \omega t + c_2 \sin \omega t\right)$$

goes to zero fast!

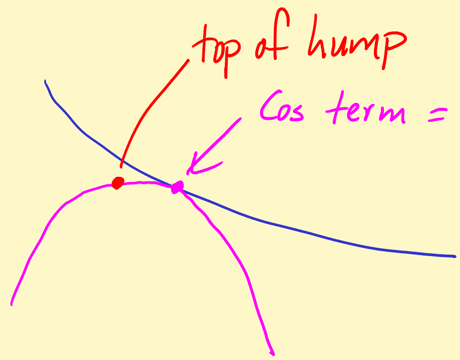
$$y = A e^{-\frac{c}{2m}t} \cos(\omega t - \varphi)$$



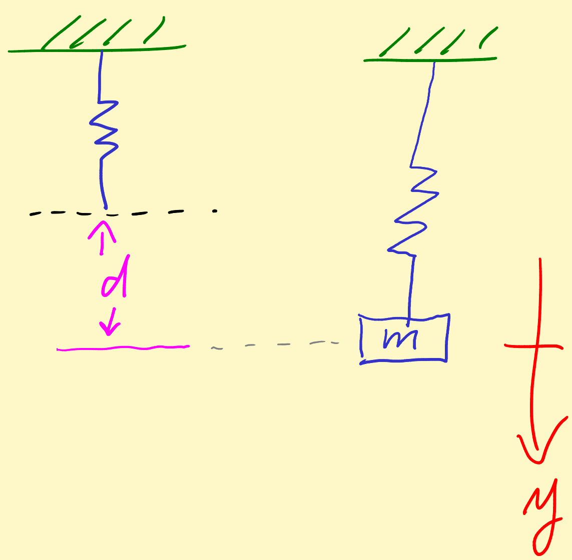
$Ae^{-\frac{c}{2m}t} \cos(\omega t - \varphi)$

Shock absorbers are shot!

Top of humps?



Vertical springs



$kd = mg$

$d = \frac{mg}{k}$

$F = ma$

$-cv + mg - k(d+y) = ma$

Get same ODE.