MA 266 - 116 ODE

Steve Bell
MATH 750
494-1497

T, Th 2:00-3:30
& via Piazza

www.math.purdue.edu/MA266
& Brightspace

First HWKs due next Mon night at 11:59 pm.

Easiest ODE $\quad \frac{dy}{dx} = f(x)$

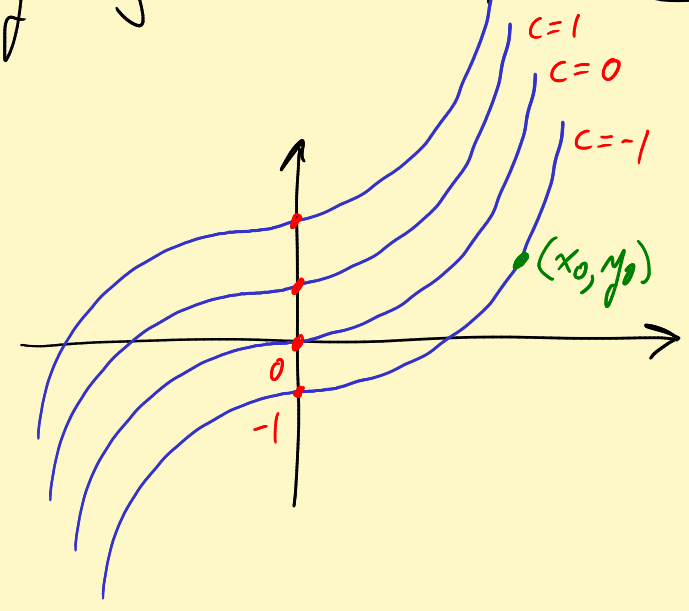
$$y = \int f(x)\,dx + C$$

∞ many sol'ns!

any antiderivative of $f(x)$
Good one: $\int_a^x f(t)\,dt$

EX $\quad \frac{dy}{dx} = 3x^2$

$$y = \int 3x^2\,dx = x^3 + C$$



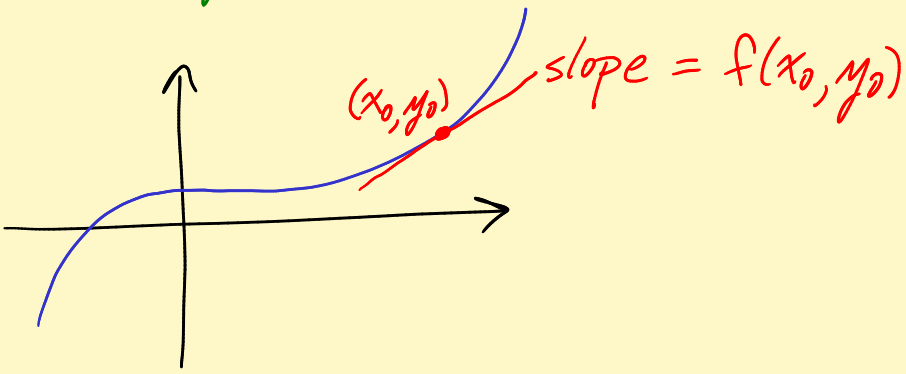
Pin one sol'n down by specifying

$y(x_0) = y_0$

Think: x = time. $\quad y(x)$ = position at time x.

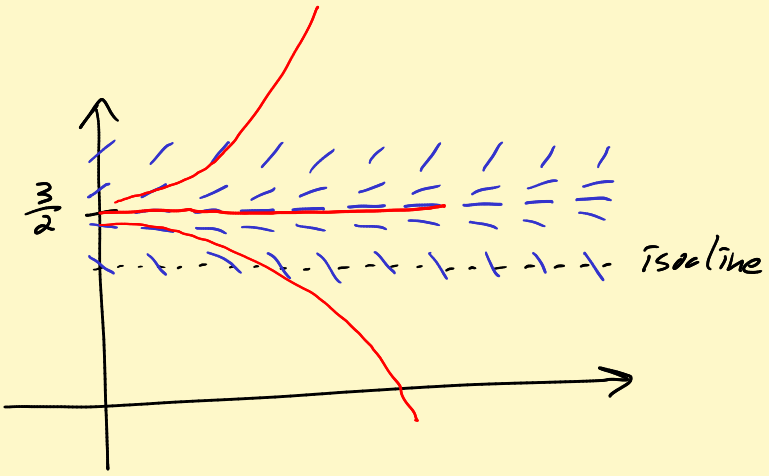
General first order ODE $\quad \frac{dy}{dx} = f(x,y)$

Soln: Is a fcn $y(x)$



EX: $\quad \frac{dy}{dx} = 2y - 3 \qquad 2y - 3 = s$ slope

Curve where $s = \text{const}$ are called "isoclines"



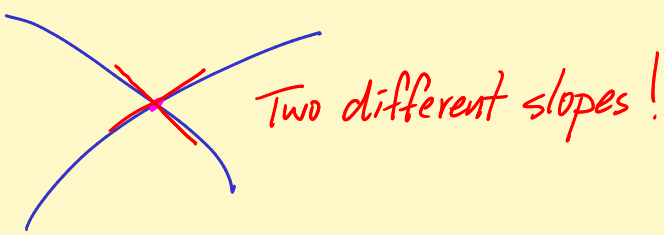
$2y - 3 = s$ horiz

$y = \frac{s}{2} + \frac{3}{2}$ lines

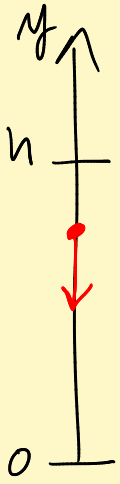
Direction field

Think: Soln's like "dropping a cork" in field.

Fact Solutions can't cross



EX Drop a rock



$F = -mg$ down

$F = ma$ ← 2nd order diff eqn

$$-mg = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -g$$

$$v = \int -g\,dt = -gt + C$$

"Drop" means $v(0) = 0$.

$$v(0) = -g \cdot 0 + C$$

$v(0) = 0$, so $C = 0$

$$\boxed{v = -gt}$$

Next: $v = \frac{dy}{dt}$: So $\frac{dy}{dt} = -gt$

$$y = \int -gt\,dt = -\frac{1}{2}gt^2 + \tilde{C}$$

Initial condition: $y(0) = h$

$y(0) = -\frac{1}{2}g \cdot 0^2 + \tilde{c}$ $\qquad$ $\boxed{\tilde{c} = h}$

$y(0) = h$

Soln

$$v = -gt$$
$$y = -\frac{1}{2}gt^2 + h$$

More generally: Soln $\quad y = -\frac{1}{2}gt^2 + v_0 t + y_0$

$v_0 < 0$: down .

$v_0 > 0$: up velocity.