Lecture 2 Examples (1.1, 1.2)

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MyLab HW01, HW02, HW03 due Monday, Aug 31, 11:59 pm

HW04 due Wed, Sept 2, 11:59 pm

Gradescope written HWK: HW02, HW03, HW04 due Wed, 9/2

Last time: Solved $\frac{dy}{dx} = x^2$ ← fcn of x

$$y = \frac{1}{3}x^3 + C$$

Pin down a unique sol^n via an initial condition like $y(0)=4$

Today $\frac{dy}{dx} =$ fcn of y ← Ouch! y is an unknown fcn of x!

EX $\frac{dy}{dx} = ky$

$k>0$: y = rabbits in Aus.

$k<0$: radioactive decay

Sol^n $\frac{1}{y}\frac{dy}{dx} = k$

Chain rule

$$\frac{d}{dx}(\sin 3x)^5 = 5(\sin 3x)^4(3\cos 3x)$$

$$\frac{d}{dx}(y^5) = 5y^4\frac{dy}{dx}$$

$$\frac{d}{dx}(\ln|y|) = \frac{1}{y}\frac{dy}{dx}$$

$$\frac{d}{dx}(\ln|y|) = k$$

Aha! $\ln|y| = \int k\,dx = kx + C$

$$e^{\ln|y|} = e^{kx+C}$$

$$|y| = e^{kx+C} = e^{kx}e^{C}$$

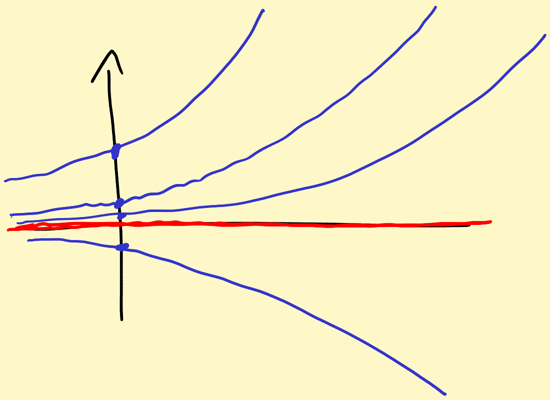
$$y = \underbrace{\left(\pm e^{c}\right)}_{\text{a constant = anything but zero.}} e^{kx}$$

Call constant $\tilde{C}$:

$$\boxed{y = \tilde{C} e^{kx}}$$

Hmmm. If $\tilde{C} = 0$, $y \equiv 0$. ← This is a sol^n: $\frac{dy}{dx} = ky$

Method missed this sol^n. Life in ODE!



$y(0) = y_0$

$\underbrace{c e^{k \cdot 0}}$ $= y_0$

$\boxed{c = y_0}$ $\quad y = y_0 e^{kx}$

Linear ODE: $\frac{dy}{dx}$ and y occur in a linear way.

Sol^{n}s exist forever!

Next $\quad \frac{dy}{dx} = y^2$ ← non-linear

$$\underbrace{\frac{1}{y^2}\frac{dy}{dx}}_{\frac{d}{dx}\left(\frac{-1}{y}\right)} = 1$$

Chain rule

$$\frac{d}{dx}\left(y^{-N}\right) = -N y^{-N-1} \cdot \frac{dy}{dx}$$

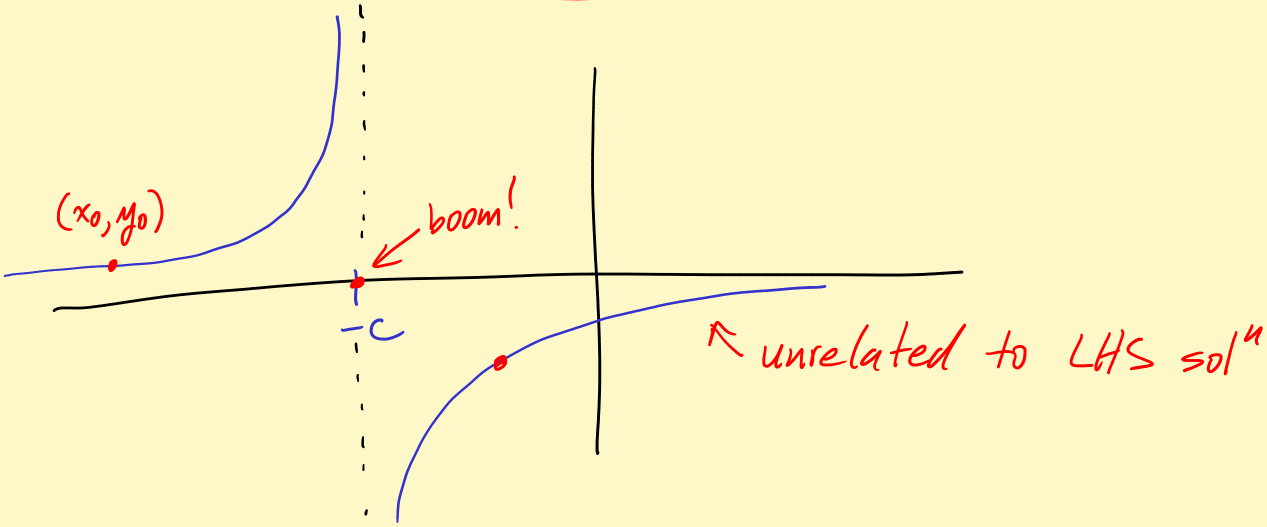
Want y^{-2} on RHS.

Need $N = 1$.

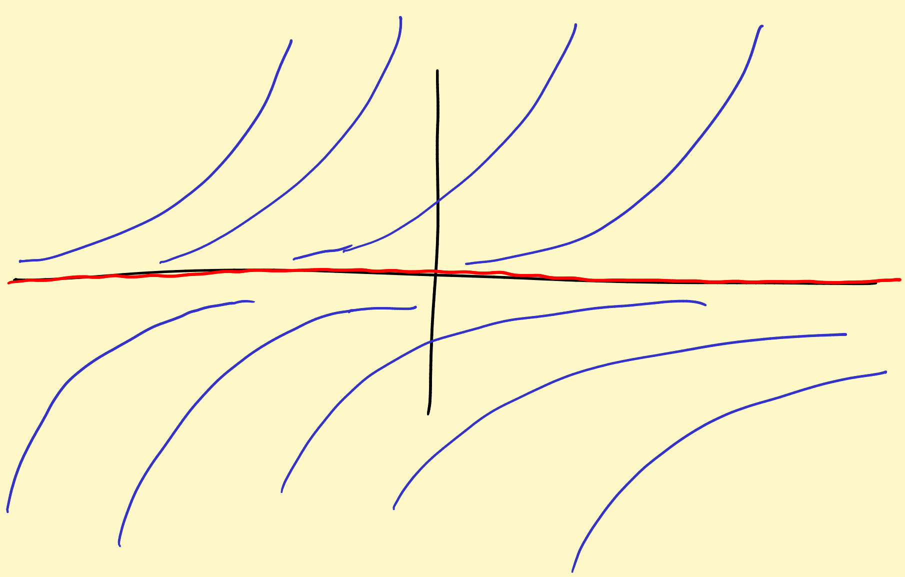
$$\frac{d}{dx}\left(-y^{-1}\right) = y^{-2}\frac{dy}{dx}$$

Aha! $-\frac{1}{y} = \int 1\,dx = x + C$

$$\boxed{y = \frac{-1}{x+c}}$$



Hmmm. $y \equiv 0$ is missing sol^n again!



Funny thing. $\lim_{c \to \infty} \frac{-1}{x+c} \equiv 0$ ← missing sol^{n}s

$\lim_{c \to -\infty} (\pm e^{c}) e^{kx} \equiv 0$

Ex: $\dfrac{dp}{dt} = 2p - 3$

Do chain rule backwards

$$\frac{1}{2p-3}\frac{dp}{dt} = 1$$

need antiderivative for $\frac{1}{2p-3}$: $\int \frac{1}{2p-3}\,dp$

$u = 2p-3$
$du = 2dp$

Get $p = \tilde{C}e^{2t} + \frac{3}{2}$

Change $\tilde{C}$ back to C.

$$p = Ce^{2t} + \frac{3}{2}$$

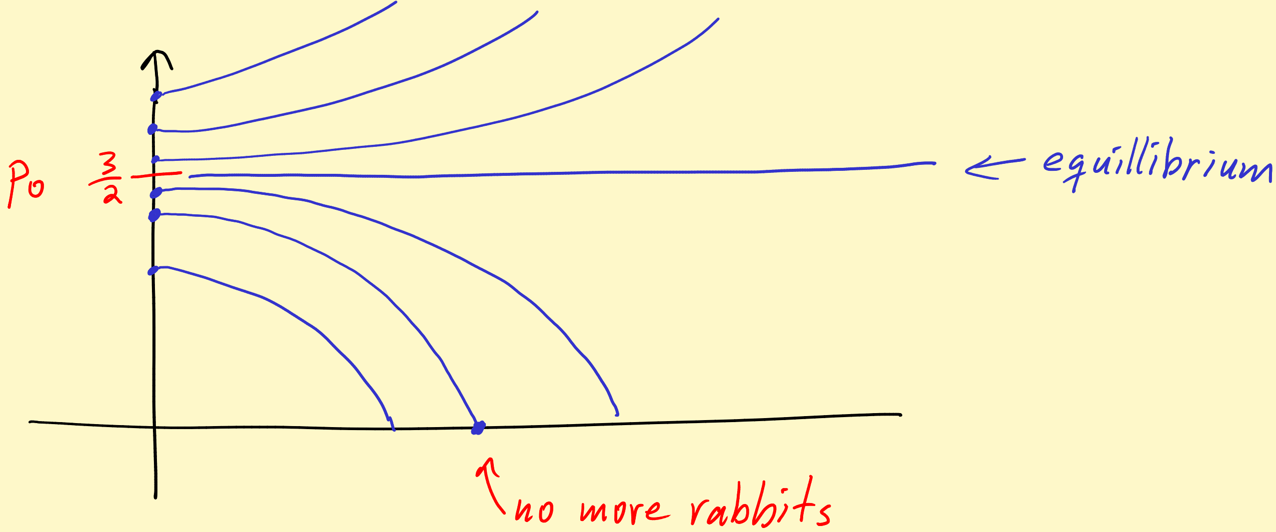
$P(0) = P_0$.

$$\underbrace{p(0)}_{P_0} = Ce^{2\cdot 0} + \frac{3}{2}$$

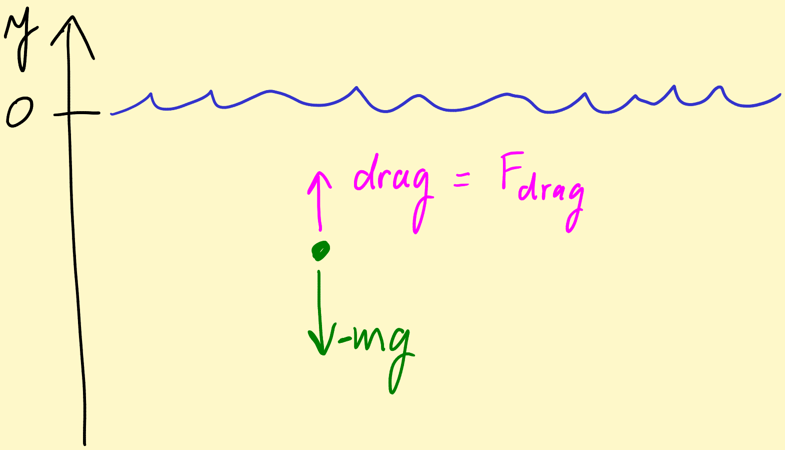
$$P_0 = C + \frac{3}{2}$$

$$\boxed{C = P_0 - \frac{3}{2}}$$

Sol^n $\boxed{p = \left(P_0 - \frac{3}{2}\right)e^{2t} + \frac{3}{2}}$



Drop a rock in ocean



Stokes' law

$$F_{drag} = -cv$$

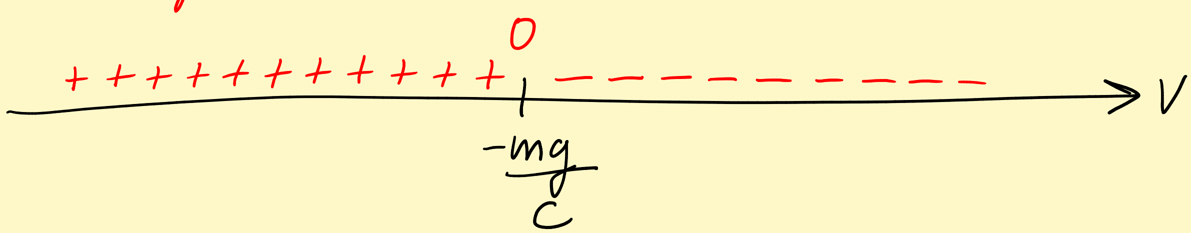
$$F = ma$$

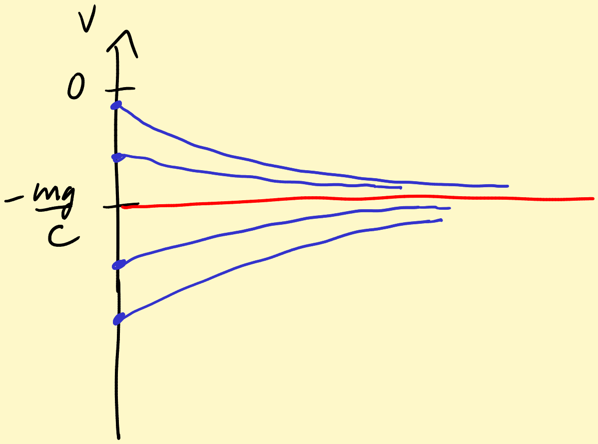
$$-mg - cv = m\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = -\frac{c}{m}v - g}$$

Same kind of problem!

Sign of $\frac{dv}{dt}$





v
0
$-\frac{mg}{c}$