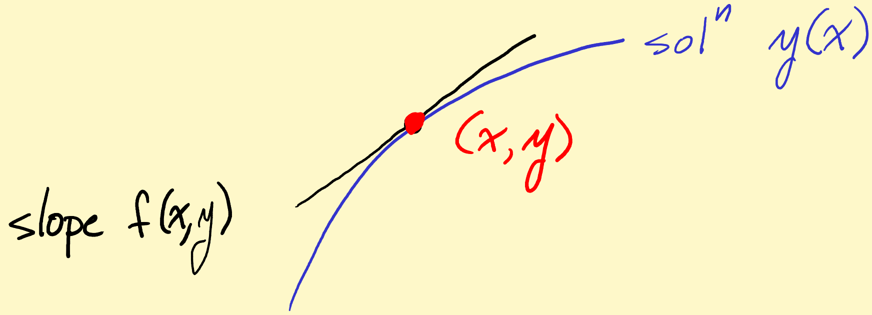
Lecture 3 Direction fields (1.3)

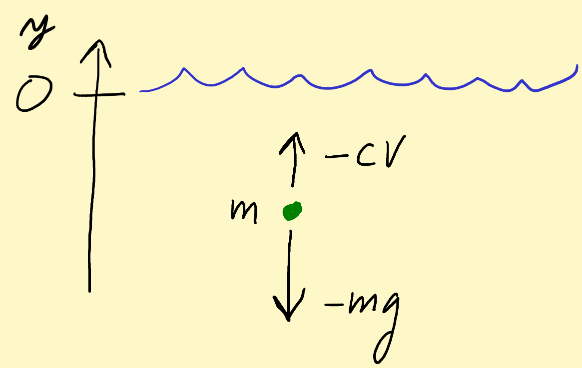
Monday: MyLab HW01, 02, 03

ODE: $\frac{dy}{dx} = f(x,y)$ IVP: $y(x_0) = y_0$



ODE: $y'(x) = f(x, y(x))$ IVP: sol^n $y(x)$ goes through pt (x_0, y_0)

Drop rock in ocean



$F = ma$

$-mg - cv = m\frac{dv}{dt}$

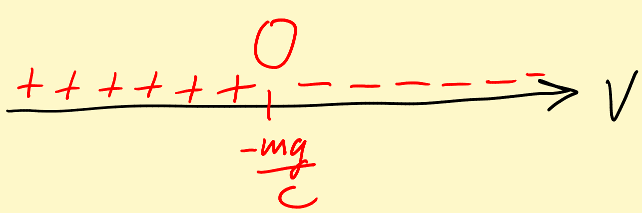
$$\boxed{\frac{dv}{dt} = -\frac{c}{m}v - g}$$

first order linear ODE

Drop from rest at surface: $v(0) = 0$

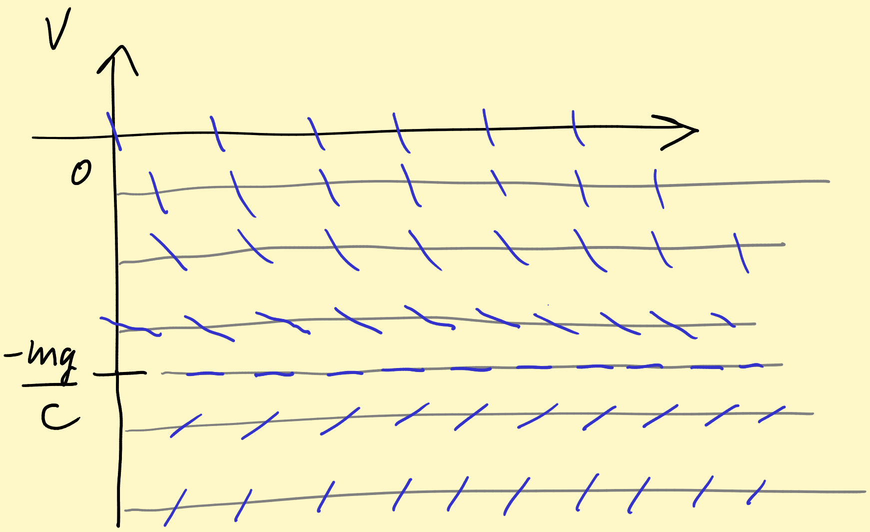
$$\frac{dv}{dt} = \underline{-\frac{c}{m}\left(v + \frac{mg}{c}\right)} = s, \text{ a slope}$$

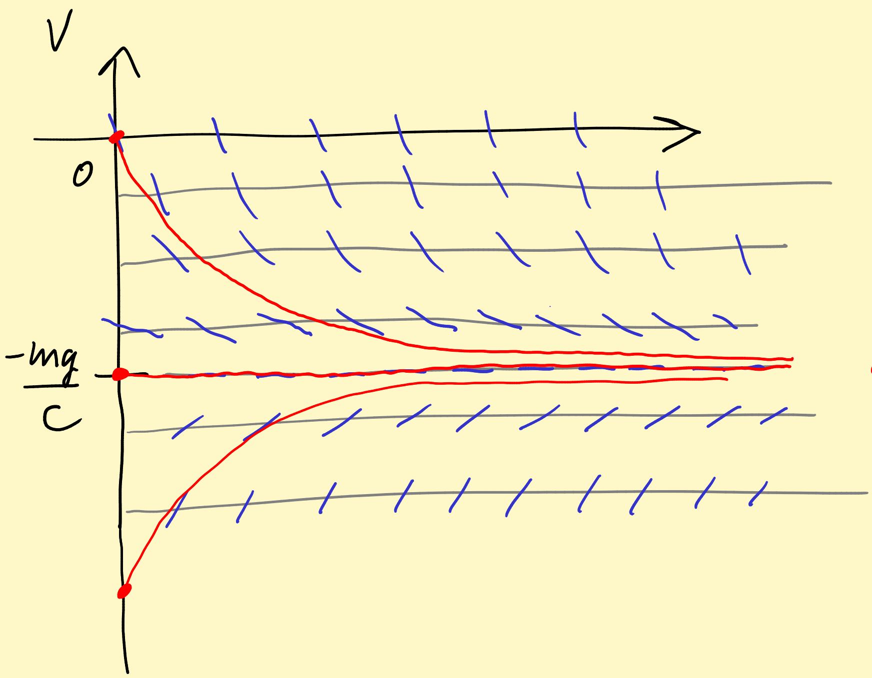
Sign of $\frac{dv}{dt}$



Curves of constant slope are "isoclines"

$$-\frac{c}{m}\left(v+\frac{mg}{c}\right) = s \quad \leftarrow \text{horizontal lines}$$

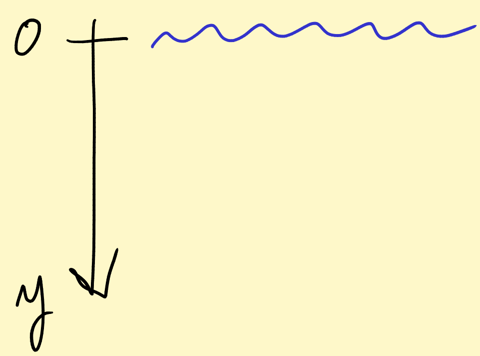




equillibrium: Terminal velocity!

(solutions can't cross!)

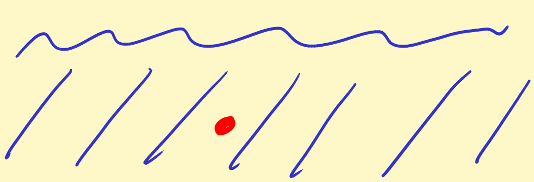
Book



$$\frac{dv}{dt} = mg - cv$$

Get same conclusions

Hot rock in the ocean



Newton's Law of Cooling

T = temp of rock at time t

A = ambient temp of ocean

$T(0) = T_0$ given.

$$\frac{dT}{dt} = -k(T - A)$$

Hot rock cools, so $\frac{dT}{dt} < 0$. const < 0.

HW: Analyze via dir. field

Today: Solve it! $\qquad \frac{dT}{dt} = -k(T-A)$

$$\underbrace{\frac{1}{T-A}}\ \frac{dT}{dt} = -k$$

need antiderivative of this wrt T

Need $\int \frac{1}{T-A}\,dT = \ln|T-A|$

LHS: $\quad \frac{d}{dt}\left[\ln|T-A|\right] = \frac{1}{T-A}\cdot\frac{dT}{dt} = -k$

$$e^{\ln|T-A|} = e^{-kt + C}$$

$$|T-A| = e^{-kt+C} = e^{-kt}e^{C}$$

$$T - A = \underbrace{\left(\pm e^{C}\right)}e^{-kt}$$

call this const little c

$$\boxed{T = A + ce^{-kt}}$$

IVP: $T(0) = T_0$

$$T_0 = A + c\underbrace{e^{-k\cdot 0}}_{1}$$

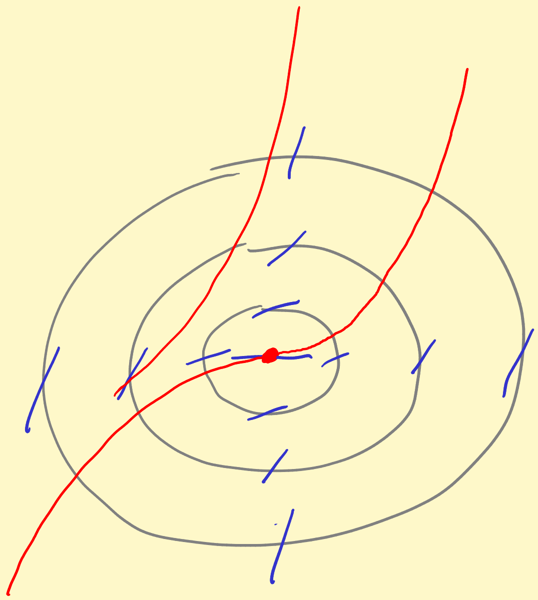
$$\boxed{c = T_0 - A}$$

Sol^n: $T = A + (T_0 - A)e^{-kt}$

D. Fields

EX $\dfrac{dy}{dx} = x^2 + y^2$

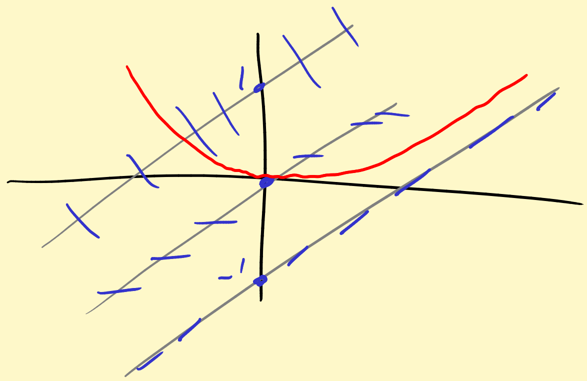
Isoclines: $\underbrace{x^2 + y^2}_{\geq 0} = s$ Circles radius $\sqrt{s}$ centered at $(0,0)$



EX $\qquad \dfrac{dy}{dx} = x - y$

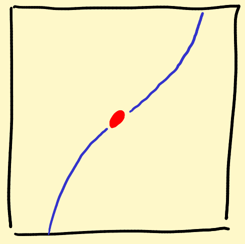
Isoclines: $\qquad x - y = s$

$$y = x - s$$ ← lines. Slope 1
y intercept $-s$.



Existence & Uniqueness Thm

$$\frac{dy}{dx} = f(x,y)$$

 ← $f(x,y)$ nice:

f continuous, $\dfrac{\partial f}{\partial y}$ continuous too

Sol^n through dot exists and is unique, at least until it leaves the box.