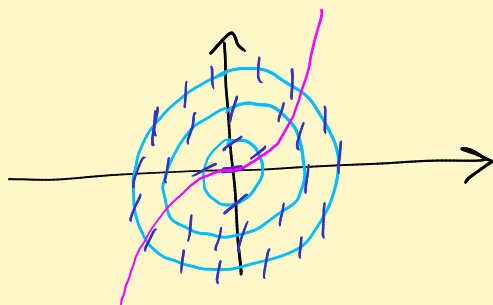


Lecture 10 Autonomous 1st order ODE (2.2)

MyLab HW 09

GS HW 08, 09

EX $\frac{dy}{dt} = t^2 + y^2 \leftarrow t \text{ on RHS. } \underline{\text{Not}} \text{ autonomous}$



Isoclines: $t^2 + y^2 = s$
Circles
↑ slope

EX $\frac{dy}{dx} = f(y) \leftarrow \text{no } t \text{ on RHS. Autonomous } \checkmark$

Isoclines: $f(y) = s \leftarrow \text{horizontal lines}$
 $y = f^{-1}(s)$

Defⁿ $\frac{dy}{dt} = f(y) \leftarrow \text{zeroes of } f \text{ important.}$

Let z_0 be a zero of f : $f(z_0) = 0$

Claim: $y \equiv z_0$ is a solⁿ.

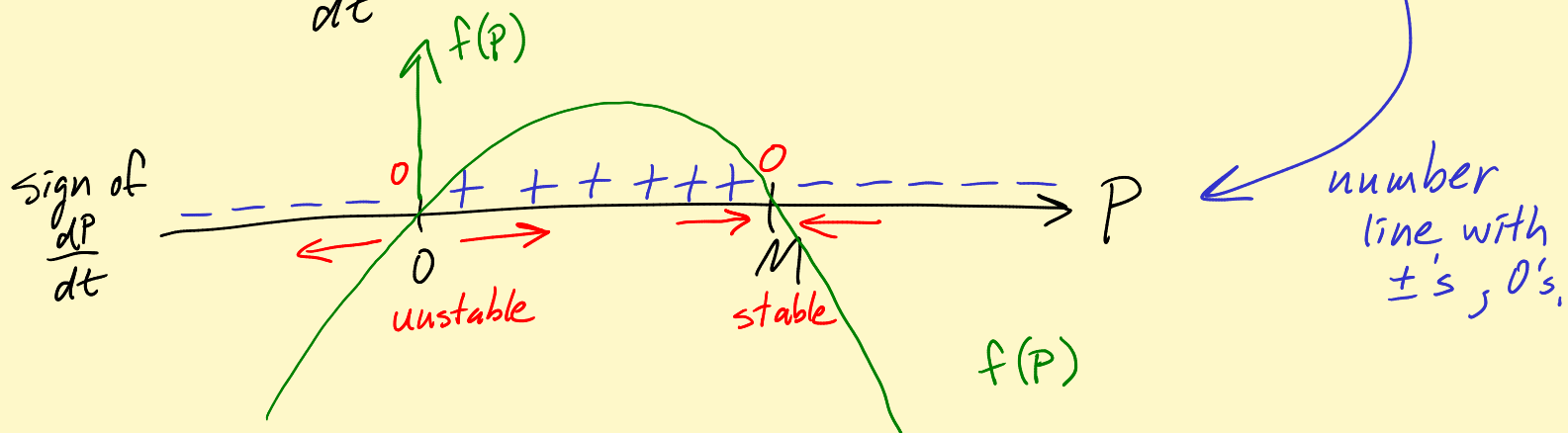
$$\underbrace{\frac{d}{dt} [z_0]}_0 \stackrel{?}{=} \underbrace{f(z_0)}_0 \quad \checkmark$$

Sometimes z_0 is called a "critical value".

$y \equiv z_0$ is called an "equilibrium solution"

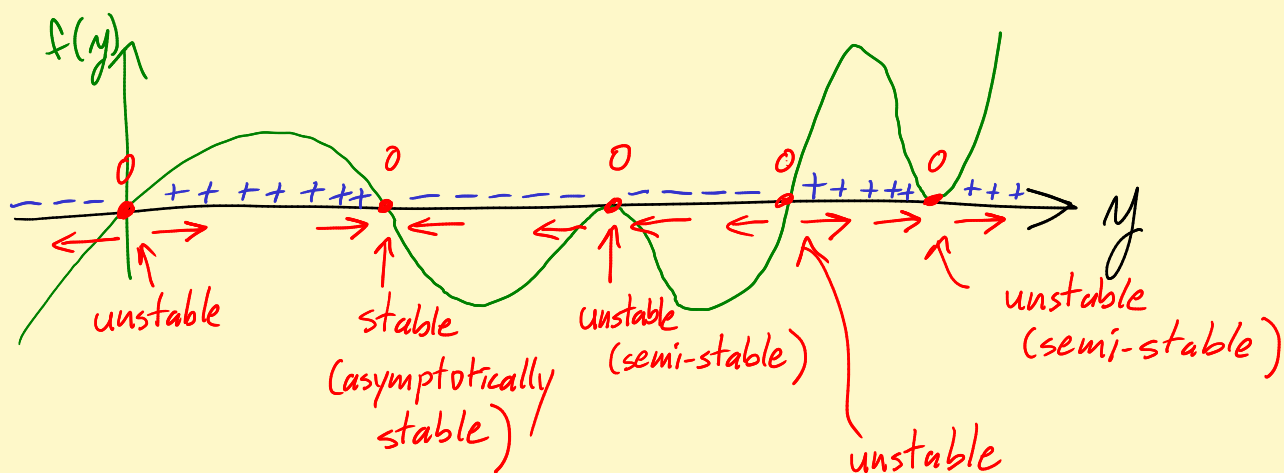
Important diagram: "Phase portrait"

EX $\frac{dP}{dt} = k(M-P)P \leftarrow f(P)$

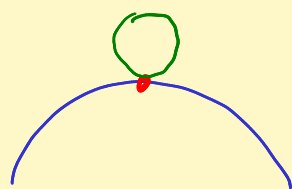


General case

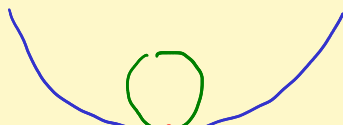
$$\frac{dy}{dt} = f(y)$$



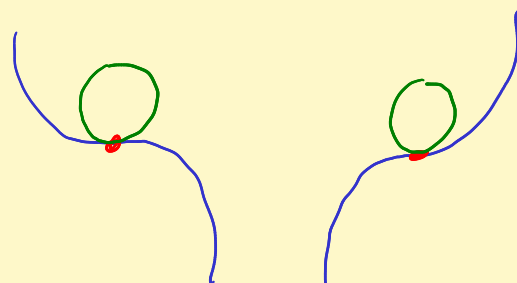
Feeling Sticky ping pong ball on a "bump"



unstable



stable
(asymptotically stable)



unstable
(semi-stable)

Something that is stable, but not asympt. stable =

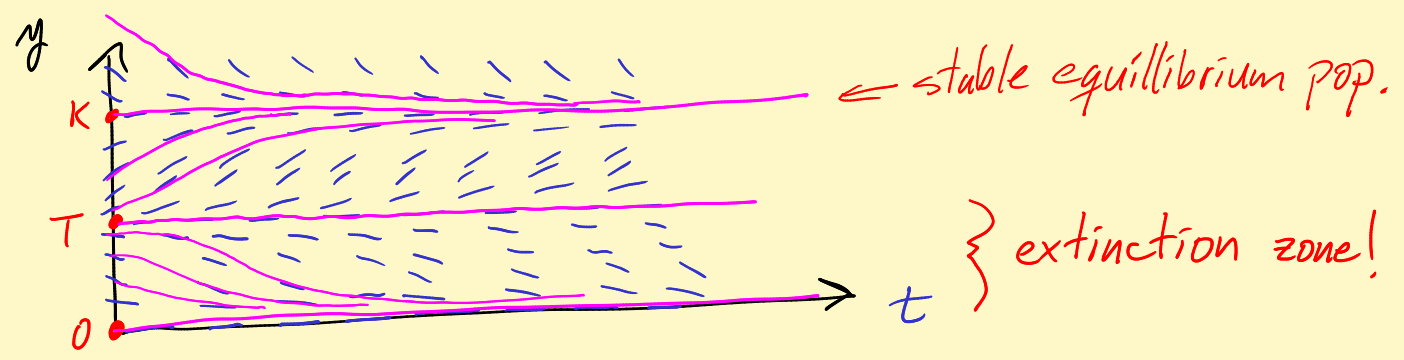
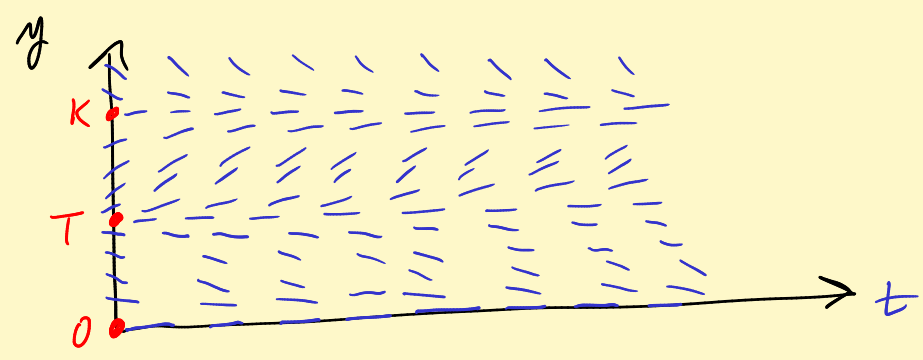
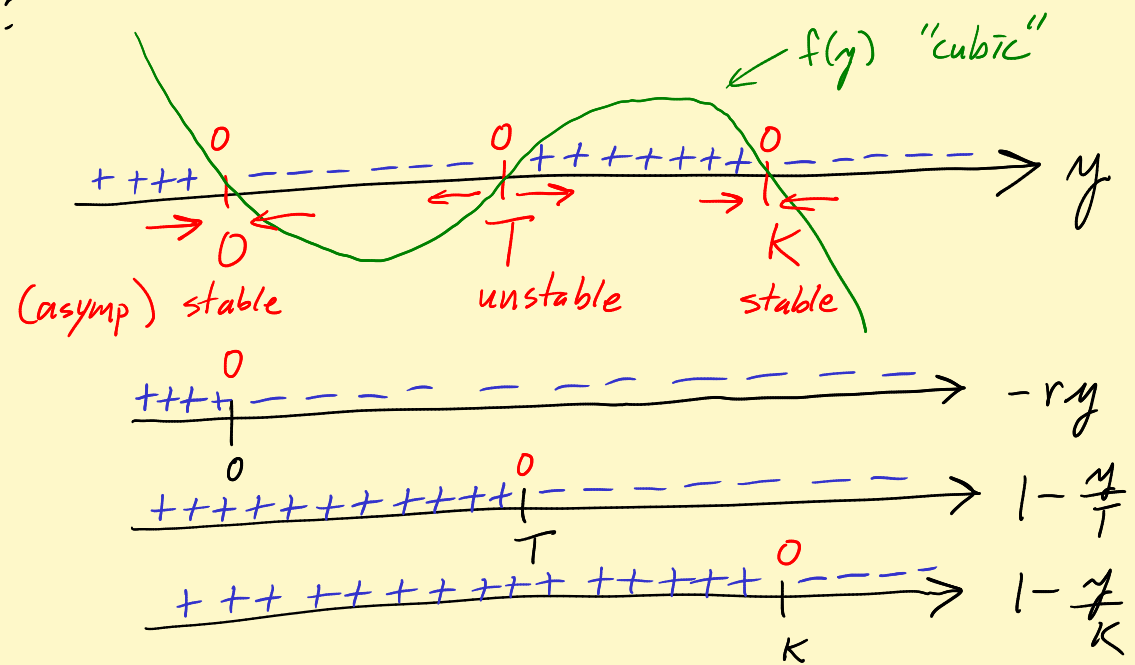
The solar system! Add a rock to the moon.
The system shifts, but it stays close to what it
it was.

EX Carrier pigeons

$$\frac{dy}{dt} = -ry \left(1 - \frac{y}{T}\right) \left(1 - \frac{y}{K}\right) = f(y)$$

Phase diagram:

Sign of $\frac{dy}{dt}$



Finding inflection pts.

$$\frac{dy}{dt} = f(y)$$

"Concavity changes"

Sign of $\frac{d^2y}{dt^2}$ > 0 Concave up
 < 0 Concave down

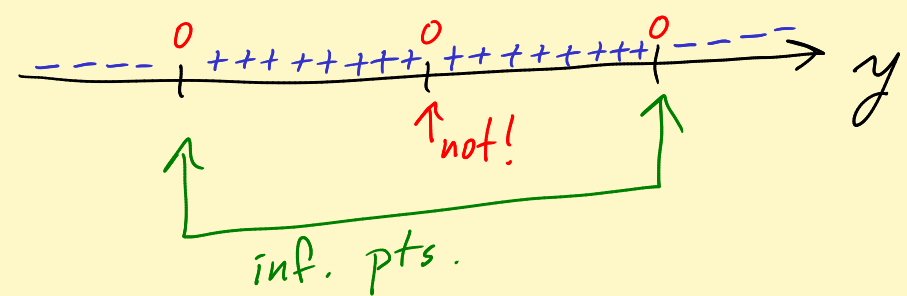
Hmmm. $\frac{d^2y}{dt^2} = \frac{d}{dt} f(y) = \underset{\substack{\uparrow \\ \text{chain} \\ \text{rule!}}}{f'(y)} \frac{dy}{dt} \underset{\substack{\uparrow \\ \text{Aha!} \\ \frac{dy}{dt} = f(y)}}{f(y)}$

So $\boxed{\frac{d^2y}{dt^2} = f'(y) f(y)}$

Potential inflection pts = zeroes of $f'(y)$ and $f(y)$.

EX

sign of $\frac{d^2y}{dt^2}$

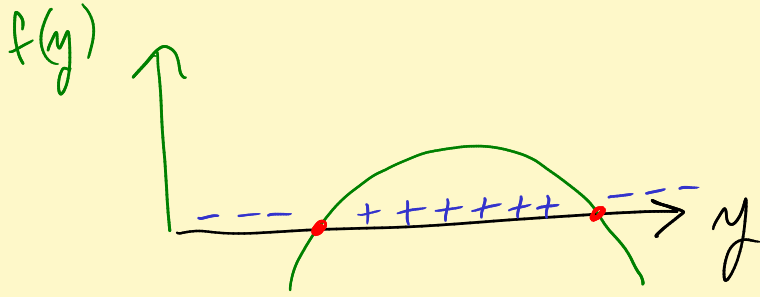


Bifurcation pts

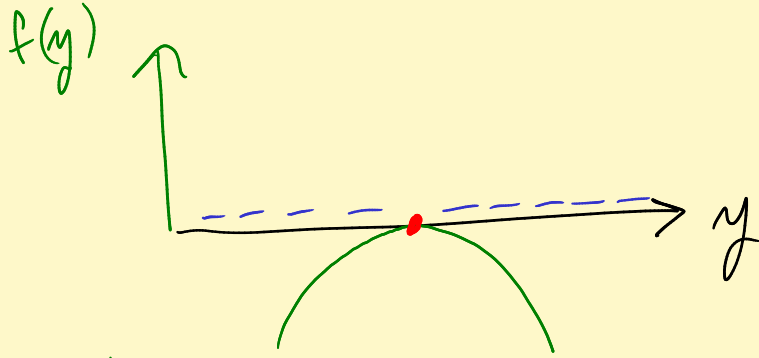
EX

$$\frac{dy}{dt} = \underbrace{y(4-y)}_{f(y) \text{ is quadratic}} - h \leftarrow h = \text{parameter}$$

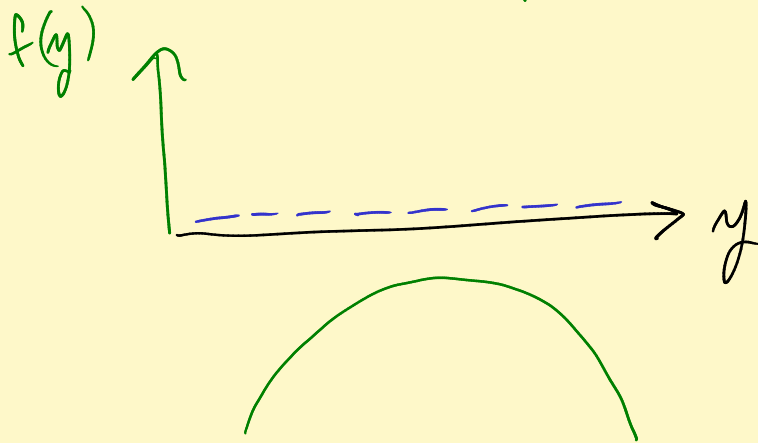
upside down parabola



Two crt. pts.
(two roots)



only one ctr. pt.
Delicate! $h=h_0$
is called the "bifurcation
pt."



no crt pts.
(complex roots)

See p. 92.