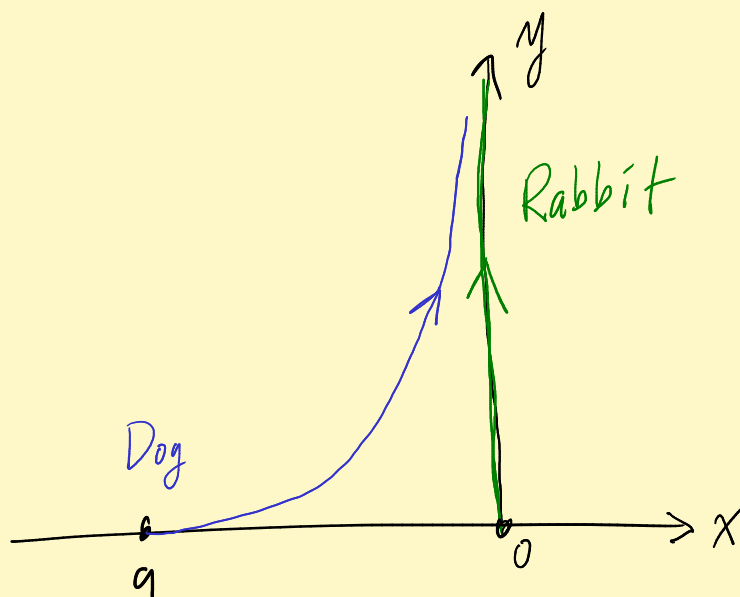


Dog/Rabbit problem

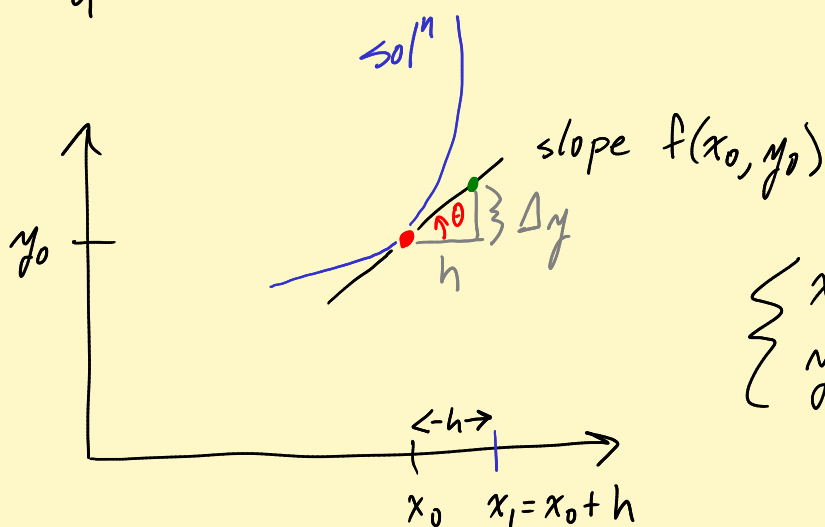
$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{a}{x} - \frac{x}{a} \right)$$

$$y = \frac{a}{2} \ln|x| - \frac{1}{4a} x^2 + K$$

Get K from $y=0$ when $x=a$.



Euler's method



$$\begin{cases} x_1 = x_0 + h \\ y_1 = y_0 + h f(x_0, y_0) \end{cases}$$

Δy

$$\text{slope} = \frac{\Delta y}{h} = f(x_0, y_0) \quad \text{so} \quad \Delta y = h f(x_0, y_0)$$

$\parallel$
Tan θ

Euler method

2

 x_0

$$x_1 = x_0 + h$$

$$x_2 = x_1 + h$$

 $\vdots$

$$x_{n+1} = x_n + h$$

 y_0

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y_2 = y_1 + h f(x_1, y_1)$$

 $\vdots$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

EX $\frac{dy}{dx} = \underbrace{1 - x + 4y}_{f(x, y)}$

$$y(0) = 1$$

$\uparrow$ $x_0 = 0$ $\uparrow$ $y_0 = 1$

Step size : $h = 0.1$

$$x_1 = .1$$

$$\begin{aligned} y_1 &= y_0 + h f(x_0, y_0) \\ &= 1 + (.1) [1 - 0 + 4(1)] \\ &= 1.5 \end{aligned}$$

$$(x_0, y_0) = (0, 1)$$

$$(x_1, y_1) = (.1, 1.5)$$

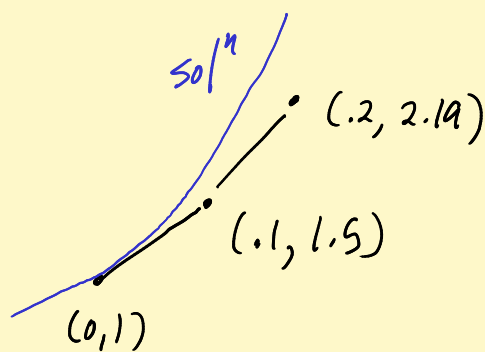
$$x_2 = .2$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$= (1.5) + (.1) \left[1 - \underbrace{(.1)}_{x_1} + 4 \underbrace{(1.5)}_{y_1} \right]$$

$$= 2.19$$

$$(x_2, y_2) = (.2, 2.19)$$



Use Excell,

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1} $y_n + h f(x_n, y_n)$
0	0	1	$1 - 0 + 4 \cdot 1 = 5$	1.5
1	0.1	1.5		2.19
2	0.2	2.19		

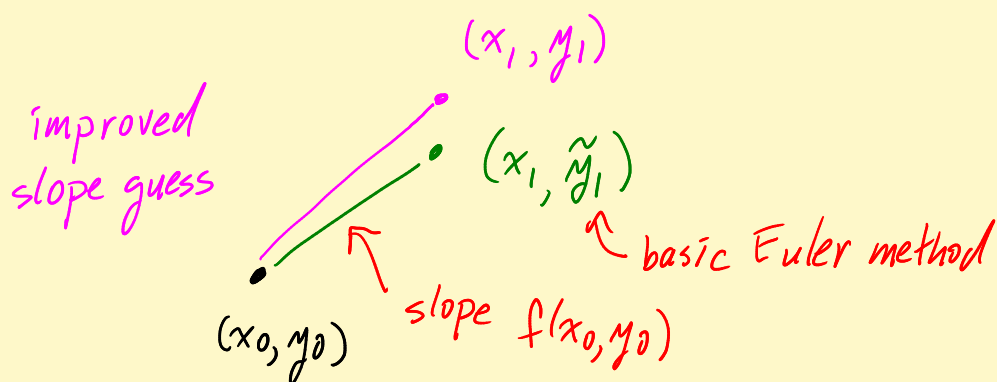
Fun fact Easy problem: $\frac{dy}{dx} = f(x)$

Euler method: $y_N = y_0 + \sum_{k=0}^{N-1} f(x_k) h$

Riemann sum!
Rectangle
method

actual $\rightarrow y = y_0 + \int_{x_0}^x f(t) dt$
solⁿ

Improved Euler method



Improved slope guess: Ave of $f(x_0, y_0)$ and $f(x_1, \tilde{y}_1)$

$$y_1 = y_0 + h \cdot \left[\frac{f(x_0, y_0) + f(x_1, \tilde{y}_1)}{2} \right]$$

where $\tilde{y}_1 = y_0 + h f(x_0, y_0)$

Method

$$x_n = x_0 + nh$$

Step 1

Slope 1
 $f(x_n, y_n)$

Slope 2

$$f(x_{n+1}, \tilde{y}_{n+1})$$

where $\tilde{y}_{n+1} = y_n + h f(x_n, y_n)$

Improved slope : $\frac{1}{2} [f(x_n, y_n) + f(x_{n+1}, \tilde{y}_{n+1})]$

Next pt : $(x_n, y_n + h \cdot \left[\frac{f(x_n, y_n) + f(x_{n+1}, \tilde{y}_{n+1})}{2} \right])$

$$x_{n+1} = x_n + h$$

$$\tilde{y}_{n+1} = y_n + h f(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2} \cdot [f(x_n, y_n) + f(x_{n+1}, \tilde{y}_{n+1})]$$

Fun fact $\frac{dy}{dx} = f(x)$

Improved Euler : Trapezoid rule for $\int$

Runge-Kutta $\sim$ Simpson's rule