Lesson 23 cont'd

EX: $y^{(5)} - 3y^{(4)} + 3y''' - 3y'' + 2y' = t$

Step 1: Solve homog: $r^5 - 3r^4 + 3r^3 - 3r^2 + 2r = 0$

Fact: If $P(r_0) = 0$, then $r - r_0$ divides $P(r)$.

$$r\underbrace{(r^4 - 3r^3 + 3r^2 - 3r + 2)}_{Q(r)} = 0$$

$Q(r)$, $Q(1) = 0$

$$
\begin{array}{r|l}
 & r^3 - 2r^2 + r - 2 \leftarrow H(r) \\
\hline
r-1 & r^4 - 3r^3 + 3r^2 - 3r + 2 \\
 & r^4 - r^3 \\
\hline
 & -2r^3 + 3r^2 - 3r + 2 \\
 & -2r^3 + 2r^2 \\
\hline
 & r^2 - 3r + 2 \\
 & r^2 - r \\
\hline
 & -2r + 2 \\
 & -2r + 2 \\
\hline
 & 0
\end{array}
$$

$H(2) = 0$

Divide by $r - 2$.

Get $P(r) = r(r-1)(r-2)(r^2+1)$

Roots $r = 0, 1, 2, \pm i$

Homog sol^n: $y = c_1 + c_2 e^t + c_3 e^{2t} + c_4 \cos t + c_5 \sin t$

Step 2: Get particular sol^n to $L[y] = t$.

Try $y_p = At + B$

Ouch! $y \equiv B$ solves homog.

Correct try: $y_p = t(At + B) = At^2 + Bt$

Fact: Hit or miss method to find roots to polynomials with rational coeff.

Step 1: Clear out all the fractions:

$\frac{2}{3}x^2 + \frac{1}{5}x + 1$ ← multiply by 15

$10x^2 + 3x + 15$

Integer coefficient poly: $a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0$

where a_n's are integers.

We want to find rational roots, if any. $r = \pm\frac{p}{q}$ ← no common factors

Plug in $r = \frac{p}{q}$ and clear fractions by multiplying by q^n:

$$\left(a_n p^n + \left(a_{n-1}p^{n-1}q + \cdots + a_1 p q^{n-1}\right) + a_0 q^n\right) = 0$$

↑ if $\frac{p}{q}$ root

any factor of q divides this

conclude that factors of q divides a_n

Green ( )'s show: any factor of p must divide a_0.

Strategy: Find all factors of a_n: $\pm m_1, m_2, \ldots, m_k$

all factors of a_0: $\pm l_1, l_2, \ldots, l_q$

Check $P\left(\frac{p}{q}\right) = 0$ for $\frac{p}{q} = \pm\frac{l_j}{m_i}$.