MA 266 Exam 1 $\text{sol}^{\text{n}}\text{s}$ (Bell)

1. Standard Form linear: $y' + \overset{P(x)=2}{2}y = 2e^{4x}$

$u = e^{\int P dx} = e^{2x}$

$\underbrace{e^{2x}(y' + 2y)}_{[e^{2x}y]'} = e^{2x} \cdot 2e^{4x}$

$e^{2x}y = \int 2e^{6x}\,dx = \frac{2}{6}e^{6x} + C$

$y(0) = 1$: $\quad e^{2\cdot 0}\cdot 1 = \frac{1}{3}e^{6\cdot 0} + C$ $\quad \boxed{C = 1 - \frac{1}{3} = \frac{2}{3}}$

$y = \frac{1}{3}e^{4x} + \frac{2}{3}e^{-2x}$

2. $\frac{\partial M}{\partial y} = 6x^2 y = \frac{\partial N}{\partial x}$ ✓ Exact $\quad$ Want φ with

A) $\frac{\partial \varphi}{\partial x} = 3x^2y^2 + 1 \implies \varphi = \int 3x^2y^2 + 1\,dx = x^3y^2 + x + C(y)$

B) $\frac{\partial \varphi}{\partial y} = 2x^3y + 2y \implies \frac{\partial}{\partial y}\left[x^3y^2 + x + C(y)\right] \underset{\text{want}}{=} 2x^3y + 2y$

$2x^3y + 0 + C'(y) = 2x^3y + 2y$

$\boxed{C'(y) = 2y}$

So $C(y) = y^2$ and $\varphi = x^3y^2 + x + y^2$.

Sol^{n} to ODE: $x^3y^2 + x + y^2 = c$

$y^2(x^3+1) = c - x$

$y = \pm\sqrt{\frac{c-x}{x^3+1}}$

3. Homogeneous. Let $v = \frac{y}{x}$. Then $y = xv$

$$\frac{dy}{dx} = 1 \cdot v + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{y}{x}\right)} + \frac{y}{x}$$

$$\overbrace{v + x\frac{dv}{dx}} = \frac{1}{v} + v$$

$$x\frac{dv}{dx} = \frac{1}{v} \leftarrow \text{Separable}$$

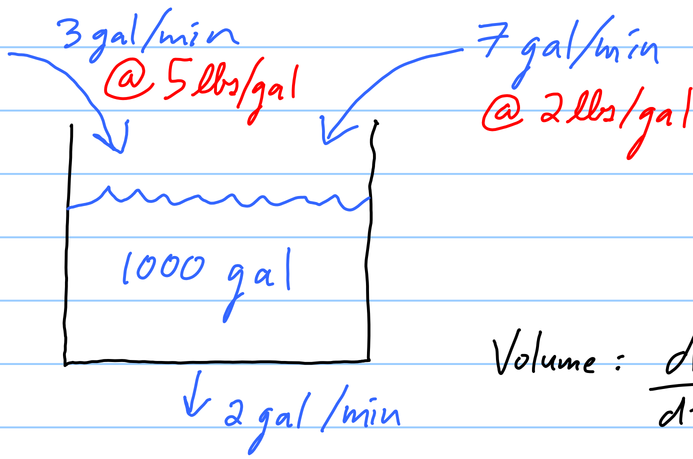
$$\int v\,dv = \int \frac{dx}{x}$$

$$\frac{1}{2}v^2 = \ln|x| + C$$

$$\underbrace{v}_{\frac{y}{x}} = \pm\sqrt{\underbrace{2\ln|x|}_{\ln|x|^2 = \ln x^2} + \underbrace{2C}_{K}}$$

$$y = \underline{\pm x\sqrt{\ln x^2 + K}}$$

4.



$$\underline{\underline{Q(0) = 0}}$$

Volume: $\frac{dV}{dt} = \underbrace{(\text{Rate in})}_{3+7} - \underbrace{(\text{Rate out})}_{2} = 8$

$V(t) = 8t + C$, $V(0) = 1000$

So $\underline{V(t) = 1000 + 8t}$

$$\frac{dQ}{dt} = (\text{Rate in}) - (\text{Rate out})$$

$$= \left[3 \cdot 5 + 7 \cdot 2\right] - 2 \cdot \frac{Q(t)}{V(t)}$$

$$\boxed{\frac{dQ}{dt} = 29 - \frac{2}{1000+8t}\,Q \qquad Q(0)=0}$$

5. $\dfrac{dy}{dx} = f(x,y)$, Sol^n y with $y(2)=3$.

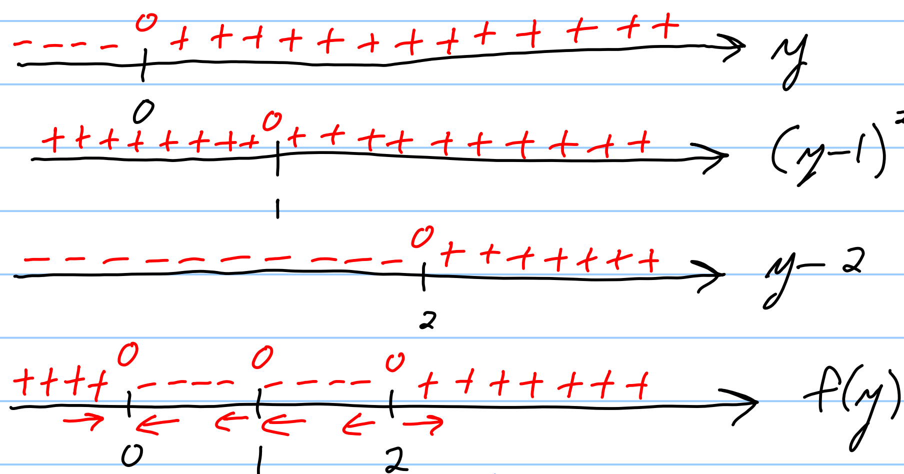
$y'(2) = f(2,3) = -4 < 0$. So y is decreasing near $x=2$.

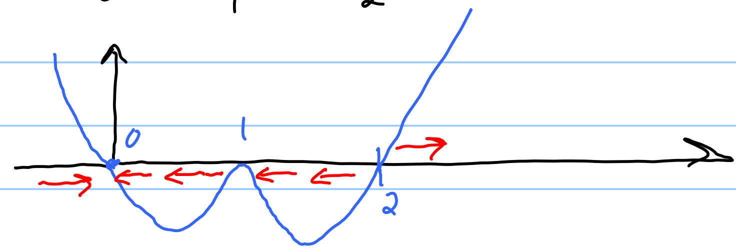
$$\frac{d^2y}{dx^2} = \frac{d}{dx} f(x,y) = \frac{\partial f}{\partial x}\underbrace{\frac{dx}{dx}}_{1} + \frac{\partial f}{\partial y}\underbrace{\frac{dy}{dx}}_{f(x,y)} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\cdot f(x,y)$$

So $\dfrac{d^2y}{dx^2}(2) = \dfrac{\partial f}{\partial x}(2,3) + \dfrac{\partial f}{\partial y}(2,3)\cdot f(2,3) = 5 + (-6)(-4) = 29 > 0$.

So graph of y is concave up near $x=2$.

6. $f(y) = y\,(y-1)^2\,(y-2)$





E) 0 - Stable, 1 - semi-stable, 2 - unstable