

MATH 266

Exam 1

1. (20 pts) Solve the initial value problem $\frac{1}{2}y' + y = e^{4x}$ with $y(0) = 1$.
2. (20 pts) Find the general solution to the exact equation

$$(3x^2y^2 + 1) + (2x^3y + 2y)\frac{dy}{dx} = 0.$$

Write your answer in the form $y = (\text{a function of } x)$.

3. (20 pts) Solve the equation

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$$

by making the substitution $v = y/x$. Write your answer in the form $y = (\text{a function of } x)$.

4. (20 pts) A large leaky fish farm tank initially containing 1,000 gallons of pure water is filled by two pumps. Pump #1 pumps in a solution containing 5 pounds of salt per gal at a rate of 3 gal/min, while Pump #2 pumps in a solution containing 2 pounds of salt per gal at a rate of 7 gal/min. The well stirred mixture leaks out a hole in the bottom at 2 gal/min. Let $Q(t)$ denote the number of pounds of salt in the tank at time t (in minutes). **SET UP BUT DO NOT SOLVE** an initial value problem for $Q(t)$.
5. (10 pts) A twice continuously differentiable function $f(x, y)$ is such that $f(2, 3) = -4$, $\frac{\partial f}{\partial x}(2, 3) = 5$, and $\frac{\partial f}{\partial y}(2, 3) = -6$. Is the solution to $\frac{dy}{dx} = f(x, y)$ that passes through $(2, 3)$ increasing or decreasing near $x = 2$? Is the graph of the solution concave up or concave down there? Explain.
6. (10 pts) Determine the stability of the equilibrium solutions to the autonomous equation $y' = y(y - 1)^2(y - 2)$.
- (A) 0 -unstable, 1 -stable, 2 -semistable,
 - (B) 0 -semistable, 1 -stable, 2 -unstable,
 - (C) 0 -unstable, 1 -semistable, 2 -stable,
 - (D) 0 -semistable, 1 -unstable, 2 -stable.
 - (E) 0 -stable, 1 -semistable, 2 -unstable.