

Problem 1.

Find the general solution of the ODE

$$y' + 4y = e^{-2t}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = \frac{1}{2}e^{-2t} + Ce^{-4t}.$$

Problem 2.

Find the general solution of the ODE

$$ty' - 2y = -2t^3e^{2t}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = -t^2e^{2t} + Ct^2.$$

Problem 3.

Find the general solution of

$$\frac{dy}{dx} = \frac{xy + 2x + y + 2}{(y + 2)^2},$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = -2 \pm \sqrt{x^2 + 2x + C}.$$

Problem 4.

Find the general solution of

$$\frac{dy}{dx} = \frac{3y - 2x}{y}.$$

Justify all your answers. Box your answers.

ANSWER:

$$y(t) = \frac{1}{2} \left(4x + C \pm \sqrt{4Cx + C^2} \right).$$

Problem 5.

A tank contains 200 *gal* (gallons) of liquid. Initially, the tank contains pure water. At time $t = 0$, brine containing 3 *lb/gal* salt begins to pour into the tank at a rate of 2 *gal/min*, and the well-stirred mixture is allowed to drain away at the same rate. How many minutes must elapse before there are 100 *lb* of salt in the tank?

- A. 600 B. $600 - 1/e$ C. $600 - 600/e$ D. $100 \ln(6/5)$ E. $-100 \ln(400)$

ANSWER: D

Problem 6.

What is the largest open interval in which a solution to the initial value problem

$$(t - 1)y' + \sqrt{t + 2}y = \frac{3}{t - 3}, \quad y(2) = -5$$

is guaranteed to exist?

- A. $(1, 3)$ B. $(2, 3)$ C. $(-2, 3)$ D. $(-2, 1)$ E. $(-2, \infty)$

ANSWER: A

Problem 7. Find all the asymptotically stable equilibrium solutions for the autonomous differential equation

$$y' = (y^2 - 1)(4 - y^2).$$

ANSWER: $y = -1$ and $y = 2$.

Problem 8. A tank with capacity 500 *gal* originally contains 100 gallons of water with a salt concentration of $1/2$ *lb/gal*. A solution containing a salt concentration of 2 *lb/gal* enters at a rate of 2 *gal/min* and the well-stirred mixture is pumped out at the rate of 1 *gal/min*. What is the amount of salt in the tank after 50 *min*.

ANSWER: $Q(50) = 200(\text{lb})$.

Problem 9. A skydiver weighing 200 *lb* (with mass $25/4$ *lb*) falls vertically downward from an altitude of 4000 *ft* and opens the parachute after 10 seconds of free fall. Assume that the force of air resistance, which is directed opposite to the velocity, is $0.8|v|$ when the parachute is closed and $12|v|$ when the parachute is open, where the velocity v is measured in *ft/sec*. Use $g = 32\text{ft/sec}^2$.

(a) Find the speed of the skydiver when the parachute opens.

(b) Find the distance fallen before the parachute opens.

ANSWER: (1) $v(10) = 250(1 - e^{-1.28})$ (2) ≈ 1089.9 .

Problem 10.

Find the implicit solution to the initial value problem

$$(e^x \sin(y) - 2y \sin(x) - 1) + (e^x \cos(y) + 2 \cos(x) + 3)y' = 0, \quad y(0) = \pi.$$

ANSWER:

$$e^x \sin(y) + 2y \cos(x) - x + 3y = C.$$

Problem 11.

What is the general solution to this differential equation?

$$\frac{dy}{dx} = (x - y)^2 + 1.$$

(Hint: use the substitution $v(x) = x - y(x)$.)

ANSWER:

$$y(x) = x - \frac{1}{x - C}.$$