MA 266 Practice problems for Exam 2 answers

On which interval is the initial value problem

$$\begin{cases} (5-t)y'' + (t-4)y' + 2y = \ln t \\ y(2) = 8 \end{cases}$$

guaranteed to have a unique solution?

$(0,5)$.

Find the general solution of the differential equation

$$y'' - 10y' + 27y = 0$$

$$y = c_1 e^{5t} \cos\sqrt{2}\,t + c_2 e^{5t} \sin\sqrt{2}\,t$$

The function $y_1 = e^x$ is a solution of

$$(x-1)y'' - 2xy' + (x+1)y = 0.$$

If we seek a second solution $y_2 = e^x v(x)$ by reduction of order, then $v(x) =$

$v = \frac{1}{3}(x-1)^3$ and

$y_2 = v y_1 = \frac{1}{3}(x-1)^3 e^x$

By the Method of Undetermined Coefficients, which of the following Y is the correct form of a particular solution to the equation

$$y^{(4)} - 3y^{(3)} + 2y^{(2)} = 4t - e^t + 3e^{3t}?$$

$$y_p = t^2[A_1 t + A_0] + t[Be^t] + [Ce^{3t}]$$

Which of the following is the correct form of a particular solution to the equation

$$y^{(4)} - y = 2 + 3te^{-t} + 2\sin t \quad ?$$

$y_p = [A] + t[(B_1 t + B_0)e^{-t}] + t[C \cos t + D \sin t]$

Find the solution of the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

$y = e^{2x} - e^{-3x}$

A spring-mass system set in motion was determined by the initial value problem

$$u'' + 100u = 0, \quad u(0) = 1, \quad u'(0) = -10.$$

What is the amplitude of the motion?

$A = \text{Amplitude} = \sqrt{c_1^2 + c_2^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

An undamped, free vibration $u'' + 9u = 0$ has initial conditions $u(0) = 4, u'(0) = 9$. The solution of this initial value problem can be written as $u = R\cos(\omega t - \delta)$. What are R and δ?

$R = \sqrt{4^2 + 3^2} = 5$ and $\delta = \tan^{-1} \frac{3}{4}$

A spring system with external forcing term is represented by the equation

$$y'' + 2y' + 2y = 4\cos(t) + 2\sin(t).$$

Then the steady state solution of the system is given by

Steady state sol^n = $2\sin t$

The homogeneous differential equation $t^2y'' - 4ty' + 6y = 0$ $(t > 0)$ has two solutions given by $y_1(t) = t^2$ and $y_2(t) = t^3$. Using the method of Variation of Parameters, find the general solution of the nonhomogeneous equation $t^2y'' - 4ty' + 6y = t^3$.

Gen^{l} sol^{n}: $y = c_1 t^2 + c_2 t^3 + (-1 + \ln|t|)\, t^3$

Find the general solution of

$$y^{(4)} - 10y'' + 9y = 0.$$

$y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{3x} + c_4 e^{-3x}$

Given that the function $y_1 = t$ is a solution to the differential equation

$$t^2y'' - ty' + y = 0, \quad t > 0,$$

choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ is a fundamental set of the solutions to the differential equation above.

$\{y_1, y_2\} = \{t, t\ln|t|\}$