MATH 341, Exam 1

Each problem is 20 points

(20) **1.** Find

$$\lim_{n\to\infty}\left(\sqrt{n^2+n} \;-\; \sqrt{n^2+1}\right).$$

(20) **2. a)** What are $\lim_{n\to\infty} 2^{1/n}$ and $\lim_{n\to\infty} 3^{1/n}$?

b) Given that $\left(1+\frac{1}{n}\right)^n$ is an increasing sequence of real numbers between 2 and 3 that converges to the famous number e as $n\to\infty$ where $2<e<3$, explain how to find

$$\lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)^n.$$

(20) **3.** Compute

$$\left(\frac{1}{2}\right)^5+\left(\frac{1}{2}\right)^6+\left(\frac{1}{2}\right)^7+\left(\frac{1}{2}\right)^8+\cdots$$

b) Let $s_n = 1+r+r^2+\cdots+r^n$ denote the partial sums of a geometric series with $0<r<1$. Show that (s_n) is a Cauchy sequence. (Start with "*Let* $\epsilon>0$...")

(20) **4.** Suppose A is a subset of the real numbers that is bounded from above. Define the *supremum* Sup A and state why it exists.

(20) **5.** Prove that $[0,1]$ is *uncountable* via Cantor's argument involving the Nested closed interval theorem.