

Math 341

Exam 2

Each problem is worth 25 points

1. A set $K \subset \mathbb{R}$ is called *compact* if every sequence (x_n) of elements in K has a subsequence (x_{n_k}) that converges to *a point in K* .
- a) Assume $a < b$. Prove that $[a, b]$ is compact
- b) Prove that if f is continuous on \mathbb{R} and $K \subset \mathbb{R}$ is compact, then $f(K) = \{f(x) : x \in K\}$ is compact.

2. Find $\lim_{x \rightarrow 0+} x^{\sin 3x}$. Explain your work.

3. Suppose that $R > 0$ and $x \in [-R, R]$. Prove that

$$0 < e^x - \sum_{n=0}^N \frac{x^n}{n!} < \frac{R^{N+1}e^R}{(N+1)!}.$$

4. Find $F'(x)$ when F is defined on $[0, 1]$ by

$$F(x) = \int_{x^2}^x \sqrt{1+t^7} \, dt.$$

Explain your work.