Practic problems for Exam 1 in class, Wed, Feb 28

1. $A, B, C$ sets. Prove that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

2. If $f: A \to B$ and $E, F$ are subsets of $A$, show that $f(E \cap F) \subseteq f(E) \cap f(F)$. Is the reverse inclusion always true?

p. 16: 18
p. 36: 18a $\max\{a,b\} = \frac{1}{2}(a+b+|a-b|)$
p. 39: 4, 7

3. What is the Archimedean property of $\mathbb{R}$?
4. Explain how to use the Nested closed interval theorem to prove that $[0,1]$ is uncountable.
5. Prove that a convergent sequence $(x_n)$ cannot have two distinct limits $L_1, L_2$, $L_1 \neq L_2$.

p. 62: 11, 12, 13, 17
p. 70: 7, 14
p. 77: 7, 12ab
p. 84: 9
p. 91: 5, 8
p. 93: 1, 4d
p. 101: 6a

6. What is $\lim_{n \to \infty} \left(1 + \frac{1}{n^2}\right)^n$? Explain.

7. Why is there no largest set in Cantor's sense?

8. Give the $\varepsilon$-definition of the limit of a sequence.

9. What is a Cauchy sequence?