MATH 366

Exam 2

1. (10 pts) Find the general solutions to $y'' + 4y' + 4y = \sin 3x$

2. (20 pts) a) Find a second order linear ODE with general solution $y = c_1x^2 + c_2x^3$.

Hint: Differentiate the expression for y once, and then once again. Solve the first two equations for c_1 and c_2 and then plug what you get into the third equation.

b) Are x^2 and x^3 linearly independent on $\mathbb{R}$?

c) What is the Wronskian of x^2 and x^3? Why doesn't what you get violate the consequence of Abel's theorem that says that the Wronskian is always zero or never zero.

3. (10 pts) Let $y_1 = x^3$ and define y_2 to be equal to $-x^3$ when $x < 0$ and equal to x^3 when $x \geq 0$. Note that y_1 and y_2 are both twice continuously differentiable functions of x.

a) Show that y_1 and y_2 are linearly independent on $\mathbb{R}$.

b) Show that the Wronskian of y_1 and y_2 is identically zero.

This is possible because $c_1y_1 + c_2y_2$ is *not* the general solution to an equation $y'' + Py' + Qy = 0$ where P and Q are continuous functions on $\mathbb{R}$.

4. (20 pts) Show that $r = 2$ is the only value of r such that x^r solves

$$x^2y'' - 3xy' + 4y = 0.$$

Let $y_1 = x^2$. Show that the method of reduction of order produces a second solution $y_2 = x^2 \ln x$ for $x > 0$. Next, use the method of variation of parameters to find a particular solution to

$$x^2y'' - 3xy' + 4y = x^2 \ln x$$

for $x > 0$. Finally, write down the general solution to that last equation.

5. (20 pts) Find the general solution to $y^{(4)} + 2y'' + y = 0$. What is the correct FORM of the particular solution to

$$y^{(4)} + 2y'' + y = x \sin x$$

used in the method of undetermined coefficients. (Just give the form.)

6. (20 pts) Find the general solution to the first order linear system

$$\frac{dx_1}{dt} = -2x_1 + x_2 + 5e^{2t}$$
$$\frac{dx_2}{dt} = x_1 - 2x_2 + 10e^{2t}$$