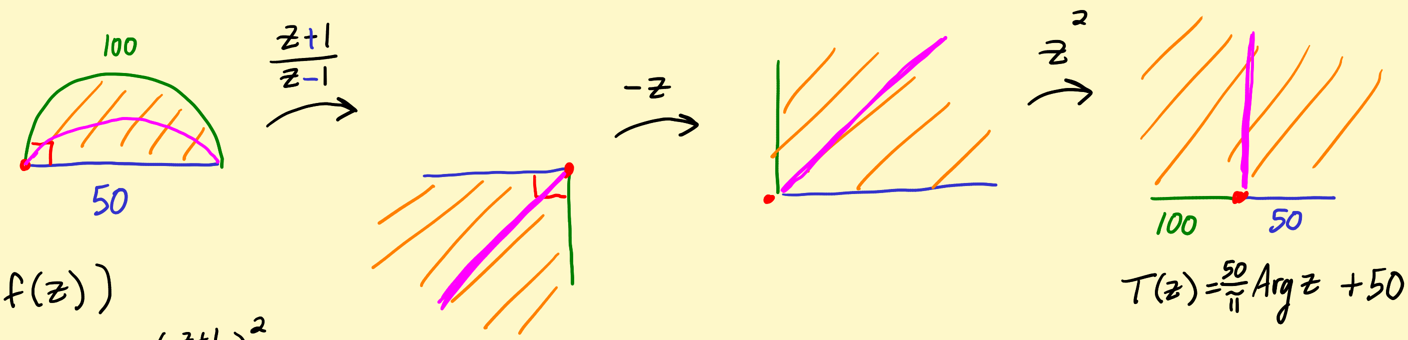
Lecture 38 Conformal mapping

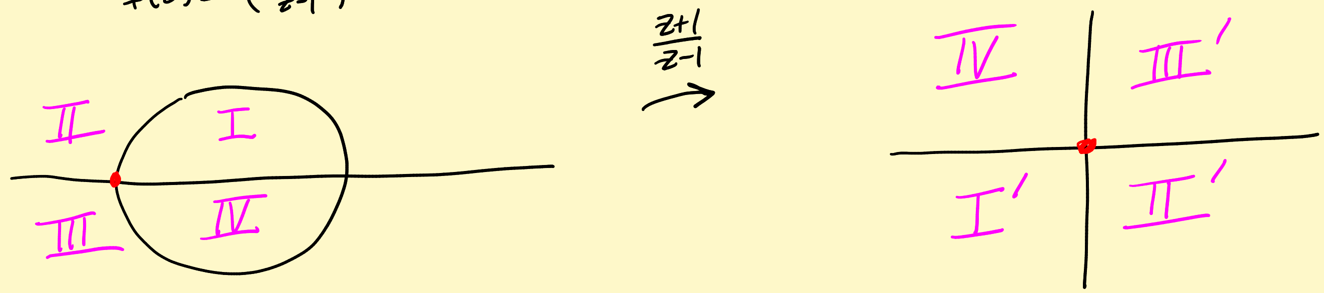
HWK 10 due Friday



$u = T(f(z))$

$f(z) = \left(\frac{z+1}{z-1}\right)^2$

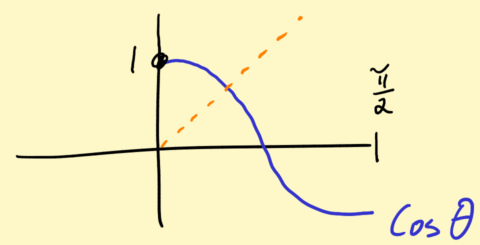
$T(z) = \frac{50}{\pi}\,\mathrm{Arg}\, z + 50$

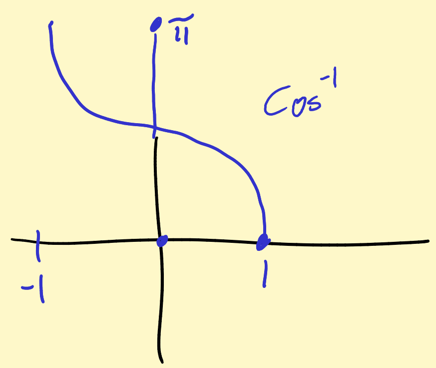


Formula in x & y for sol^n to heat prob

$\mathrm{Arg}\, z = \mathrm{Arg}\,(x+iy) = \mathrm{Tan}^{-1}\frac{y}{x}$ ← ouch when $x=0$!

Cos^{-1} is the best choice for UHP

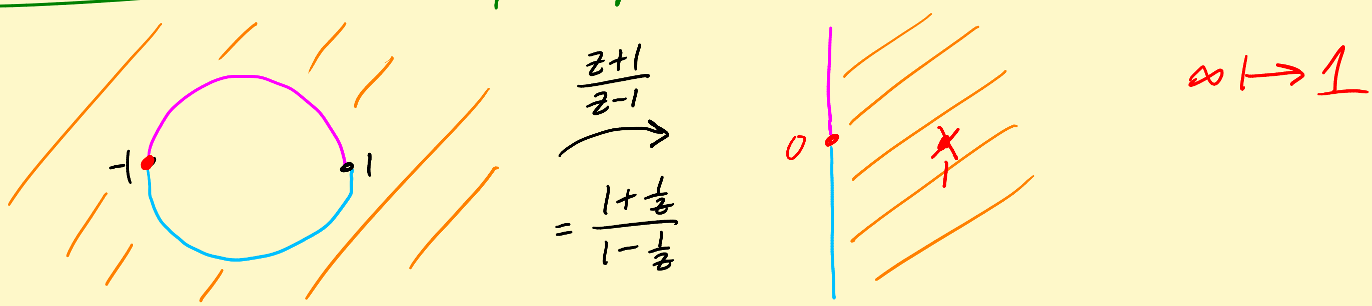


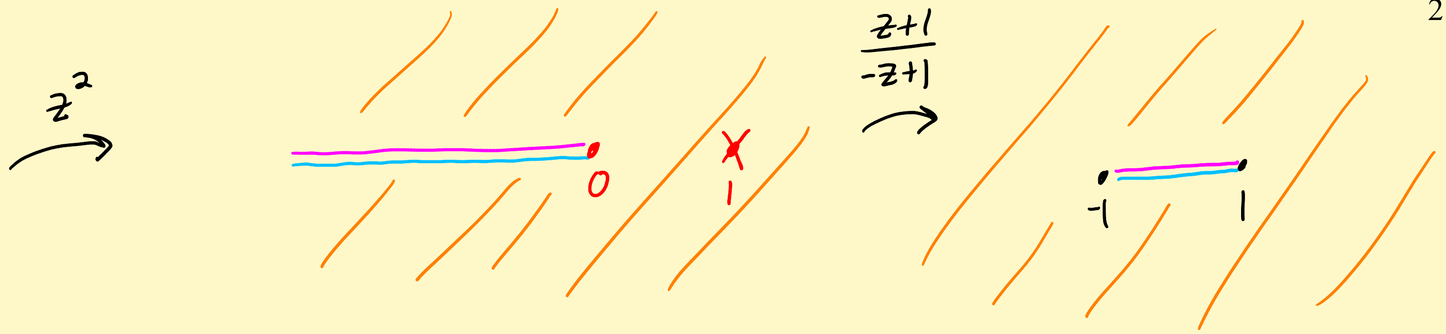


$$\mathrm{Arg}\, z = \mathrm{Cos}^{-1}\frac{x}{\sqrt{x^2+y^2}}$$

How to cook up Jukovsky map

$J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$

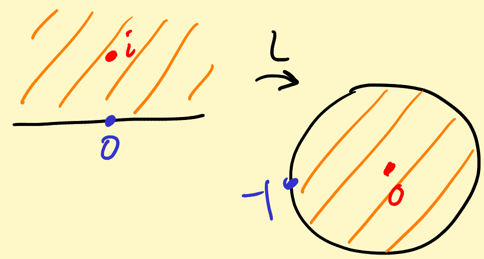




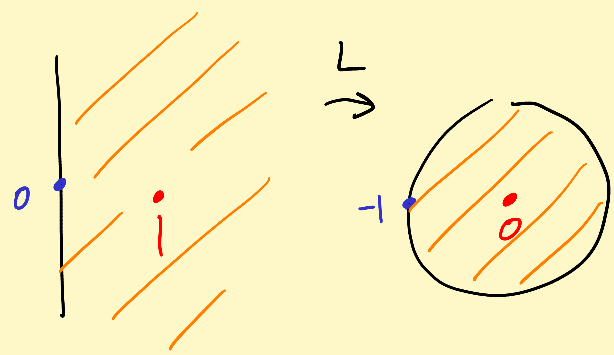
Composition $= J(z)$

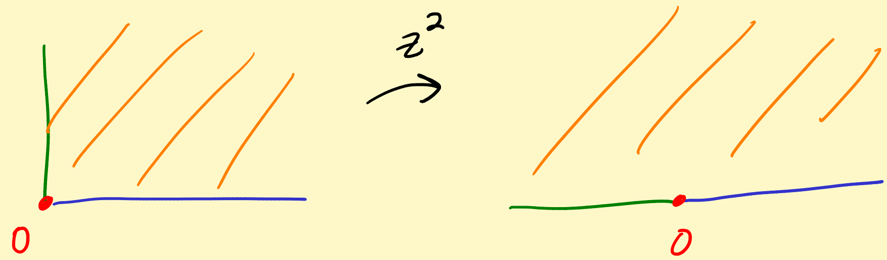
Comformal mapping toolbox

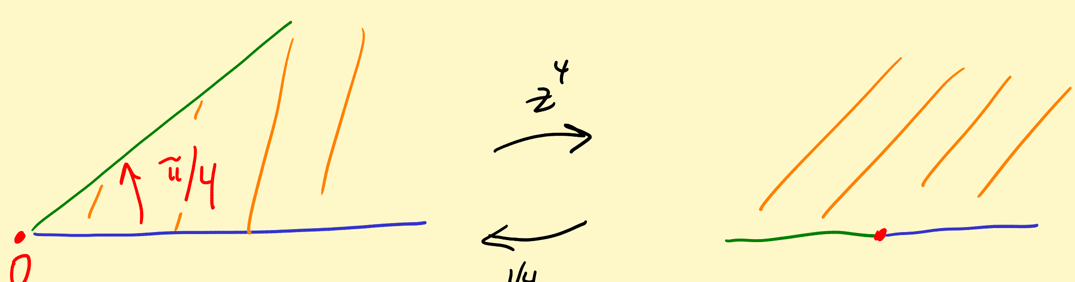
Favorite LFT's $\qquad L(z) = \frac{z-i}{z+i}$



$L(z) = \frac{z-1}{z+1}$





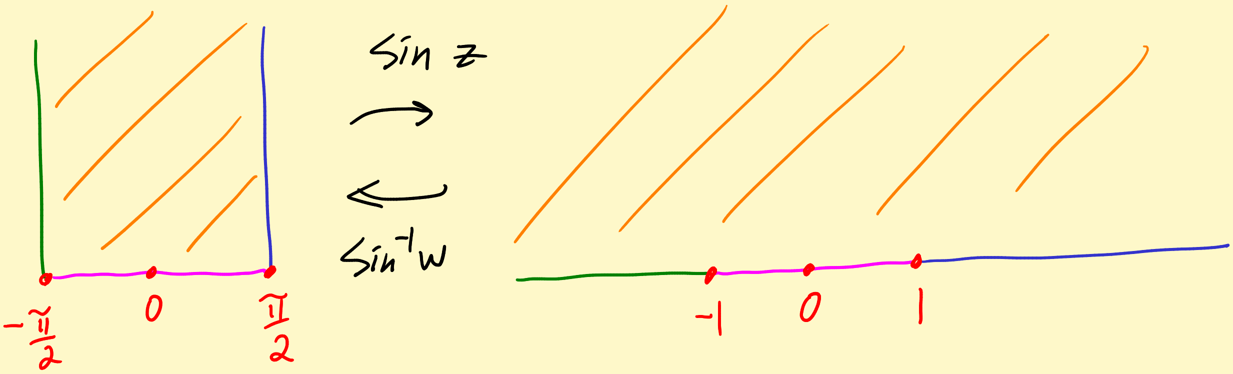


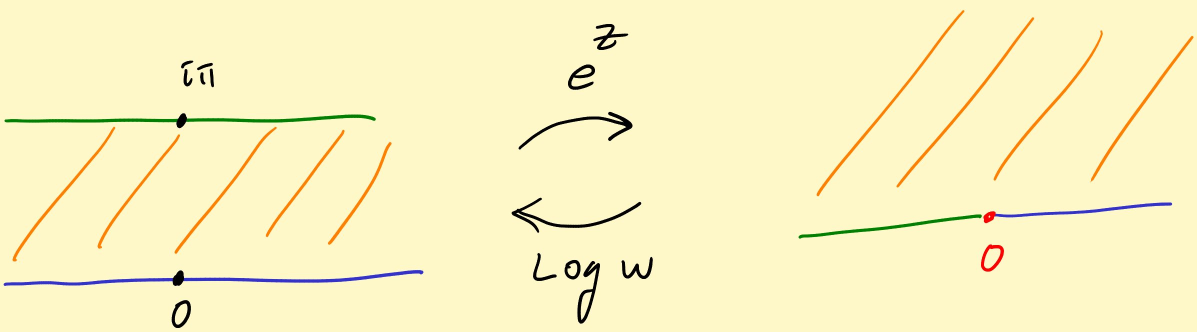
$z^{1/4}$

$= e^{\frac{1}{4}\mathrm{Log}\, z}$

$z^\alpha = e^{\alpha \operatorname{Log} z}$ opens angles by a factor of α at origin.

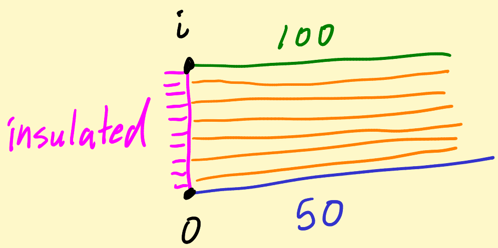
$(\alpha > 0)$





Heat problem

Easy case:

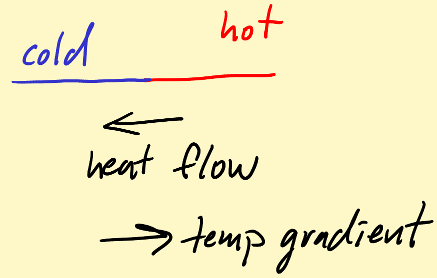


Insulated means no heat flow.

Kelvin's law:

(Rate of heat flow)

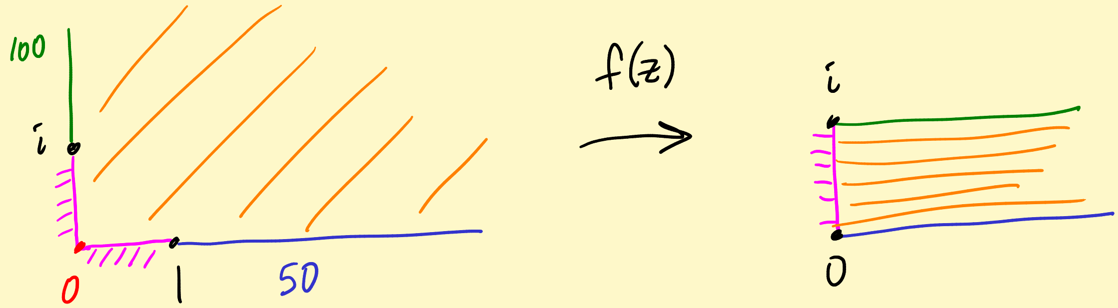
$= -k$ (temp gradient)



No heat flow: $\nabla u \cdot \hat{N} = 0$

meaning that level sets of temp fcn are $\perp$ to boundary.

Aha! Sol^n: $u(x+iy) = 50 + 50y = 50 + 50 \operatorname{Im} z$



Sol^n: $u(f(z))$

Important fact Ω_1, Ω_2 domains.

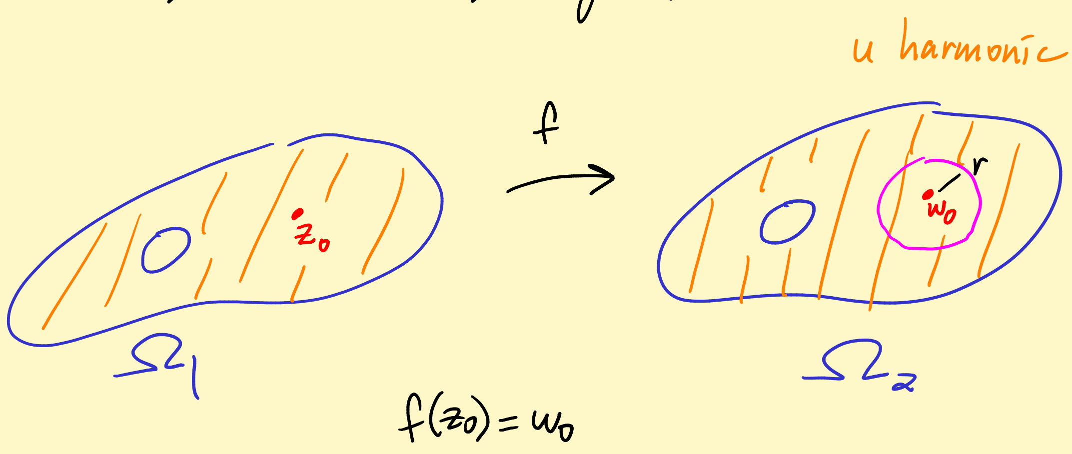
$f: \Omega_1 \to \Omega_2$ analytic, 1-1, onto (conformal).

u is harmonic on Ω_2.

Then $u \circ f$ is harmonic on Ω_1.

Pf: Chain rule, C-R eqns, ugh!

Better:



Continuity of f: $\exists\ \delta > 0$ such that $f: D_\delta(z_0) \subset D_r(w_0)$.

u harm on $D_r(w_0)$. So it has a harmonic conjugate v.

Write $F = u + iv$ on $D_r(w_0)$.

See $F \circ f$ is <u>analytic</u> on $D_\delta(z_0)$!

Aha! $u \circ f = \mathrm{Re}(F \circ f) = \mathrm{Re}$ (analytic fcn)

is harmonic! ✓