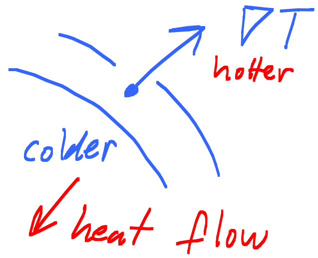
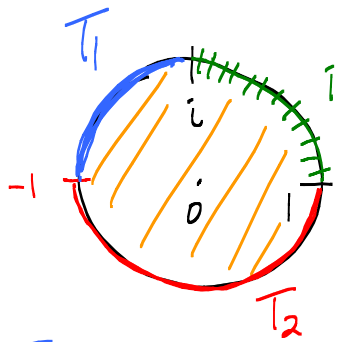
Lesson 39 Insulated parts of the boundary

Kelvin's Law: (Rate of heat flow) is proportional to the (temperature gradient)

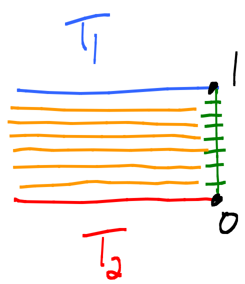
negative const.



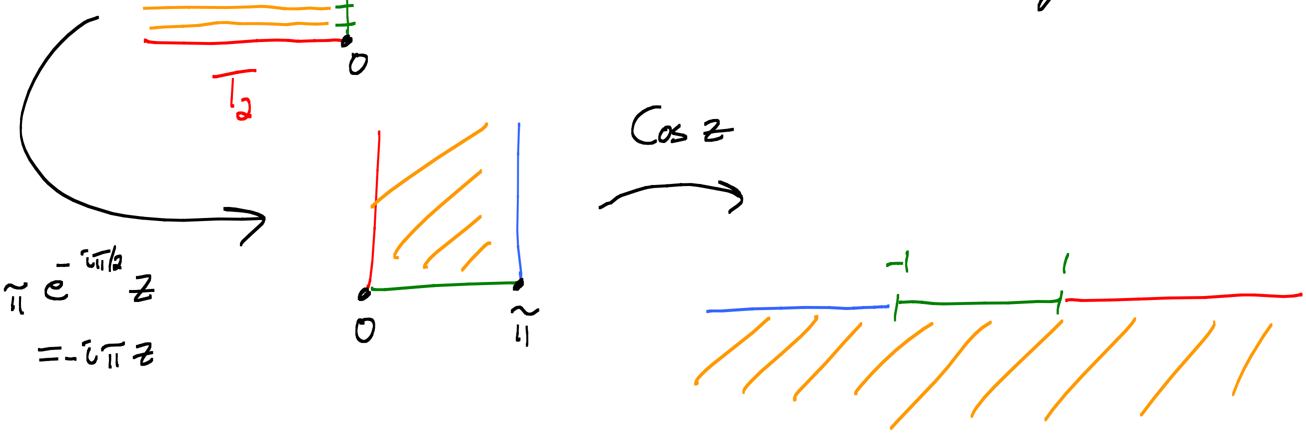
Prob:

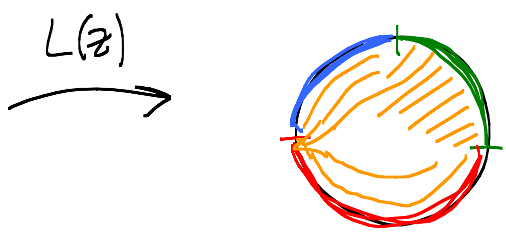


insulated, meaning normal derivative of u is zero



$$u = A\,\mathrm{Im}\, z + C = Ay + C$$





Sol^n

$u(F(w))$

where

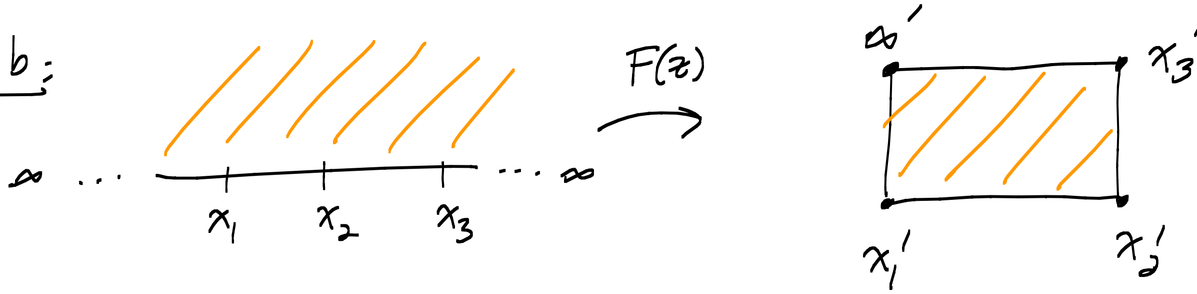
F = inverse map going to

Fact: Conformality preserves: Isotherms $\perp$ boundary.

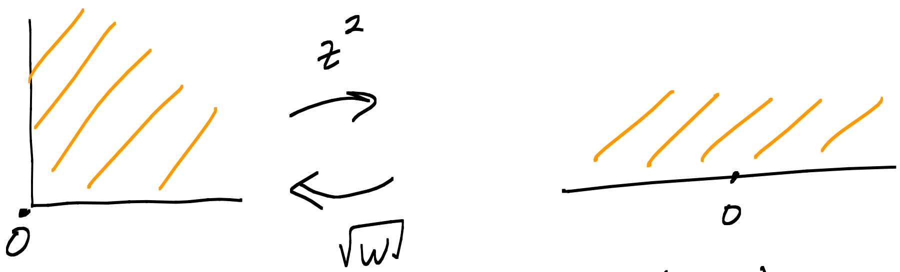
Take inverses of all maps to solve prob on disc.

Schwarz-Christoffel Maps

Prob:



Hmmm.

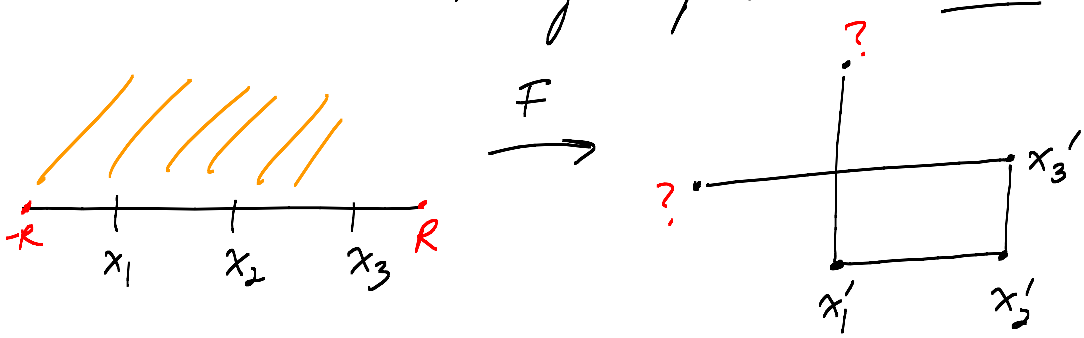


$= e^{1/2 \, \mathrm{Log}\, w}$ $\qquad \frac{d}{dw}\left(w^{1/2}\right) = \frac{1}{2} w^{-1/2}$

Big Idea: Try $\quad F'(z) = A(z-x_1)^{-1/2}(z-x_2)^{-1/2}(z-x_3)^{-1/2} + B$

It works. Why. On $\mathbb{R}$ $F'(t) = \ldots$ system of ODEs

Passing x_j leads to a sign change underneath a square root – kicking in a mult. by i meaning tangent vector <u>turns</u> 90 degrees!



Miracle: Those ? hook up! Bug test shows map goes to inside of rectangle!

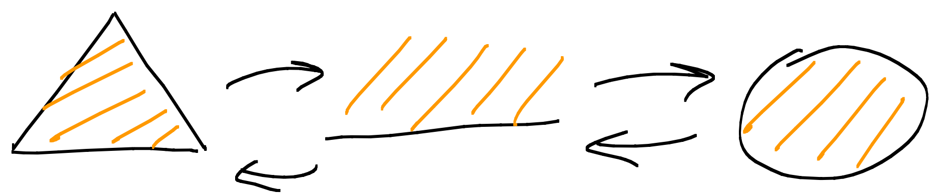
S-Ch. Formula: $\quad F(z) = \int_{L_i^z} (z-x_1)^{-1/2}(z-x_2)^{-1/2}(z-x_3)^{-1/2}\, dz$

$$|F(-R)-F(R)| = \left|\int_{C_R} F'(z)\,dz\right| \le \left(\max_{C_R}|F'|\right)\pi R$$

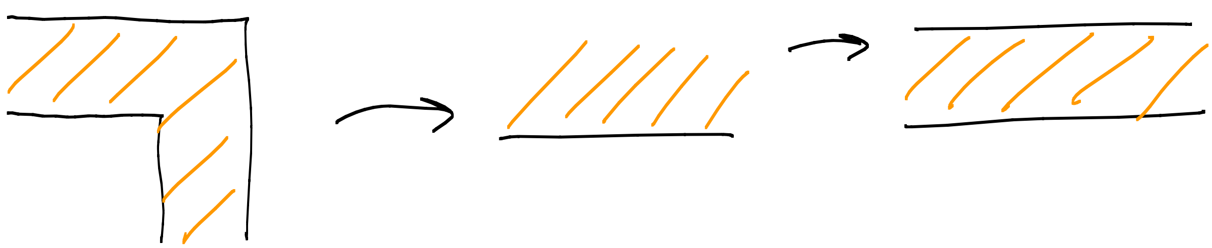
and $$|F'(z)| = \frac{1}{\sqrt{|z-x_1||z-x_2|\,|z-x_3|}} = \frac{1}{(\text{poly of deg } 3)^{1/2}}$$

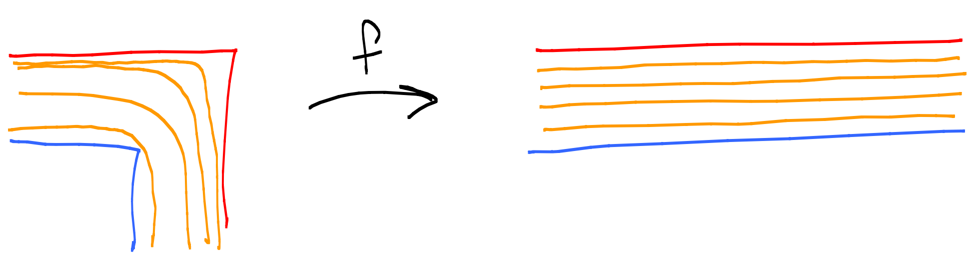
$$\sim \frac{1}{R^{3/2}}$$

Big ones:

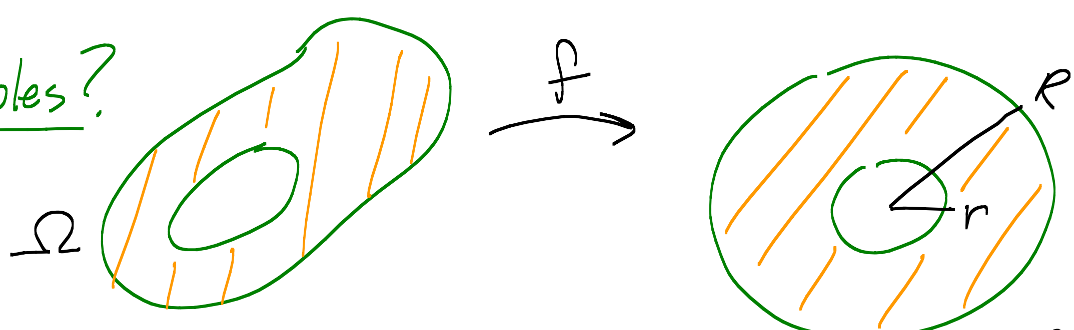


or any polygon

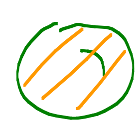




What about holes?



or



Map exists, but ratio $\frac{R}{r}$ depends on Ω!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