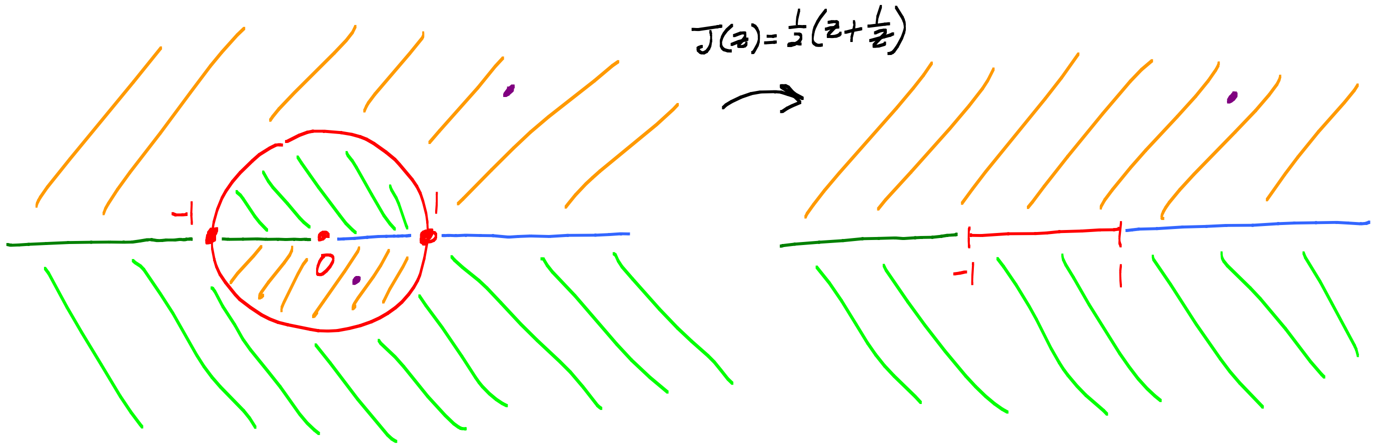
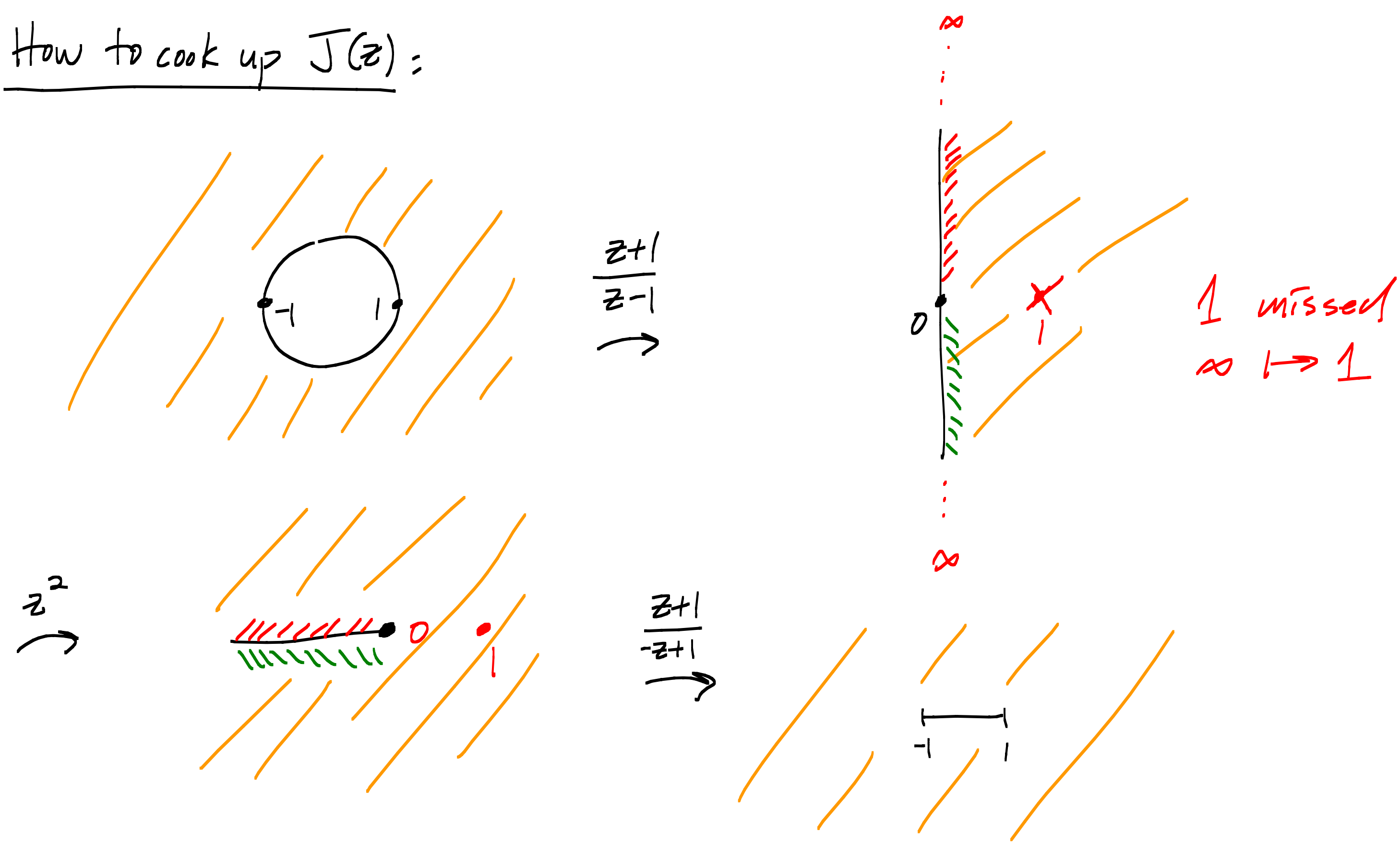
Lesson 38 Applications of conformal mapping

HWK 11 due Fri
HWK 12 due Dec. 4

(Skipping lesson 37 on Schwarz-Christofel maps today. Will do later.)

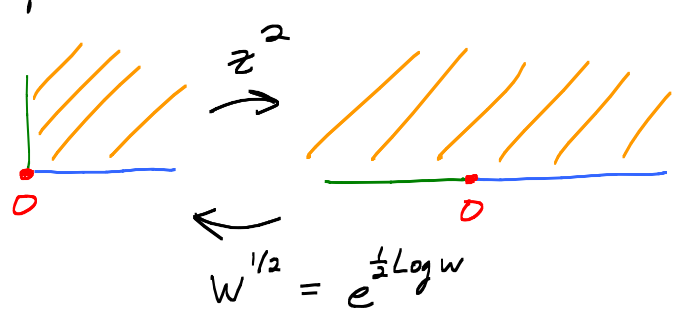


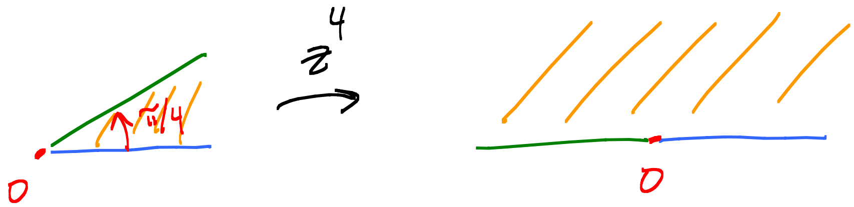
How to cook up $J(z)$:



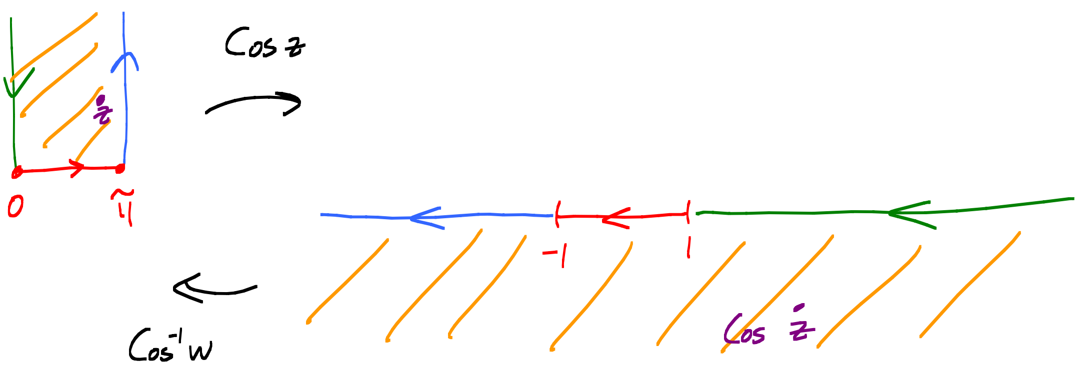
Composition is the Jukovsky map!

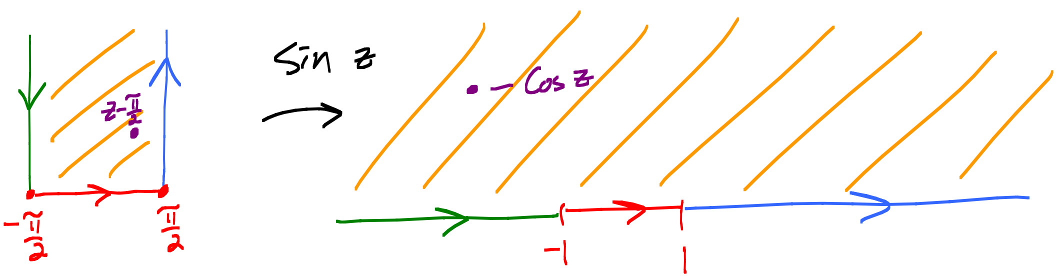
Conformal map toolbox



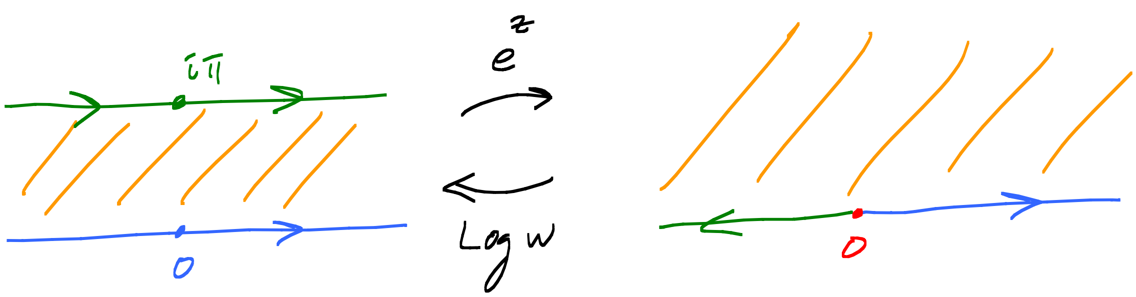


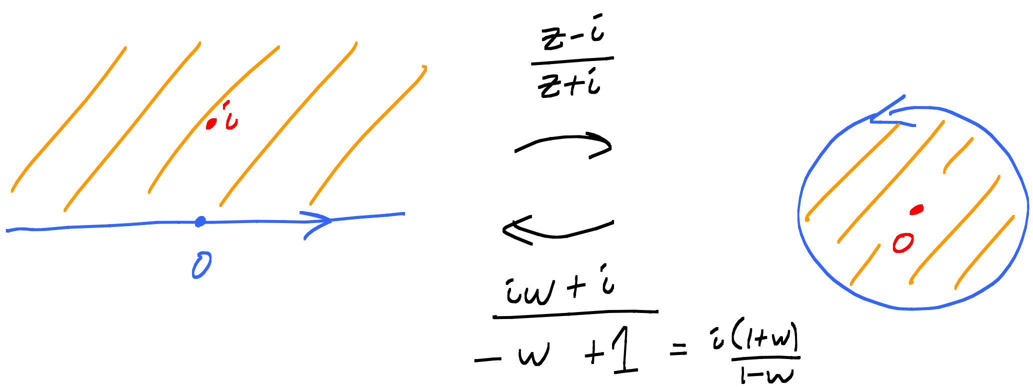
$z^\alpha = e^{\alpha \operatorname{Log} z}$ opens angle at origin by a factor of α.





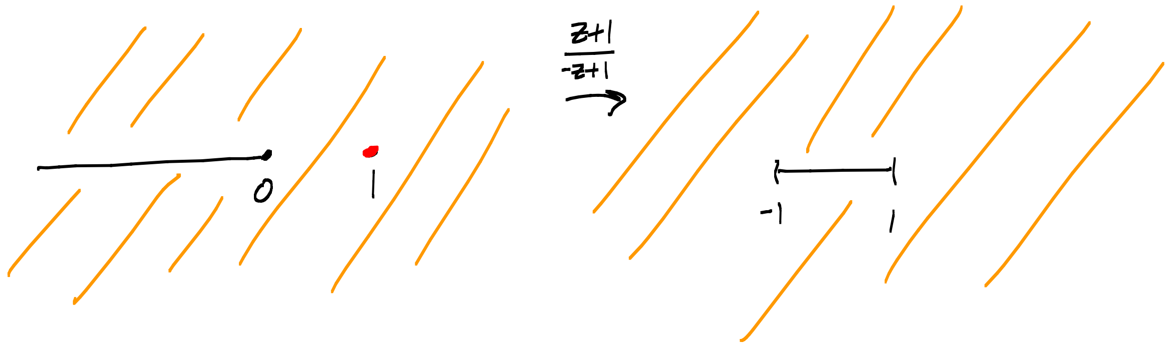
$\operatorname{Sin}(z-\frac{\pi}{2})$
$= -\operatorname{Cos} z$





LFTs : $\hat{\mathbb{C}} \xrightarrow[\text{onto}]{1\text{-}1} \hat{\mathbb{C}}$ Looking $\mathbb{C} \to \mathbb{C}$, need to be aware of where ∞ goes and where it comes from

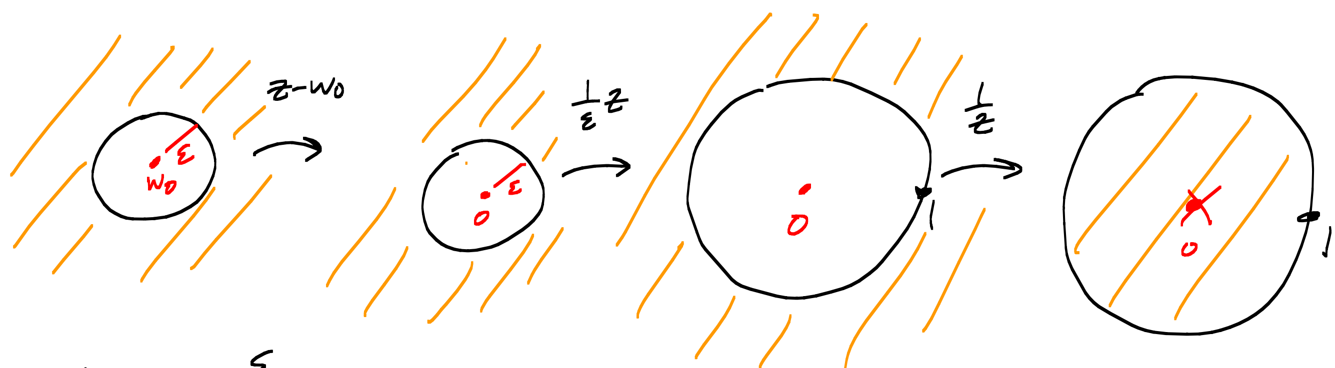
EX



Casorati–Weierstraß Thm An entire fcn that misses an open set in $\mathbb{C}$ must be constant.

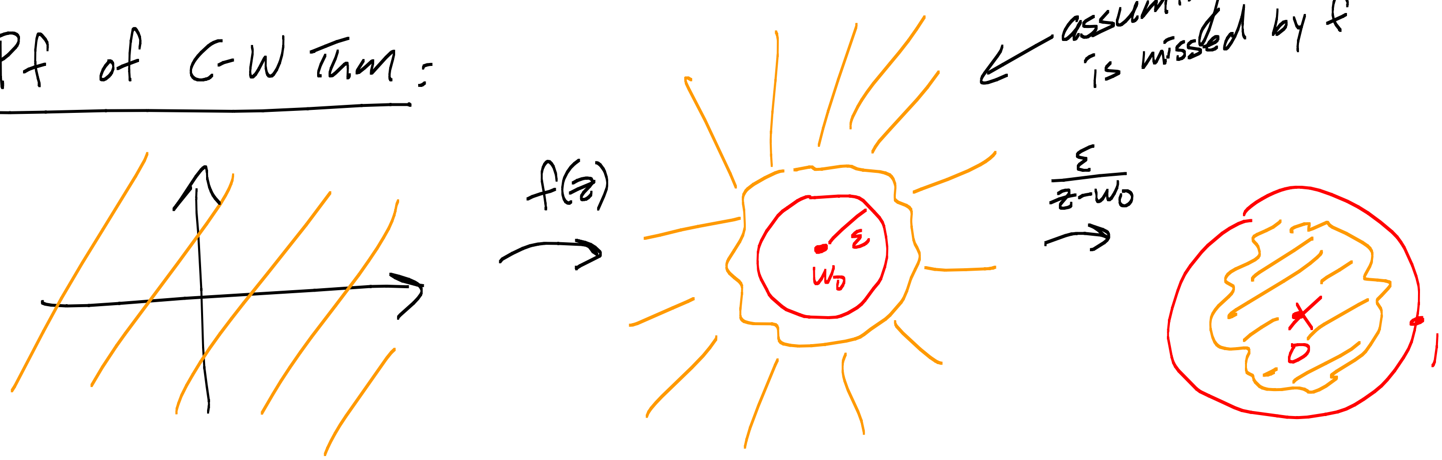
f "misses" w_0 if $w_0 \notin f(\mathbb{C}) = \{f(z) : z \in \mathbb{C}\}$

Key:



Composition : $\frac{\varepsilon}{z - w_0}$

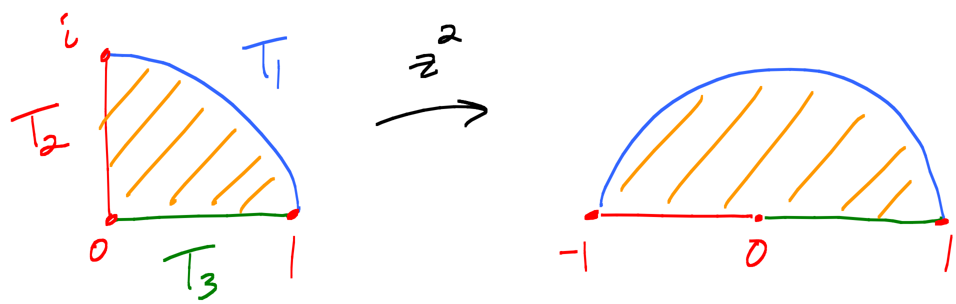
Pf of C-W Thm:

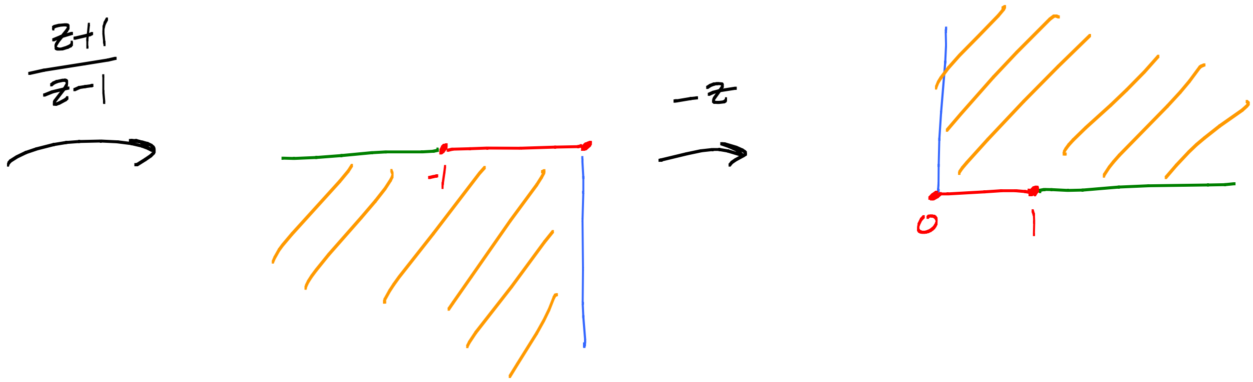


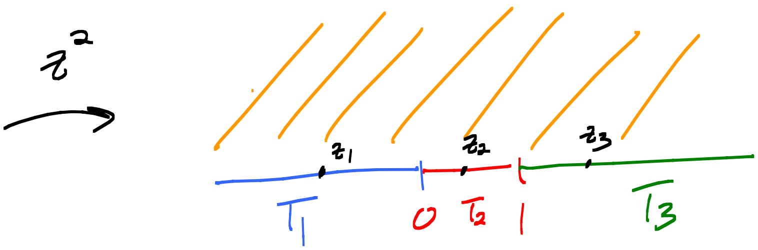
Aha! $\frac{\varepsilon}{f(z) - w_0}$ is a bnded entire fcn, so constant. (Liouville)

$\Longrightarrow$ f const.

Heat problem







$u = A \operatorname{Arg} z + B \operatorname{Arg}(z-1) + C$

At z_1: Need

3. Get A $\longrightarrow$ $T_1 = A \cdot \pi + B \cdot \pi + C$

2. Get B $\longrightarrow$ $T_2 = A \cdot 0 + B \cdot \pi + C$

1. Get C $\longrightarrow$ $T_3 = A \cdot 0 + B \cdot 0 + C$

Composition: $f(z)$

Sol^n to orig prob: $u(f(z))$

Key: Harmonic fcns preserved under conformal maps.

analytic
1-1, onto