Math 425

Exam 2

1. *(20 pts)* a) State Liouville's Theorem.
b) Suppose f is an entire function such that $|f(z)| \to \infty$ as $|z| \to \infty$. Show that f must have zeroes.

2. *(20 pts)* Use the Ratio Test to find the radius of convergence about $z = 0$ of the power series

$$\sum_{n=1}^{\infty} \frac{n^n}{2^n n!} z^{2n}.$$

3. *(20 pts)* Compute

$$\int_0^{\infty} \frac{1}{x^5 + 1}\, dx$$

by integrating $f(z) = 1/(z^5+1)$ around the contour that follows the real line from zero to R, then follows the circle Re^{it} from $t = 0$ to $t = 2\pi/5$, and then follows the line $te^{i2\pi/5}$ from $t = R$ back to $t = 0$. Use the Residue Theorem and let $R \to \infty$.

4. *(20 pts)* Convert the integral

$$\int_0^{2\pi} \frac{d\theta}{(2 + \sin\theta)^2}$$

into a contour integral of the form $\int_C f(z)\, dz$ where f is a rational function and C is the unit circle parametrized in the standard sense. DO NOT COMPUTE THE VALUE of the integral and don't even bother to use any algebra to clean up the function.

5. *(20 pts)* Compute

$$\mathrm{Res}_i \frac{\mathrm{Log}\ z}{(z^2+1)} \qquad \text{and} \qquad \mathrm{Res}_i \frac{e^{2z}}{(z^2+1)^2}.$$