MATH 425, Practice Problems

1. Use the residue theorem to compute the integral

$$\int_0^\infty \frac{1}{x^3+1}\,dx$$

by integrating along a contour that follows the real axis from 0 to R and then the circular arc from R to $Re^{i2\pi/3}$, and finally the line from $Re^{i2\pi/3}$ back to 0. Let R go to infinity. Be sure to explain why the integral along the circular arc goes to zero.

Let Log z denote the principal branch of the complex logarithm of z. If you use a different branch, be sure to define it carefully.

2. Find an analytic function that maps the quarter disc

$$\{re^{i\theta} \,:\, 0 < r < 1, \quad 0 < \theta < \pi/2\}$$

one-to-one onto the strip

$$\{z \,:\, 0 < \mathrm{Re}\, z < 1\}.$$

3. What is the image of the unit disc under the linear fractional transformation $T(z) = z/(1-z)$?

5. Suppose g is an analytic function near zero that has a zero of multiplicity 2 at zero, i.e., $g(0) = 0$, $g'(0) = 0$, but $g''(0) \neq 0$. Use only power series to find a formula involving derivatives of g at zero that gives the residue at zero of $1/g(z)$. (Don't use the formula I gave you for f/g to do this problem.)

6. Show that the Maximum Principle implies the Fundamental Theorem of Algebra.

7. Problems from the book:

p. 430: 5, 6
p. 440: 3