Math 428

Homework 3

1. Find the full Fourier series

$$a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

on $[-\pi, \pi]$ for the function $f(t) = t$. Also, express your answer in the equivalent complex form $\sum_{n=-\infty}^{\infty} c_n e^{int}$. Graph the function to which the full Fourier series converges on the whole real line. What happens at $\pm\pi$?

2. Find all real valued solutions to the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

having the form $u(x, y) = X(x)Y(y)$. (This is called the method of "separation of variables.")

3. Show that $r^n \sin n\theta$ and $r^n \cos n\theta$ are harmonic in the plane by showing that they satisfy the Laplace equation in polar coordinates. Use the complex exponentials $e^{in\theta}$ and $e^{-in\theta}$ to show that $r^n \sin n\theta$ is a polynomial of x and y when converted to cartesian coordinates.

4. Find a harmonic function $u(r, \theta)$ on the unit disc that has boundary values given by

a) $u(1, \theta) = 1$
b) $u(1, \theta) = \sin\theta$
c) $u(1, \theta) = \sin^2\theta = (\frac{1}{2})(1 - \cos 2\theta)$

Write your final answers without the appearance of any complex numbers. (Note that infinite sums are not needed for these problems.)

5. Rewrite the answers for the last problem in terms of x and y.

6. Exercise 6 on page 59 of Stein. In part (d), verify only the sum of odd terms. Challenge: How can you use your knowledge of the sum of the odd terms to deduce the sum of the even terms and the sum of all the terms? Hint: The sum of the even terms is 1/4 the sum of all the terms!

7. Exercise 15 on page 63 of Stein.