Lecture 27

Ch. 2: Ex. 18:

$$P_r(\theta) = \frac{1-r^2}{1-2r\cos\theta + r^2}$$

$$u = \frac{\partial}{\partial\theta} P_r(\theta) = \ldots$$

Why is u harmonic?

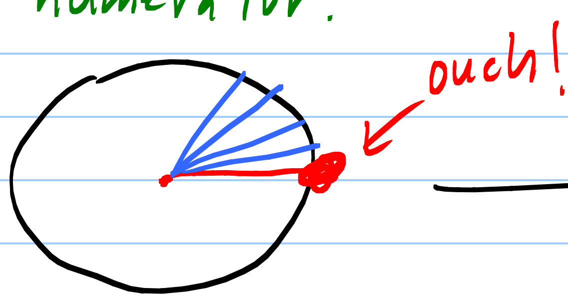
$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

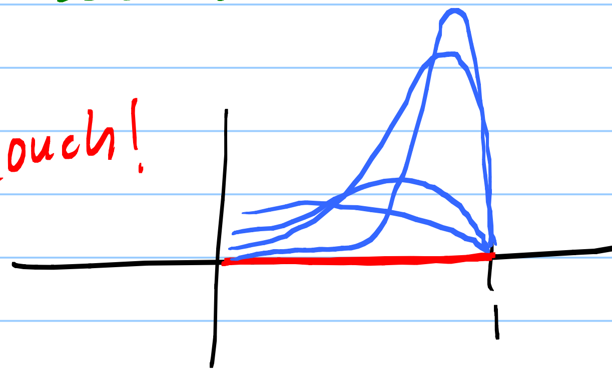
Aha! $\Delta \frac{\partial}{\partial\theta} P_r = \frac{\partial}{\partial\theta} \underbrace{\Delta P_r}_{\equiv 0} \equiv 0$

$\lim\limits_{r\to 1} \frac{\partial}{\partial\theta} P_r(\theta)$ $\quad \theta \neq 0$ case. Do it.

$\longrightarrow 0$ if $\theta \neq 0$.

$\theta = 0$ case is easy because $\sin\theta = 0$ in numerator.





Max Princ. fails? No, because u is not bounded on $\overline{D_1(0)}$, bad singularity at 1.

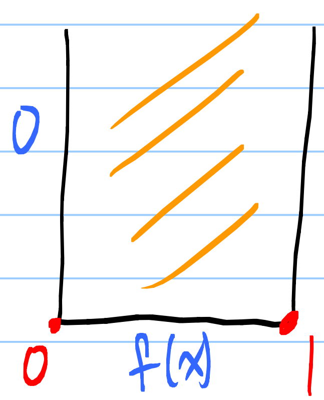
Ch. 2: Ex. 19: Separation of variables.

$$\left.\begin{aligned} &X'' - \lambda X = 0 \\ &X(0)=0,\ X(1)=0 \end{aligned}\right\}$$ know the solutions, λ, etc.

$$Y'' + \lambda Y = 0$$

Get $u(x,y) = \sum_{n=1}^{\infty} A_n e^{-n\pi y} \sin n\pi x$

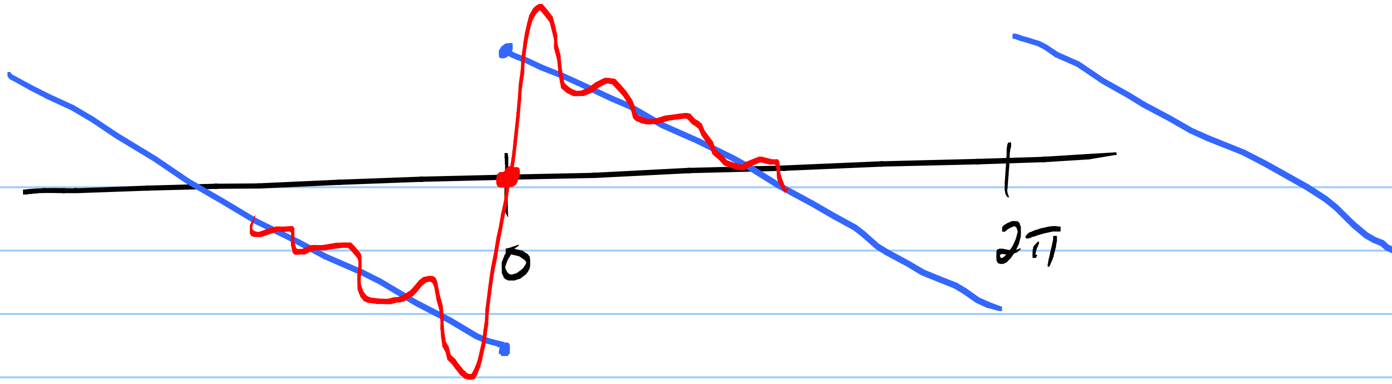
$A_n = \int_0^1 \cdots\cdots dt$



$$= \int_0^1 \underbrace{\left(\sum_1^{\infty} \qquad\qquad \right)}_{K(x,y,t)} f(t)\,dt$$

Poisson kernel

Ch. 3: Ex. 19. $\quad f(x) = \dfrac{\pi - x}{2}$



$$f(x) \sim \frac{1}{2i} \sum_{n \neq 0} \frac{e^{inx}}{n}$$

$$S_N(x) = \sum_{\substack{-N \\ n \neq 0}}^{N} = \sum_{n=1}^{N} \frac{e^{inx} - e^{-inx}}{n \cdot 2i}$$

$$= \sum_{n=1}^{N} \frac{\sin nx}{n}$$

$$D_N(x) = \sum_{n=-N}^{N} e^{inx}$$

Aha! $\frac{d}{dx} S_N(x) = \frac{1}{2}\left(D_N(x) - 1\right)$

So $S_N(x) = \frac{1}{2}\int_0^x \left(D_N(t) - 1\right) dt$

Big trick: Hint for Ex. 12.

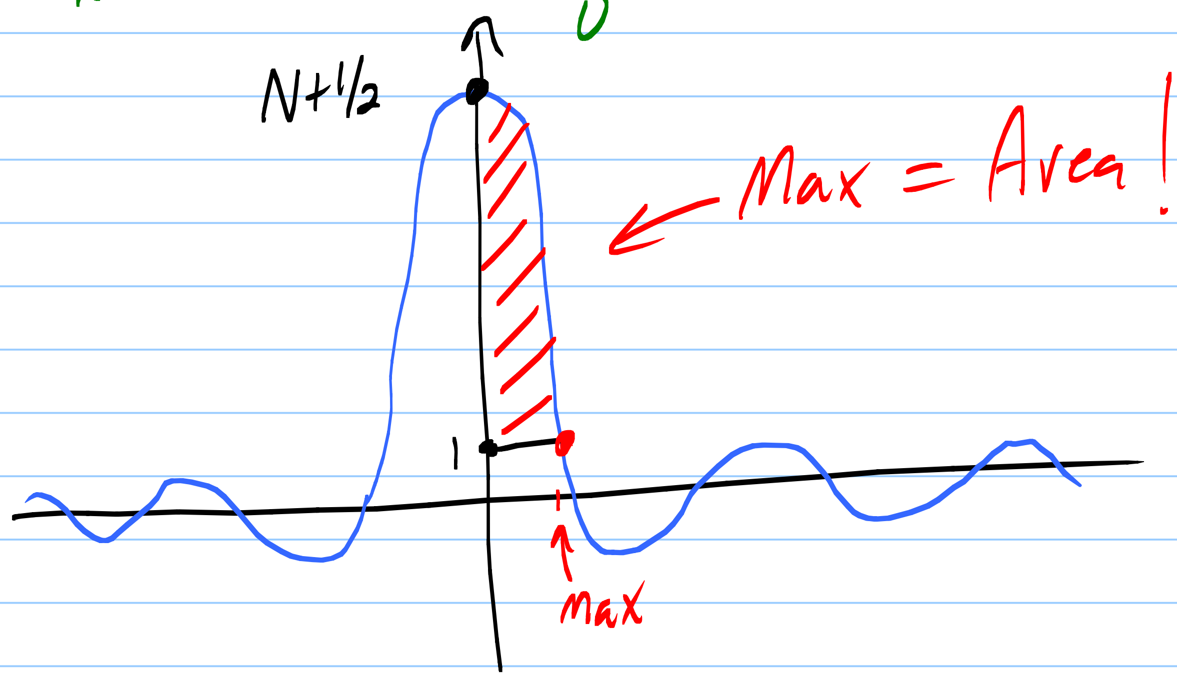
$$\frac{1}{(t/2)} - \frac{1}{\sin\frac{t}{2}} = h(t)$$

L'H twice shows $h(t) \to 0$ as $t \to 0$.

$$\boxed{\frac{\sin(N+\frac{1}{2})t}{t/2} - D_N(t) = \underbrace{h(t)\sin(N+\tfrac{1}{2})t}_{\text{bounded!}}}$$

Ch. 3.: Ex. 20:

$$S_N(x) - \frac{\pi}{2} = \frac{1}{2}\int_0^x \left(D_N(t) - 1\right) dt - \frac{\pi}{2}$$



Let $\sigma_N(x) = \int_0^x \frac{\sin(N+\frac{1}{2})t}{t}\,dt$

$$\max_{[0,\frac{\pi}{N}]} S_N(x) - \max_{[0,\frac{\pi}{N}]} \sigma_N(x) \to 0$$ as $N \to \infty$.

Show $S_N(x) - \sigma_N(x) = H(x)$ where H is a continuous fcn and $H(x) \to 0$ as $x \to 0$.

$$\max_{[0,\frac{\pi}{N}]} S_N(x) - \frac{\pi}{2} \xrightarrow[N\to\infty]{} \int_0^{\pi} \frac{\sin t}{t}\,dt - \frac{\pi}{2}$$

Stein =, not true.