MATH 428 Exam 1 solutions

1. $$\cos 3\theta + i\underline{\sin 3\theta} = e^{i3\theta} = \left(e^{i\theta}\right)^3 = \left(e^{i\theta}\right)^2 \cdot e^{i\theta}$$

$$= \left[\left(\cos^2\theta - \sin^2\theta\right) + 2i\cos\theta\sin\theta\right]\left(\cos\theta + i\sin\theta\right)$$

$$= (\text{Real part}) + i\left[\underbrace{\left(\cos^2\theta - \sin^2\theta\right)\sin\theta + 2\cos^2\theta\sin\theta}\right]$$

$$= \underline{\underline{3\cos^2\theta\sin\theta - \sin^3\theta}}$$

$$= \sin 3\theta$$

2. a) Functions of the form

$$a_0 + \sum_{n=1}^{N}\left(a_n\cos nx + b_n\sin nx\right) = \sum_{n=-N}^{N} c_n e^{inx}$$

b) We will choose A so that

$$f(-\pi) - A(-\pi) = f(\pi) - A\pi, \quad \text{i.e.,}$$

$$A = \frac{f(\pi) - f(-\pi)}{2\pi}. \qquad \text{Then } f(x) - Ax$$

can be extended as a 2π-periodic function and we can use Féjer's theorem to find a trig poly $P(x)$ so that

$$\left|(f(x)-Ax)-P(x)\right|<\varepsilon \text{ on } [-\pi,\pi].$$

3. $\lambda>0$, so $\lambda=k^2$ for a $k>0$.

Characteristic polyomial $r^2+k^2=0$.

$r=\pm ki$. So $X(x)=A\cos kx+B\sin kx$.

$$X(0)=A\cdot 1+B\cdot 0 \overset{\text{want}}{=} 0. \text{ So } \boxed{A=0}$$

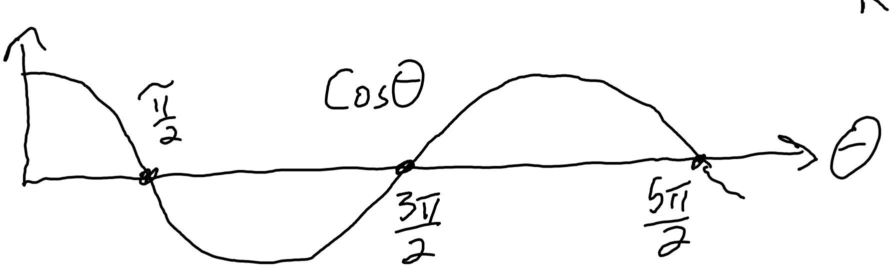
$$X'(L)=Bk\underbrace{\cos kL}\overset{\text{want}}{=}0$$

Don't want $B=0$ too.

need $\cos kL=0$

$$kL=\frac{2n-1}{2}\cdot\pi$$

$n=1,2,3,\ldots$



$$\lambda_n = k^2 = \left(\frac{(2n-1)\pi}{2L}\right)^2 \qquad n=1,2,3,\ldots$$

$$X_n(x) = B \sin\left(\frac{2n-1}{2L}\pi x\right)$$

take $B=1$

4. $$\int_{-\pi}^{\pi} \cos Nt \left[\underbrace{\frac{(x-t)^2 - x^2}{\sin \frac{t}{2}}}\right] dt$$

$g(t)$ looks like $\frac{0}{0}$ at $t=0$.

$$\lim_{t\to 0} g(t) \overset{L'H}{=} \lim_{t\to 0} \frac{-2(x-t)}{\frac{1}{2}\cos\frac{t}{2}} = -4x$$

So g becomes continuous on $[-\pi,\pi]$ if we set it equal to $-4x$ at $t=0$. The integral is π times the Fourier cosine coefficient for g. Bessel's ineq $\Rightarrow \int \to 0$ as $N \to \infty$.

(or Riemann-Lebesgue)