Formulas

Fourier sine series for $f(x)$ on $[0,\pi]$ is $\sum_{n=1}^{\infty} B_n \sin nx$ where

$$B_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx \; dx$$

Full Fourier series for $f(x)$ on $[-\pi,\pi]$ is $a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\; dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \; dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \; dx$$

The complex version is $\sum_{n=-\infty}^{\infty} c_n e^{inx}$ where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\; dx$

Facts: $c_0 = a_0$, and if $n \geq 1$: $c_n = \frac{a_n - ib_n}{2}$ and $c_{-n} = \frac{a_n + ib_n}{2}$

Parseval's identities (Bessel's inequality with $\leq$ in place of $=$)

$$2a_0^2 + \sum_{n=1}^{\infty}({a_n}^2 + {b_n}^2) = \frac{1}{\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx \quad \text{and} \quad \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx$$

Fourier transform of f is $\hat{f}(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-isx}\; dx$

Inverse Fourier transform of g is $\check{g}(x) = \int_{-\infty}^{\infty} g(s)e^{isx}\; ds$

Fourier cosine transform of f is $(\mathcal{F}_c f)(s) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos(sx)\; dx$

Fourier sine transform of f is $(\mathcal{F}_s f)(s) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin(sx)\; dx$

The Plancherel formula is $\int_{-\infty}^{\infty} |f(x)|^2\; dx = 2\pi\int_{-\infty}^{\infty} |\hat{f}(s)|^2\; ds$