Math 428

Review problems

1. Use D'Alembert's method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

for the *infinite* vibrating string problem with initial position $u(x,0) = e^{-x^2}$ and zero initial velocity. The solution has one "hump" at time zero. Is that true of the solution for all time?

2. Find the complex Fourier transform and the Fourier cosine transform of e^{-4x^2}.

3. Solve $\dfrac{\partial^2 u}{\partial x \partial y} = 0$.

4. Let $f(x) = \begin{cases} 1 & 0 < x < \pi/2 \\ 2 & \pi/2 < x < \pi. \end{cases}$, and let $a_0 + \sum_{n=1}^{\infty} a_n \cos nx$ be the Fourier cosine series for f on $[0, \pi]$. Then $a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi/2 =$

5. Given that $f(x) = \displaystyle\int_0^{\infty} \frac{\cos sx}{1+s^2}\, ds$, what is the Fourier transform of $f(x)$? Explain.

Hint: Even, odd functions. Inverse Fourier Transform.

6. The Legendre polynomials

$$p_n(x) = \frac{d^n}{dx^n}(1-x^2)^n$$

are orthogonal on $[-1, 1]$. Let

$$A_n = \int_{-1}^{1} p_n^2 \; dx.$$

To expand a function f in terms of Legendre polynomials, $f = \sum_{n=0}^{\infty} c_n p_n$, what must the coefficients c_n be? What is Bessel's inequality in this context? It can be shown that the linear span of the Legendre polynomials of degree N or less is equal to the vector space of polynomials of degree N or less. Explain why this implies that Bessel's inequality must be an equality when f is assumed to be continuous.