MA 527 Steve Bell

www.math.purdue.edu/~bell/MA527

Collaborative homework area:

www.projectrhea.org/rhea/index.php/2013_Fall_MA_527_Bell

On campus student video lecture access:

https://engineering.purdue.edu/ProEd/OnCampus

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Exam 1,	Sept. 27	100
Exam 2,	Nov. 11	100
Final Exam		200
Homework		100

On campus: Paper – 3 lessons stapled

Off campus: Blackboard, Filename: BellSteveHWK1.pdf

HWK 1: Lessons 1, 2, 3 due Wed., Aug 28

Linear Alg. Chap 7 ,

$$\begin{cases} x_1 + x_2 + x_3 = 4 \\ 2x_1 \qquad - x_3 = 5 \\ -x_1 + 2x_2 + 4x_3 = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 0 & -1 & 5 \\ -1 & 2 & 4 & 6 \end{array}\right]$$

$$\underbrace{\begin{matrix} \qquad\qquad\qquad \end{matrix}}_{\mathbb{A}} \qquad \vec{b}$$

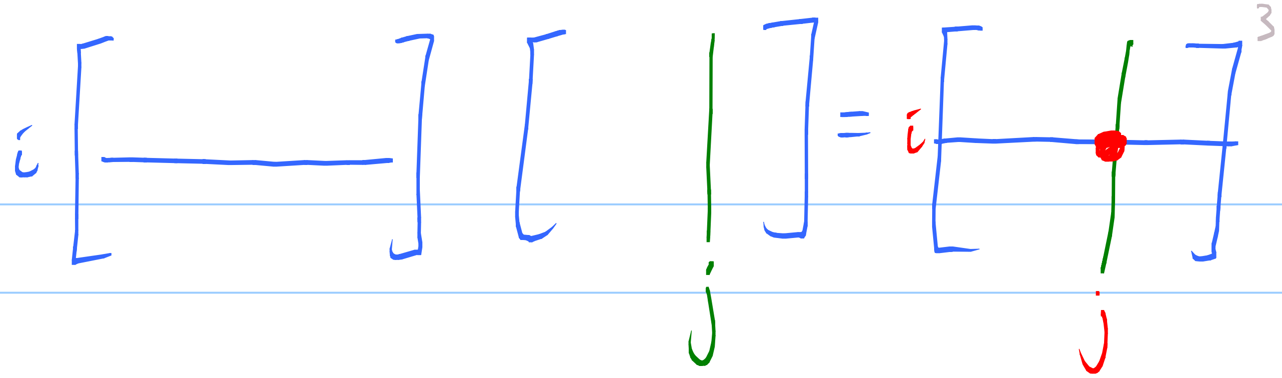
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 2 & 4 \end{bmatrix} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\vec{x}} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

$$\mathbb{A}\vec{x} = \vec{b}$$

$$\mathbb{A} \underbrace{\left[\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m\right]}_{\mathbb{X}} = \left[\vec{b}_1, \vec{b}_2, \dots, \vec{b}_m\right]$$

$$\mathbb{A} \quad \mathbb{X} \qquad = \mathbb{B}$$

7.1 easy , 7.2 harder



$$\mathbb{A} = [a_{ij}]_{n \times m}$$

rows cols

$$\mathbb{A}\mathbb{B} = \mathbb{C} = [c_{ij}]$$

$$c_{ij} = (i\text{th row } \mathbb{A}) \cdot (j\text{th col } \mathbb{B})$$

$$\mathbb{A}_{n \times k}\mathbb{B}_{k \times m} = \mathbb{C}$$

$=$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} a & b & e \\ \alpha & \beta & \pi \\ A & B & \sqrt{2} \end{bmatrix} =$$

$$\begin{bmatrix} 1\cdot a + 2\cdot\alpha + 3\cdot A & \sim & \sim \\ \sim & \sim & 4e + 5\pi + 6\sqrt{2} \\ \sim & \sim & \sim \end{bmatrix}$$

Big matrices:
$$\int_a^b f(x,t)\, y(t)\, dt = g(x)$$

$$\sum \underbrace{\Delta t f(x_i, t_j)}_{a_{ij}} \underbrace{y(t_j)}_{x_j = y(t_j)} = \underbrace{g(x_i)}_{b_i}$$

Way cool:
$$\mathbb{A} \underbrace{\left[\vec{x}_1, \ldots, \vec{x}_n\right]}_{\mathbb{A}^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 & & 0 \\ 0 & 1 & & \vdots \\ \vdots & \vdots & & \\ 0 & 0 & & 1 \end{bmatrix}}_{\mathbb{I}}$$

identity matrix

$$\mathbb{A}\mathbb{B} \stackrel{?}{=} \mathbb{B}\mathbb{A}$$ No! (Sometimes)

$$(\mathbb{A}\mathbb{B})\mathbb{C} = \mathbb{A}(\mathbb{B}\mathbb{C})$$ Yes!

$$\mathbb{A}\mathbb{A}^{-1} = \mathbb{A}^{-1}\mathbb{A}$$ Yes! lucky

$$\mathbb{A}^{-1}(\mathbb{A}\vec{x}) = \mathbb{A}^{-1}\vec{b}$$

$$\underbrace{(\mathbb{A}^{-1}\mathbb{A})}_{\mathbb{I}}\vec{x} = \mathbb{A}^{-1}\vec{b}$$

$$\vec{x} = \mathbb{I}\vec{x} = \mathbb{A}^{-1}\vec{b}$$

$$\left[\begin{array}{cc|c} 1-\lambda & 2 & 5 \\ 3 & 4-\lambda & 6 \end{array}\right]$$

$$\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\boxed{(\mathbb{A} - \lambda\mathbb{I})\vec{x} = \vec{b}}$$

$$10\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 20 \\ 30 & 40 \end{pmatrix}$$

$\mathbb{A}^T$ = "transpose of $\mathbb{A}$"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$\underbrace{[a_{ij}]}_{n\times m}^T = \underbrace{[\alpha_{ij}]}_{m\times n} \qquad \alpha_{ij} = a_{ji}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}^T = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

$$A = \begin{bmatrix} \vec{a}_1^{\,T} \\ \vec{a}_2^{\,T} \\ \vdots \\ \vec{a}_n^{\,T} \end{bmatrix} \cdot \begin{bmatrix} \vec{b}_1, \ldots, \vec{b}_m \end{bmatrix} = \underbrace{[c_{ij}]}_{n\times m}$$

$$c_{ij} = \vec{a}_i \cdot \vec{b}_j$$

End 7.2: Linear Transf.

$$T(c_1\vec{x}_1 + c_2\vec{x}_2) = c_1 T\vec{x}_1 + c_2 T\vec{x}_2$$

$$T\left(\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right) = T\left(a_1\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\vec{e}_1} + a_2\underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\vec{e}_2} + a_3\underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{\vec{e}_3}\right)$$

$$= a_1 T\vec{e}_1 + a_2 T\vec{e}_2 + a_3 T\vec{e}_3$$

$$= \underbrace{\left[T\vec{e}_1 \; , \; T\vec{e}_2 \; , \; T\vec{e}_3 \right]}_{\mathbb{B}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

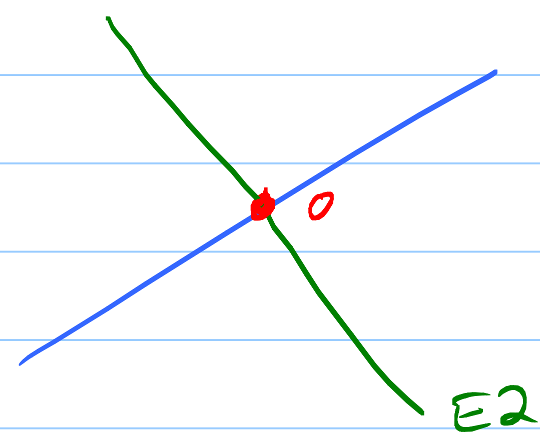
$T\vec{a} = \mathbb{B}\vec{a}$ Aha!

Linear Trans $\sim$ Matrix Mult.

$$T_1 \circ T_2 \sim \mathbb{B}_1 \mathbb{B}_2$$

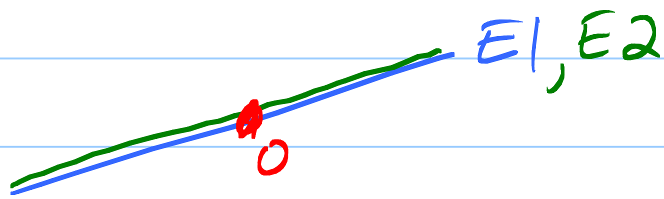
$$\begin{cases} ax + by = A & E1 \\ cx + dy = B & E2 \end{cases}$$

<u>$A=0,\ B=0$</u>



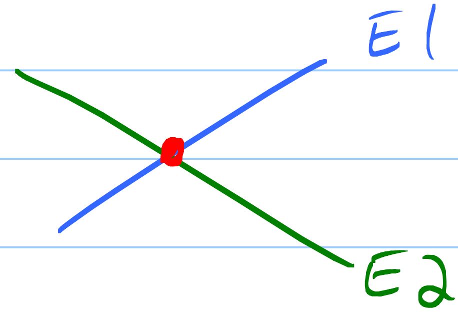
unique sol^n $(x,y)=(0,0)$

<u>or</u>

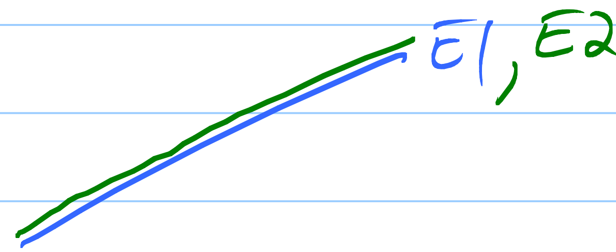


∞ many sol^{n}s

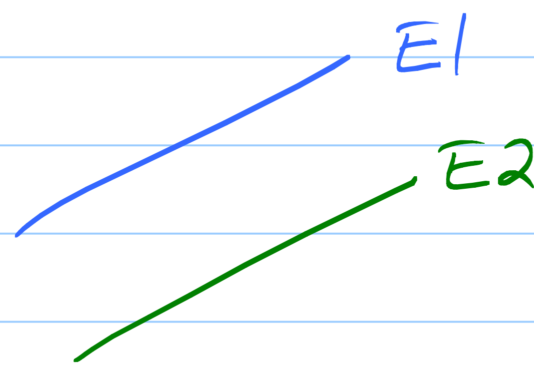
A, B not both zero



unique solⁿ



∞ many sol^{n}s



no sol^{n}s !