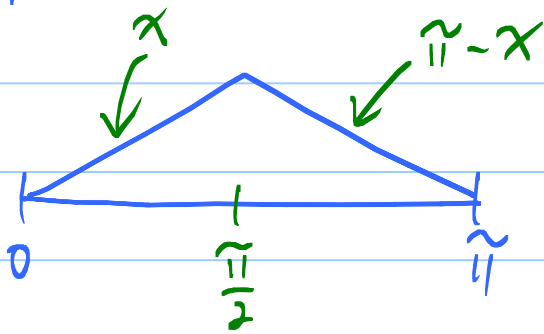
Lesson 26 11.2 Fourier Series, HWK 8 23,24,25 due W.

Harpsichord: $f(x) = \sum_{n=1}^{\infty} c_n \sin nx$ ← Fourier Sine series



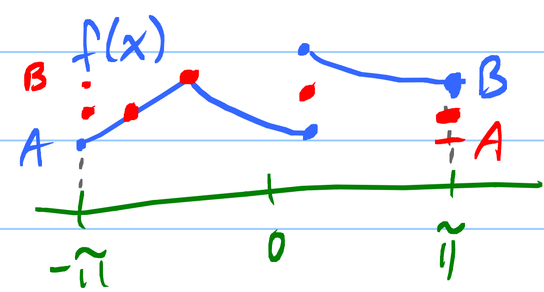
$$c_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$c_n = \frac{2}{\pi}\left[\int_0^{\pi/2} \underset{\uparrow\,u}{x}\, \underbrace{\sin nx\,dx}_{dv} + \int_{\pi/2}^{\pi}(\pi - x)\sin nx\,dx\right]$$

$c_1 = 1,\ c_2 = 0,\ c_3 = -\frac{1}{3^2},\ c_4 = 0,\ c_5 = \frac{1}{5^2},\ c_6 = 0, \ldots$

$$f(x) \overset{?}{=} \sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \frac{1}{7^2}\sin 7x + \cdots$$

Full Fourier Series:



$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

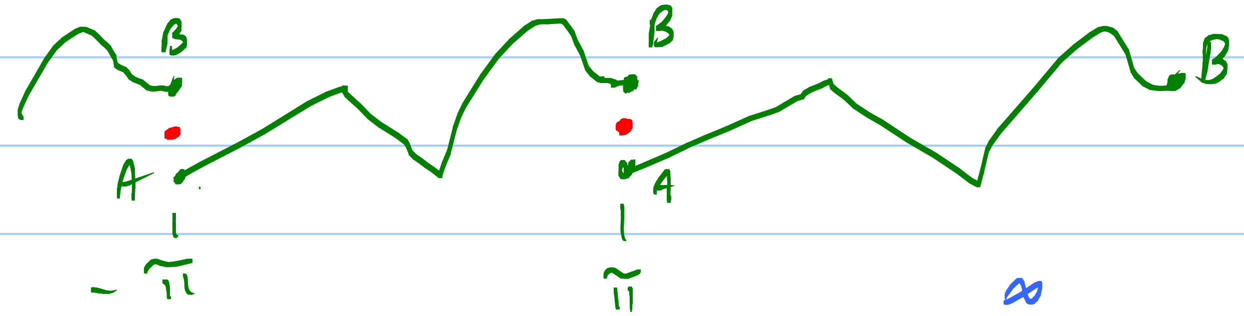
$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \text{Ave of } f \text{ on } [-\pi, \pi]$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

Why: 1, $\cos nx$, $\sin nx$ are orthog on $[\pi,\pi]$

Mind blowing fact: Fourier Series converges to f where f is nice (even at kinks), converges to midpoint of jumps. At $\pm\pi$, converges to $\frac{A+B}{2}$. [Note: every term of Fourier Series is 2π periodic.

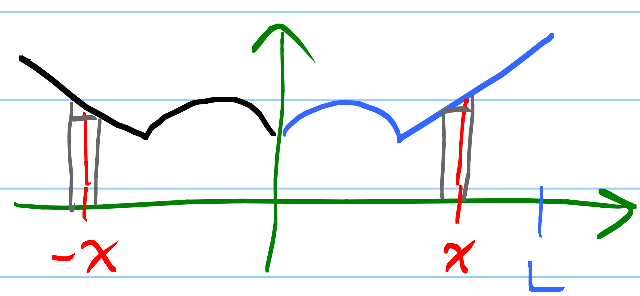


Fourier Series on $[-L,L]$: $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$

a_0, a_n, b_n see book.

Even & Odd fcns:

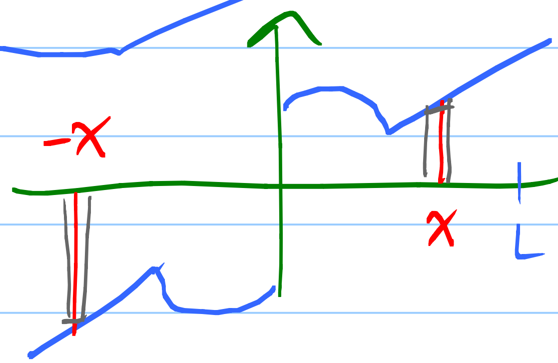
$f(-x) = f(x)$ EVEN



Areas $=$: $\int_{-L}^{0} f(x)\,dx = \int_{0}^{L} f(x)\,dx$

$$\int_{-L}^{L} f(x)\,dx = 2\int_0^L f(x)\,dx$$

$f(-x) = -f(x)$ <u>ODD</u>



$$\int_{-L}^{0} f(x)\,dx = -\int_0^L f(x)\,dx$$

$$\int_{-L}^{L} f(x)\,dx = 0$$

EVEN $f(-x) = f(x)$

ODD $g(-x) = -g(x)$

EVEN · ODD $f(-x)g(-x) = -f(x)g(x)$ $h(x) = f(x)g(x)$

h is odd

<u>Fact</u>: EVEN · ODD = ODD

ODD · ODD = EVEN

EVEN · EVEN = EVEN

<u>Fact</u>: 1, $\cos\frac{n\pi x}{L}$ are all <u>even</u>.

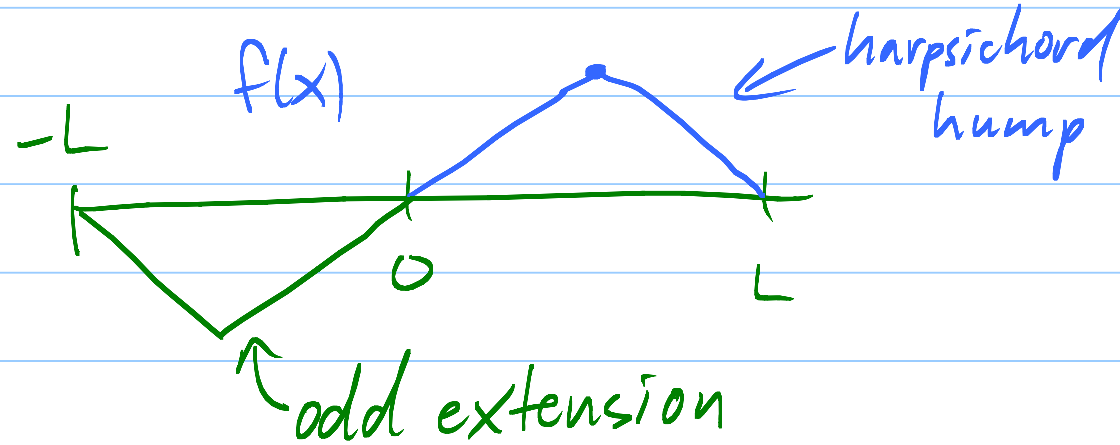
$\sin\frac{n\pi x}{L}$ are <u>odd</u>.

<u>Fact</u>: 1) Odd functions only have <u>Sine</u> terms in Fourier Series. [No even fcns allowed!]

2) Even fcns only have a_0 and cosine terms.

[No odd fcns allowed!]

EX:



$$a_0 = \frac{1}{2L}\int_{-L}^{L} \underset{\text{odd}}{f(x)}\,dx = 0$$

$$a_n = \frac{1}{L}\int_{-L}^{L} \underbrace{\underset{\text{odd}}{f(x)}\ \underset{\text{even}}{\cos\frac{n\pi x}{L}}}_{\text{odd}}\,dx = 0$$

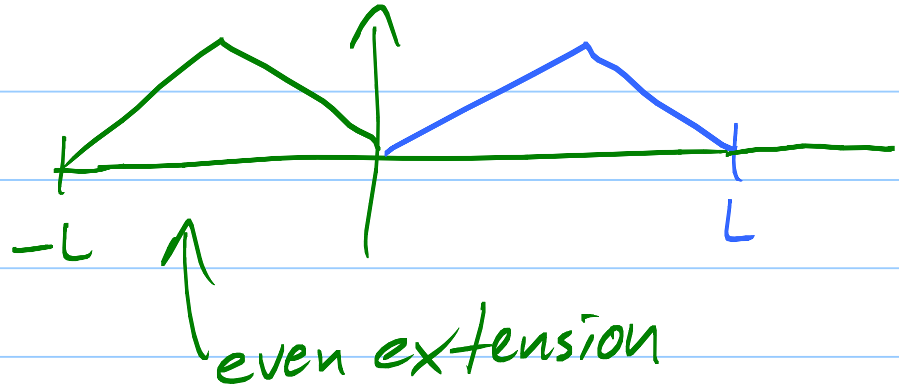
$$b_n = \frac{1}{L}\int_{-L}^{L} \underbrace{\underset{\text{odd}}{f(x)}\ \underset{\text{odd}}{\sin\frac{n\pi x}{L}}}_{\text{even}}\,dx = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Aha! Fourier Sine Series is the Full Fourier Series for the odd extension of f.

Freaky fact: Can expand fcns in F.

Sine or Cosine Series on [0,L]!

Cosine Series:

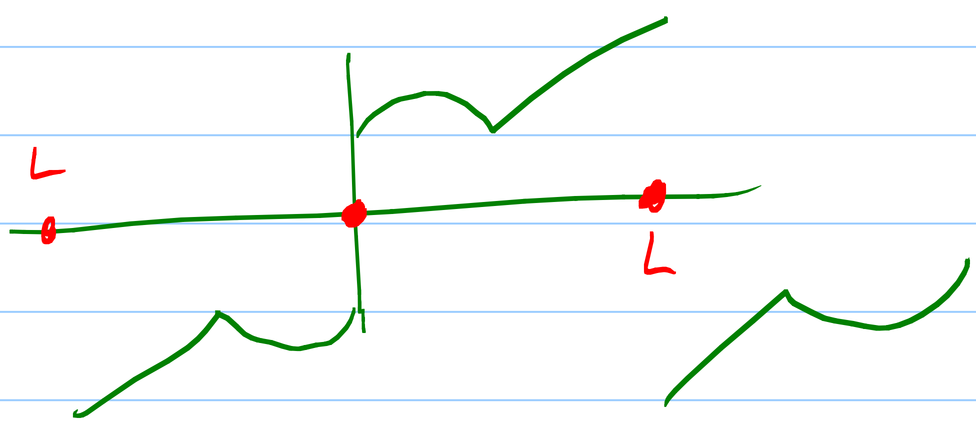


Full Fourier: Get all b_n's are zero!

On [0,L]: $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

Converges even at endpoints, since no jumps.

What about odd extension?



Converges to zero at $\pm L$, 0,