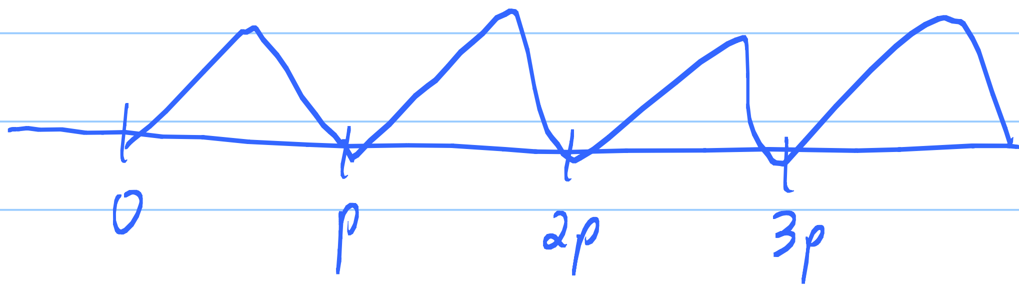
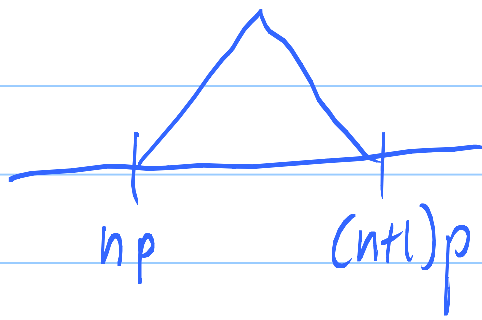
247. 16.



$$\mathcal{L}[f] = \int_0^\infty e^{-st} f(t)\, dt = \sum_{n=0}^{\infty} \int_{np}^{(n+1)p} e^{-st} f(t)\, dt$$



$$\int_{np}^{(n+1)p} e^{-st} f(t)\, dt$$

$\tilde{\tau} = t - np \qquad d\tilde{\tau} = dt$

$t = \tilde{\tau} + np$

when $t = np$, $\tilde{\tau} = 0$

$t = (n+1)p$, $\tilde{\tau} = p$

$$\int_0^p e^{-s(\tilde{\tau}+np)} \underbrace{f(\tilde{\tau}+np)}_{f(\tilde{\tau})}\, d\tilde{\tau}$$

$$= \int_0^p e^{-s\tilde{\tau}} e^{-snp} f(\tilde{\tau})\, d\tilde{\tau} = (e^{-sp})^n \int_0^p e^{-s\tilde{\tau}} f(\tilde{\tau})\, d\tilde{\tau}$$

$$\mathcal{L}[f] = \sum_{n=0}^{\infty} \left(e^{-sp}\right)^n \int_0^p e^{-s\tilde{\tau}} f(\tilde{\tau})\, d\tilde{\tau}$$