# MA 528 Steve Bell

HWK 1 : Lessons 1, 2, 3
due Wed., Jan 20
11:59 pm on Blackboard

www.math.purdue.edu/~bell/MA528
Office Hours: T, W 2-3pm in MATH 750
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|---|---|
| Exm 1 | 100 |
| Exm 2 | 100 |
| HWK | 100 |
| F. Exam | 150 |
| | 450 |

## Lesson 1 : 9.6, 9.7

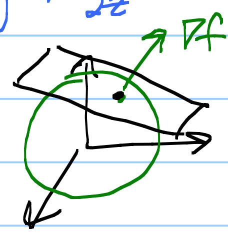
EX: $f(x,y,z) = x^2 + y^2 + z^2$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$f(x,y,z) = c$

level sets

spheres, radius $\sqrt{c}$




Fact: $\nabla f$ is a normal vector $\perp$ tangent plane.

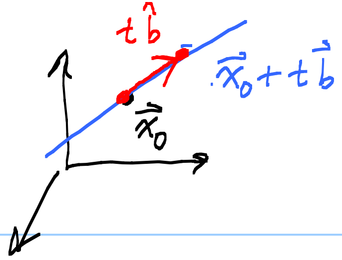
EX: $f(x,y) = e^{x \sin y} + y^2$

$$\begin{cases} \dfrac{\partial f}{\partial x} = e^{x \sin y}(\sin y) + 0 \\ \dfrac{\partial f}{\partial y} = e^{x \sin y}(x \cos y) + 2y \end{cases}$$

so at $(2, \frac{\pi}{4})$, $\nabla f = \frac{\sqrt{2}}{2} e^{\sqrt{2}}\hat{i} + \left(\sqrt{2}e^{\sqrt{2}} + \frac{\pi}{2}\right)\hat{j}$

Directional derivative: $\hat{b}$ a unit vector.

$$D_{\hat{b}} f = \lim_{t \to 0} \frac{f(\vec{x}_0 + t\hat{b}) - f(\vec{x}_0)}{t}$$




$$= \lim_{t\to 0} \frac{f(x_0+tb_1, y_0+tb_2, z_0+tb_3) - f(x_0,y_0,z_0)}{t}$$

$$= \frac{d}{dt} f(\underbrace{x_0+tb_1}_{u}, \underbrace{y_0+tb_2}_{v}, \underbrace{z_0+tb_3}_{w}) \Big|_{t=0}$$

$$= \frac{\partial f}{\partial u}\frac{du}{dt} + \cdots + \frac{\partial f}{\partial w}\frac{dw}{dt}$$

$$= \frac{\partial f}{\partial x}\Big|_{(x_0,y_0,z_0)} \cdot b_1 + \cdots + \frac{\partial f}{\partial z}\Big|_{(x_0,y_0,z_0)} \cdot b_3 = \hat{b}\cdot\nabla f \Big|_{\vec{x}_0}$$

If $\vec{a}$ $(\neq 0)$ is not a unit vector

$$D_{\vec{a}} f = \frac{\vec{a}}{\|\vec{a}\|}\cdot \nabla f$$

Fact: The directional deriv of $f$ is max in $\nabla f$ dir and min in $-\nabla f$ dir.

Why: $D_{\hat{b}} f = \hat{b}\cdot\nabla f = \|\hat{b}\|\cdot\|\nabla f\| \cos\theta$

$\text{Max} = \|\nabla f\|$ when $\theta = 0$, $\hat{b} = \frac{\nabla f}{\|\nabla f\|}$.

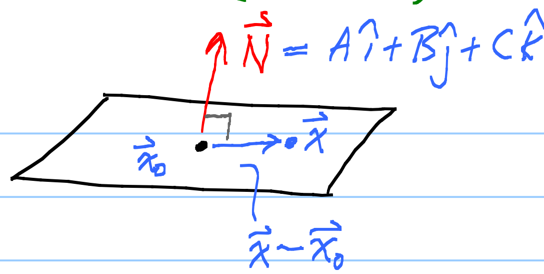
$\text{Min} = -\|\nabla f\|$ $\theta = \pi$, $\hat{b} = -(\quad)$

EX: from above: At $(2, \frac{\pi}{4})$, the unit vector in dir. of largest rate of change of $f$ is

$$\hat{b} = \frac{\nabla f}{\|\nabla f\|} = \frac{\frac{\sqrt{2}}{2}e^{\sqrt{2}}\,\hat{i} + \left(\sqrt{2}e^{\sqrt{2}} + \frac{\pi}{2}\right)\hat{j}}{\sqrt{\frac{5}{2}e^{2\sqrt{2}} + \sqrt{2}\pi e^{\sqrt{2}} + \frac{\pi^2}{4}}}$$

and $D_{\hat{b}} f = \|\nabla f\| = \sqrt{\text{ee}}$ (in denom)

## Finding tangent planes:




$$\vec{N} \cdot (\vec{x} - \vec{x}_0) = 0$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

EX: Let $f(x,y,z) = x^2 + 2y - xe^z = 2$

Find tang plane at $(2,0,0)$.

$$\nabla f = (2x - e^z)\hat{\imath} + 2\hat{\jmath} - xe^z\hat{k}$$

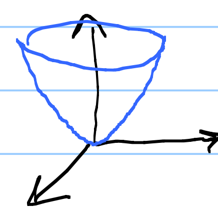
$$\nabla f \Big|_{(2,0,0)} = 3\hat{\imath} + 2\hat{\jmath} - 2\hat{k} = \vec{N}$$

Eqn of tang. plane: $\vec{N} \cdot (\vec{x} - \vec{x}_0) = 0$

$$3(x-2) + 2(y-0) - 2(z-0) = 0$$

Common prob: Surface in $\mathbb{R}^3$ given as a graph:

$z = f(x,y)$. EX: $z = \underbrace{x^2 + y^2}_{r^2}$



Aha! Write $F(x,y,z) = f(x,y) - z = x^2 + y^2 - z$

Then surface is a level set $F(x,y,z) = 0$.

So $\nabla F = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} - 1 \cdot \hat{k} = \vec{N}$

Eqn of tangent plane at $(x_0, y_0, z_0)$ is

$$\boxed{\frac{\partial f}{\partial x}(x_0,y_0,z_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0,y_0,z_0)(y - y_0) - (z - z_0) = 0}$$

EX: $(2,3,13)$ is on $z = x^2 + y^2$

$$\vec{N} = 2x\hat{i} + 2y\hat{j} - \hat{k}\Big|_{(2,3,13)} = 4\hat{i} + 6\hat{j} - \hat{k}$$

Tang. plane = $\boxed{4(x-2) + 6(y-3) - (z-13) = 0}$