Lesson 12 Review for Exam 1 (Lessons 1-11) Tues, Feb. 9, 8-9pm EE-129

One crib sheet: regular sized paper handwritten on both sides. (Do not turn in.)

Office Hours this week: M, T, W 2-3 pm MATH 740 (765-494-1497)

WebEx Office Hour: M 8-9 pm

HWK 4: 9, 10, 11 due Wed, 11:59 pm

Prac Exam: 1. $z = \underbrace{e^{x\cos y} + \frac{1}{x^2+1}}_{f(x,y)}$

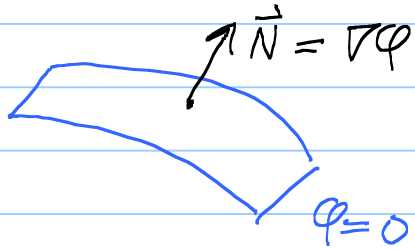
Find normal at $(0,0,2)$.

Could: $\vec{r}(x,y) = x\hat{\imath} + y\hat{\jmath} + f(x,y)k$

$$\vec{N} = \vec{r}_x \times \vec{r}_y \Big|_{(0,0)}$$

Or: $\varphi(x,y,z) = z - e^{x\cos y} - \dfrac{1}{x^2+1}$

S is a level set of φ.



Unit normal

$$\hat{n} = \pm\frac{\vec{N}}{\|\vec{N}\|}$$

Tangent plane: $N_1(x-0) + N_2(y-0) + N_3(z-2) = 0$

2. $\displaystyle\int_C \underbrace{F_1\,dx + F_2\,dy + F_3\,dz}$

Is it exact differential?

$F_1\,dx + F_2\,dy + F_3\,dz$ is exact: $= df$

Same as: $F_1\hat{\imath} + F_2\hat{\jmath} + F_3\hat{k}$ is conservative: $= \nabla f$

Test: Fields: $\vec{F}$. Curl $\vec{F} = 0$

Forms: $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial F_2}{\partial x}$, $\dfrac{\partial F_1}{\partial z} = \dfrac{\partial F_3}{\partial x}$, $\dfrac{\partial F_2}{\partial z} = \dfrac{\partial F_3}{\partial y}$

$\uparrow x_2 \quad \uparrow x_1 \quad \uparrow x_3 \quad \uparrow x_1 \quad \uparrow x_3 \quad \uparrow x_2$

$i \neq j$: $\dfrac{\partial F_i}{\partial x_j} = \dfrac{\partial F_j}{\partial x_i}$

EX: $\overset{A}{(\underbrace{yz+2x}_{\partial f/\partial x})}\hat{i} + \overset{B}{(\underbrace{xz+2y}_{\partial f/\partial y})}\hat{j} + \overset{C}{(\underbrace{xy+2z}_{\partial f/\partial z})}\hat{k}$ is conservative. [So Curl $\vec{F} \equiv 0$.] Find a potential fcn: f with $\nabla f = \vec{F}$

(A): $\dfrac{\partial f}{\partial x} = yz + 2x$

$$f = \int (yz + 2x)\,dx$$

$$= \boxed{x(yz) + x^2 + \underbrace{g(y,z)}_{\text{arbitrary fcn of other vars.}}}$$

(B): $\dfrac{\partial f}{\partial y} = xz + 2y$

$$\frac{\partial}{\partial y}\left[xyz + x^2 + g(y,z)\right] \overset{\text{want}}{=} xz + 2y$$

$$xz + 0 + \frac{\partial g}{\partial y} = xz + 2y$$

$$\frac{\partial g}{\partial y} = 2y$$

So $g(y,z) = \int 2y\,dy = \boxed{y^2 + h(z)}$

So $f = xyz + x^2 + y^2 + h(z)$

(c): $\frac{\partial f}{\partial z} = xy + 2z$

want

$$\frac{\partial}{\partial z}\left[xyz + x^2 + y^2 + h(z) \right] = xy + 2z$$

$$xy + 0 + 0 + h'(z) = xy + 2z$$

$$h'(z) = 2z$$

So $h(z) = z^2 + C$

Finally $f(x,y,z) = xyz + x^2 + y^2 + z^2 + C$

<u>A</u> potential fcn. Take $C=0$.

<u>All</u> potential fcns. Keep the C.

2. ii) Evaluate $\int_{(0,1,2)}^{(1,0,3)} \vec{F} \cdot d\vec{r} = f(1,0,3) - f(0,1,2)$

Fund. Thm. Calc for line integrals:

"Curl free" means $\text{Curl}\,\vec{F} = 0$. So $\int$ is IoP.

<u>Or</u> take $L: \vec{r}(t) = (0,1,2) + t\left[(1,0,3) - (0,1,2)\right]$

$0 \le t \le 1$

and compute $\int_L \vec{F} \cdot d\vec{r}$.

Cylindrical coords: $\iiint_\Omega f \, dV$

$$\int_0^H \int_0^{2\pi} \int_0^R f(r,\theta,z) \underbrace{\underline{r}\,dr\, d\theta\, dz}_{dV}$$

$$\begin{cases} x = u\cos v \\ y = u\sin v \end{cases} \qquad \|\vec{r}_u \times \vec{r}_v\| = u$$

$\uparrow$ = the r in $r\,dr\,d\theta$

Prob: S': $z = 2\sqrt{1-x^2-y^2}$ over $x^2+y^2 \le 1$

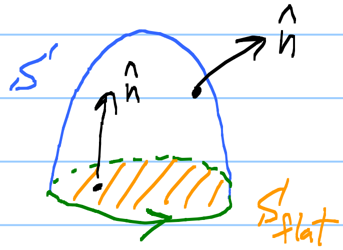
Upward normal.

$$\vec{F} = x^2\,\hat{\imath} + y^2\,\hat{\jmath} + z^2\tan(xy)\,\hat{k}$$

Compute $\displaystyle\iint_{S'} (\text{Curl}\,\vec{F})\cdot\hat{n}\,dA \underset{\text{Stokes}}{=} \int_C \vec{F}\cdot d\vec{r}$

Hmmm: $\text{Curl}\,\vec{F} = \det\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2\tan(xy) \end{bmatrix}$

$$= xz^2\sec^2(xy)\,\hat{\imath} - yz^2\sec^2(xy) + \underset{\text{Aha!}}{0}\,\hat{k}$$



$$\iint_{S'} (\text{Curl}\,\vec{F})\cdot\hat{n}\,dA = \int_C \vec{F}\cdot d\vec{r} = \iint_{S_{flat}} (\text{Curl}\,\vec{F})\cdot\hat{n}\,dA$$

$(\text{Curl}\,\vec{F})$: 0 in $\hat{k}$ component; $\hat{n}$: $\hat{k}$

$$= \iint_{S_{flat}} 0\,dA = 0$$

Prob: Compute $\int_\gamma \underset{F=-y}{-y}\,dx + \underset{G=5x}{5x}\,dy$

where γ bounds a circle of radius 7 centered at $(\sqrt{2}, \sqrt{3})$.

$$\int_\gamma F\,dx + G\,dy = \iint_\Omega \left(\frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \right) dx\,dy$$

$$\begin{cases} x(t) = \sqrt{2} + 7\cos t \\ y(t) = \sqrt{3} + 7\sin t \end{cases} \qquad 0 \le t \le 2\pi$$

Better: $\displaystyle\iint_\Omega \frac{\partial}{\partial x}(5x) - \frac{\partial}{\partial y}(-y)\,dx\,dy$

$$= 6 \iint_\Omega 1\,dA = 6\,(\text{Area})$$

$$= 6 \cdot \pi \cdot 7^2$$

Prob: Show that the potential fcn for an electrostatic field is harmonic on regions with no charge.

Fact: $\displaystyle\iint_S \vec{E} \cdot \hat{n}\,dA = c(\text{net charge inside } S)$

Region with no charge $\displaystyle\iint_S \vec{E} \cdot \hat{n}\,dA = 0$ $\quad \vec{E} = \nabla\varphi$

Divergence Thm $\displaystyle\iiint_\Omega \big(\underbrace{\text{Div}\,\nabla\varphi}_{\Delta\varphi}\big)\,dV = 0$

If $(\Delta\varphi)(\vec{x}_0) \neq 0$, take $\Omega = B_\varepsilon(\vec{x}_0)$.
For small $\varepsilon > 0$, $\displaystyle\iiint_{B_\varepsilon} \Delta\varphi\,dV \neq 0$ $\lightning$.