Lesson 27 Review for Exam 2 : Tues., March 22, 8-9 pm in EE 129

Office Hours : M, T 2-3 pm in MATH 750 (765-494-1497)
WebEx Office Hour: M 8-9 pm

Exam 2 : Closed books, no notes, no calculators, no phones or tablets
Crib sheet: One regular sized sheet handwritten on both sides.

Chap 14: Analytic fcns.

C-R Equs: Ex: Is $e^{\bar{z}}$ analytic ?

$$e^{\bar{z}} = e^{x-iy} = e^{x} e^{-iy} = e^{x}\cos(-y) + i e^{x}\sin(-y)$$
$$= \underbrace{e^{x}\cos y}_{u} + i\underbrace{(-e^{x}\sin y)}_{v}$$

C-R Equs: $u_x = v_y$ fail! Not analytic.
$u_y = -v_x$

Finding harmonic conjugates: Given a harmonic fcn u, find v such that $f(x+iy) = u(x,y) + i v(x,y)$ is analytic.

Important fcns: e^{z}, $\operatorname{Log} z$, $\sin z$, $\cos z$

principal $\operatorname{Log} z = \ln|z| + i \operatorname{Arg} z$

$-\pi < \operatorname{Arg} z \le \pi$

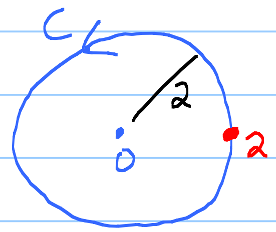
$$\frac{d}{dz} e^{kz} = k e^{kz}$$

$$\frac{d}{dz} \operatorname{Log} z = \frac{1}{z} \leftarrow \text{true for any branch of log.}$$

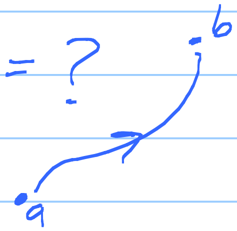
Fundamental Theorem of Calculus :

$$\int_{\gamma_a^b} F' \, dz = F(b) - F(a) \leftarrow$$ F analytic on open set containing γ_a^b

EX:



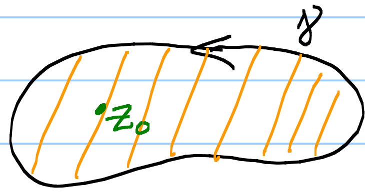
$$\int_C \frac{1}{(z+1)^2}\, dz = ?$$



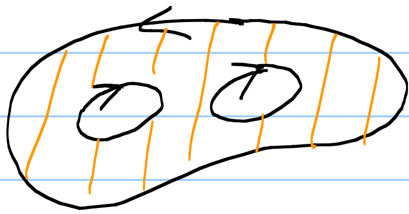
$$\int_C = \int_C \frac{d}{dz}\left[\frac{1}{-2+1}(z+1)^{-2+1}\right] dz$$

$$= \int_C \frac{d}{dz}\underbrace{\left[\frac{-1}{z+1}\right]}_{F(z)} dz = F(\text{END}) - F(\text{START}) = 0$$

<u>Cauchy Theorem:</u>



$$\int_\gamma f\,dz = 0$$



$$\int_\gamma f\,dz = 0$$

<u>Cauchy Integral Formulas:</u>

$$f(z_0) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-z_0}\,dz$$

$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\int_\gamma \frac{f(z)}{(z-z_0)^{n+1}}\,dz$$

$f(z) \equiv 1$

$$\int_{C_2} \frac{1}{(z+1)^2}\,dz = \frac{2\pi i}{1!}\underbrace{f'(-1)}_{=0} = 0$$

$-(-1)$ $\quad 2 = n+1 \quad \boxed{n=1}$

$z_0 = -1$

Radius of Convergence via Ratio Test

Geometric Series: $\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$

$R = 1.$

<u>Facts:</u> R of C of series for f' has same R of C as series for f.

<u>Fact:</u> Multiplying series by z^k does not change R of C.