# Lesson 43 Review 1

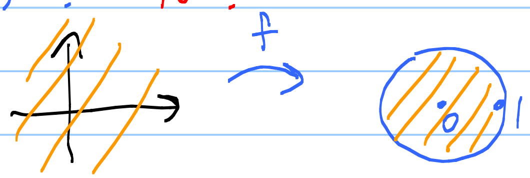


HWK 13: Lessons 39, 40, 41 due tonight
Lesson 42: Read 18.5 (no homework)

Regular Office Hourss next week
WebEx Office Hour next Wed. 5/4, 8-9 pm

Final Exam: Thurs, May 5
1:00-3:00 pm in PHYS 112
15 problems (multiple choice)
A-E, 10 points. 2 hrs.

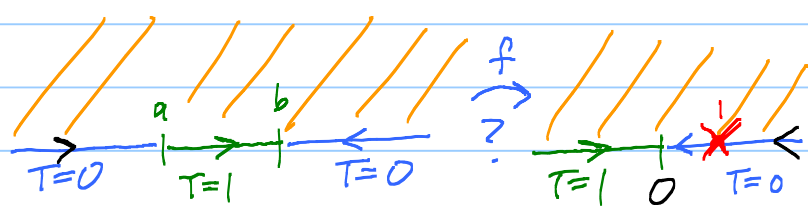
Prob: Is there a conformal ← analytic mapping 1-1 from $\mathbb{C}$ onto $D_1(0)$? No!



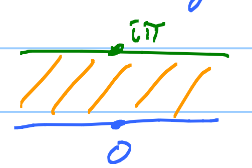

If we had one, it would be a bounded entire function, so Liouville's $\Rightarrow f \equiv$ const.

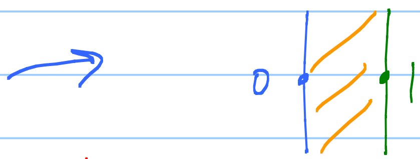
18.3: 9.

Sol'n →
Re $F(z)$



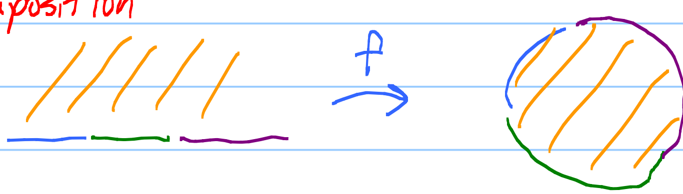

$$f(z) = \frac{z-b}{z-a}$$



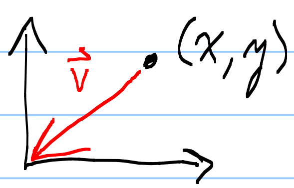

← sol'n $u = x = \operatorname{Re} z$

F composition




Use Poisson Formula!

1. $\nabla f$ points in dir of fastest increase
$-\nabla f$ " " " " decrease




$\vec{v} = -x\hat{\imath} - y\hat{\jmath}$

Direction: $\hat{u} = \vec{v}/\|\vec{v}\|$

$\hat{u} = \dfrac{-x\hat{i} - y\hat{j}}{\sqrt{x^2+y^2}}$ Ans: $D_{\hat{u}} = \nabla f \cdot \hat{u}$

3. $\varphi(x,y) = 1 - 2\cos(\pi x y) = 0$

level set

$\nabla\varphi$ is $\perp$ to level set.

Ans: $\vec{N} = \nabla\varphi$ at $\left(\frac{1}{6}, 2\right)$

So is $c\vec{N}$ where $c \neq 0$.

Upward: $(\ell\ell\ell)\,\hat{i} + (\text{positive})\,\hat{j}$

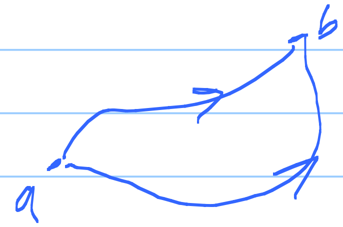
Downward: $(\ell\ell\ell)\,\hat{i} + (\text{negative})\,\hat{j}$

$\nabla\varphi$ is $\perp$ to level sets $\varphi \equiv c$.

4. Tangent plane: $\vec{N} \cdot (\vec{x} - \vec{p}_0) = 0$

$\nabla\varphi\big|_{(1,1,-1)}$ $(1,1,-1)$

6. 1. Conservative $\Longleftrightarrow$ 2. $\int_c = 0$



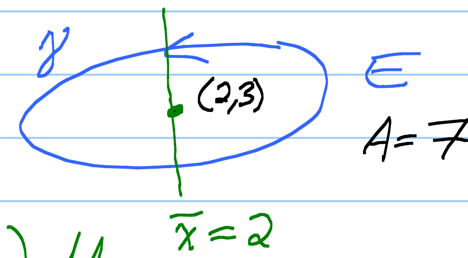

Need Curl $\vec{F} = 0$

(3) $\vec{F} = \nabla\varphi$, $\varphi$ potential

$(1)$: $\displaystyle\int_C F\,dx + G\,dy = \int_C \vec{F}\cdot d\vec{r}$

All True: Curl $\vec{F} = 0$. $\qquad \vec{F} = F\hat{i} + G\hat{j}$

Exam 1: 3. $\displaystyle\int_\gamma \underbrace{-xy}_{P}\,dx + \underset{Q}{x^2}\,dy$



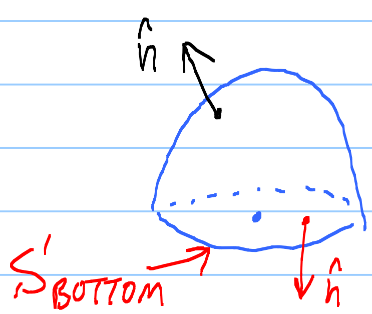

Green's! $\displaystyle\int_\gamma P\,dx + Q\,dy = \iint_E \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

$\displaystyle = \iint_E 2x - (-x)\,dA = 3\iint_E x\,dA$

$\displaystyle \bar{x} = \frac{\iint_E x\,dA}{\left(\iint_E dA\right) = A = 7}$ $\qquad$ Ans: $3\cdot\bar{x}\cdot A$

$3\cdot 2\cdot 7 = 42$

5. $\vec{F} = P_1(y)\hat{i} + P_2(z)\hat{j} + z\hat{k}$




$S'$: $z = 4 - x^2 - y^2$

Want $\displaystyle\iint_{S'} \vec{F}\cdot\hat{n}\,dA$

Div $\vec{F} = 0 + 0 + 1 \equiv 1 \qquad = 0$

Divergence Theorem $\displaystyle\iint_S \vec{F}\cdot\hat{n}\,dA + \cancel{\iint_{S'_{BOTTOM}} \vec{F}\cdot\hat{n}\,dA} = \iiint_V \operatorname{div}\vec{F}\,dV$

$\displaystyle = \iiint_V 1\,dV$

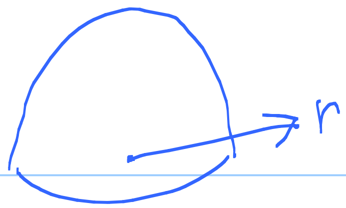
$=$ Volume

$\vec{F} = (ee)\hat{i} + (mm)\hat{j} + z\hat{k}$

$z = 0$ on bottom

Bottom: $\hat{n} = -1\cdot\hat{k}$

$\vec{F}\cdot\hat{n} = 0$ on $S_{BOTTOM}$




$z = 4 - x^2 - y^2 = 4 - r^2$

$$V = \int_0^{2\pi} \int_0^{2} (4 - r^2)\, r\, dr\, d\theta$$

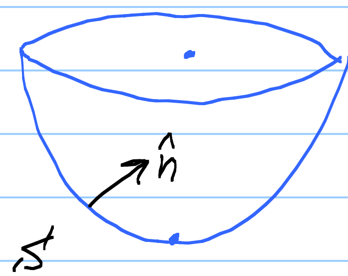
$$= 8\pi$$

7. $\displaystyle \int_C \vec{F} \cdot d\vec{r} = \int_C F_1\, dx + F_2\, dy$

$\vec{F} = F_1 \hat{\imath} + F_2 \hat{\jmath}$

Green's !

8. $z = x^2 + y^2 = r^2$




$$\iint_S \vec{F} \cdot \hat{n}\, dA$$

$$\left( - \iint_S \right) + \iint_{S_{top}} = \iiint_V \mathrm{Div}\, \vec{F}\, dV$$