Lesson 44 Review 2

Final Exam: Thurs., May 5
1:00-3:00 pm in PHYS 112

Office Hours: T, W 2:00-3:00 pm
in MATH 750 (765-494-1497)
WebEx Office Hour: Wed, 8:00-9:00 pm

15.1: 20. $\displaystyle\sum_{n=0}^{\infty} \frac{n+i}{3n^2-2i} \cdot \frac{3n^2+2i}{3n^2+2i}$

$\displaystyle\sum_{1}^{\infty} \frac{1}{n} = \infty$

$$= \sum \left[\underbrace{\frac{3n^3-2}{9n^4+4}}_{\sim \frac{1}{n} \text{ bad!}} + \underbrace{\frac{3n^2+2n}{9n^4+4}\, i}_{\sim \frac{1}{n^2} \text{ nice!}} \right]$$

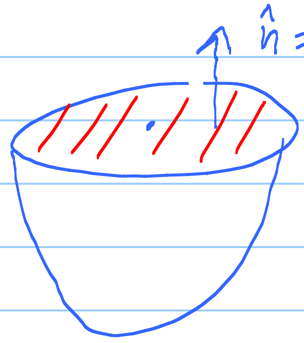
Limit Comparison Test: Compare $\frac{1}{n}$. $\quad \dfrac{a_n}{\left(\frac{1}{n}\right)} \to L > 0$

$9n^4 + 4 < 10n^4$ if $n \geq 2$

$$\frac{3n^3-2}{9n^4+4} > \frac{3n^3-2}{10n^4} = \underbrace{\frac{3}{10}\frac{1}{n}}_{\text{bad!}} - \underbrace{\frac{1}{5n^4}}_{\text{Conv.}}$$

8.

$z=4$

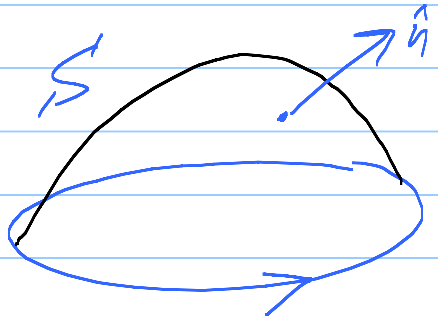


$\vec{F} \cdot \hat{n} = 4 \cdot 1$

$$\iint_{Top} \vec{F} \cdot \hat{n}\, dA = (4) \underbrace{\text{Area}}_{\pi 2^2}$$

Use Divergence Thm.

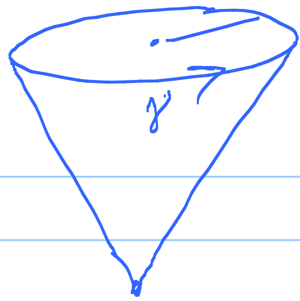
10.



$$\int_{\gamma} \vec{F} \cdot d\vec{r} = \iint_{S} \text{Curl}\, \vec{F} \cdot \hat{n}\, dA$$

Stokes' Thm.

$z = 3$



Cone

$$= \int_\gamma \vec{F} \cdot d\vec{r}$$

$$\gamma: \ \vec{r}(t) = 3\cos t \,\hat{i} + 3\sin t \,\hat{j} + 3\hat{k}$$

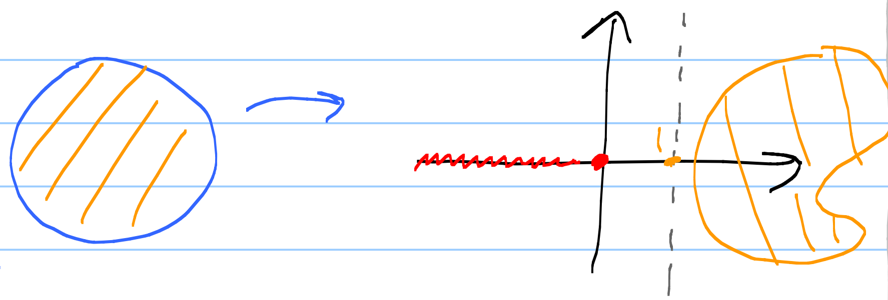
$$0 \le t \le 2\pi$$

12. Cauchy Int. Form. OR Residue Thm.

$$\operatorname{Res}_a \frac{f}{g} = \frac{f(a)}{g'(a)} \leftarrow$$ g simple zero at a, f analytic

$$\int_C = 2\pi i \sum \operatorname{Res} = 2\pi i \cdot \frac{e^{2(2-2)} \cos \pi 2}{1}$$

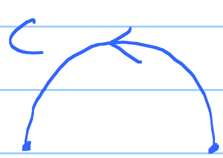
13.



1. ✓
2. $\bar{z}$ bad. $f = u + iv$, $\bar{f} = u - iv$ → oops! $\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$ ← minus v's botch this up.
3. ✓
4. ✓
5. Real valued analytic fcn ≡ const. (C-R Eqns)

14. $z^2 = (x^2 - y^2) + 2xyi$

15.



$C: \underbrace{1 \cdot e^{it}}_{z(t)}$ $\quad 0 \le t \le \pi$

$z'(t) = i e^{it}$

$$\int_0^{\pi} \underbrace{\overline{\left(e^{it}\right)}}_{f(z(t))} \underbrace{\left[i e^{it}\, dt \right]}_{z'(t)\, dt} = \int_0^{\pi} i\, dt = i\pi \qquad f(z) = \bar{z}$$

Facts: $\quad \underbrace{\overline{e^{it}}}_{\cos t + i \sin t} = e^{-it} = \dfrac{1}{e^{it}}$

$$\cos t - i \sin t = \cos(-t) + i \sin(-t)$$

16. $$f^{(n)}(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}}\, dz$$

$$(z-5)^2 \leftarrow n = 1, \quad a = 5$$

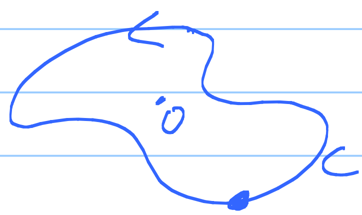
or Residue Thm. $$\frac{\sin z^2}{(z-5)^2} = \frac{A_0 + A_1(z-5) + A_2(z-5)^2 + \cdots}{(z-5)^2}$$

$$= \frac{A_0}{(z-5)^2} + \frac{A_1}{(z-5)} + A_2 + \cdots$$

$A_0 \neq 0$ Pole of order 2

Taylor's: $$A_1 = \left. \frac{\frac{d}{dz}\left[\sin z^2\right]}{1!} \right|_{z=5}$$

17.

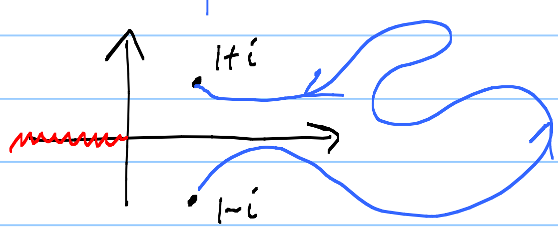


$$\int_C \frac{1}{z^2}\, dz = 0$$

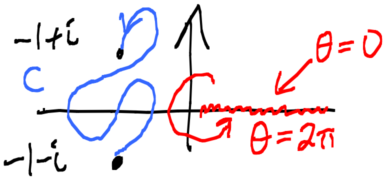
$$\int_C \left[\frac{d}{dz}\left(\frac{-1}{z} \right) \right] dz$$

$$\left. \left(\frac{-1}{z} \right) \right|_{START}^{END} = 0$$

Ans: $\left[-\frac{1}{z} \right]_1^{-1} = 2$



$$\int_\gamma \frac{1}{z}\, dz = \left. \operatorname{Log} z \right|_{1-i}^{1+i}$$



$$\log z = \text{Ln}|z| + i\theta$$

$$\theta \in \arg z \qquad 0 < \theta < 2\pi$$

$$\int_C \frac{1}{z}\,dz = \log z \Big|_{-1-i}^{-1+i}$$

19. $$\frac{1}{(z^2+1)^3} = \frac{1}{\left[(z-i)(z+i)\right]^3}$$

$$= \frac{1}{(z-i)^3}\left[\underbrace{1/(z+i)^3}_{A_0 + A_1(z-i) + A_2(z-i)^2 + \cdots}\right]$$

$$= \frac{A_0}{(z-i)^3} + \cdots \frac{A_2}{(z-i)} + A_3 + \cdots$$

$$A_2 = \frac{\frac{d^2}{dz^2}\left[\frac{1}{(z+i)^3}\right]}{2!}\Bigg|_{z=i}$$

20. $$\int_{-\infty}^{\infty} \sin x\,(\text{Rational})\,dx = \text{Im}\int_{-\infty}^{\infty} e^{ix}\,(R)\,dx$$

$$= 2\pi i \sum \text{Res}\; e^{iz} R(z)$$

$$\frac{1}{(\cancel{z}^2+1)}\left[\frac{\cancel{z}\,e^{iz}}{z^2+4}\right], \qquad \frac{f}{g}$$