Practice Problems for Exam 2

1. (20)   i) Determine all values of $|(i-1)^i|$.

Answer:

ii) Determine all values z such that the real part of $\cos z$ is 0. (Also draw a graph of the solution set.)

Answer:

2. (20)   Find a harmonic conjugate of $y + e^x \cos y$.

Answer:

3. (20)   i) Let Γ be the circle of radius 1 centered at the origin, and traversed once in the counterclockwise direction. Evaluate

$$\int_\Gamma \frac{(e^{z^3} + e^{|z|})}{z}\,dz.$$

ii) Let L be the line segment from $3 + 3i$ to $1 + i$. Writing your answer in $a + bi$ form, evaluate

$$\int_L \bar{z}\,dz.$$

3

4. (20)   i) Let Γ be the ellipse $x^2/4+y^2 = 1$ traversed once in the counterclockwise direction. Evaluate

$$\int_\Gamma \frac{\sin(\pi z^2)}{z(z+1)^2} dz.$$

Answer:

4

5. (20) Find the radii of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{2^n}{n!}(z-1)^n$$

Answer:

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}(3+4i)^n z^{2n}$$

Answer:

Problem 1:

a) Find the principal value of $(-2)^{(-i)}$. Express your answer in the form $x + iy$. *[10 points]*

b) Determine all values of z such that $sin(z) = 3$. *[10 points]*

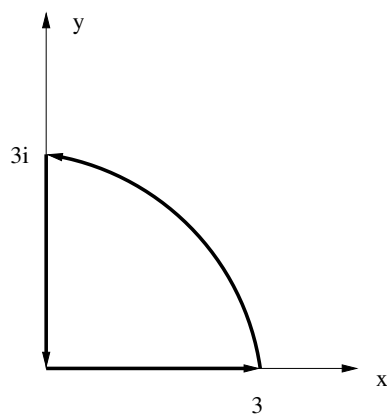**Problem 2:**

a) For which values of a is the function
$$u(x, y) = e^{\pi x} \cos(ay)$$
harmonic? Here a is supposed to be real **positive** parameter.
[8 points]

b) For the choice of the parameter a for which $u(x, y)$ from part a) is harmonic, find a harmonic conjugate $v(x, y)$ of $u(x, y)$, i.e. find a harmonic function $v(x, y)$ such that $f(x, y) = u(x, y) + iv(x, y)$ is analytic.
[12 points]

Problem 3:

Calculate

$$\int_L |z| \, dz$$

on the closed contour L starting at $z = 0$, then going along the real axis to $z = 3$, then following a quarter of a circle with radius 3 to $z = 3i$ and returning to $z = 0$ along the imaginary axis. (see figure above). Is Cauchy's integral theorem valid in this case? Why or why not?
[20 points]

Problem 4: Let C be the circle with radius 5 centered at $z = i$, oriented counterclockwise. Compute

$$\oint_C \frac{z^2}{(z-1)^2(z-i)}dz$$

Please show the details of your work. *[20 points]*

Problem 5:

a) Find the radius of convergence of the following series. Show the details of your work. *[10 points]*

$$\sum_{n=0}^{\infty} \left(\frac{4-i}{5-2i} \right)^n \frac{n}{2n+1} (z-i)^n$$

b) Is the following series convergent or divergent? Give a reason for your answer. *[10 points]*

$$\sum_{n=0}^{\infty} \frac{(3+i)^{(2n+1)}}{(2n)!}$$