NAME ______________________________ **SHOW YOUR WORK!**

1. Let $\vec{F} = \langle\, 3xy,\ -1,\ 1/(z+1)\,\rangle$ be a force field.

(15) (i) Compute the work done in displacement along the helix given by $\vec{r}(t) = \langle\, \sin t, \cos t, t\,\rangle$ for $0 \le t \le \pi/2$.

Answer:

(15) (ii) Compute the work done by the same force $\vec{F}$ in displacement along the line from (1,2,3) to (2,3,3).

Answer:

2. Let $P = (0, 1, 0)$ be a point on a surface S given by $y^4 z = \ln(x + y)$.

(15) (i) Find the upward unit normal $\vec{n}$ to S at P.

Answer:

(10) (ii) Find the equation of the tangent plane to S at P.

Answer:

3. Let $\vec{F} = \langle\, x,\, y,\, z \,\rangle$ and T be the solid region given by $x^2 + y^2 \leq z \leq 2$.

(15) (i) Without using the divergence theorem, compute the surface integral $\iint_S \vec{F} \cdot \vec{n} dA$, where S is the entire boundary of T, and $\vec{n}$ is the outward unit normal. Note that S is the sum of two smooth surfaces.

Answer:

(15) (ii) Using the divergence theorem, set up but do not evaluate, the volume integral corresponding to (i).

$$\int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \; dz\, dr\, d\theta$$

4. (15) Let $\vec{F} = \langle y, 0, 2 \rangle$, and S be the surface given by $z = x + y$ for $0 \le x \le 1$ and $0 \le y \le 1$. Let Cbe the curve bounding S and traversed in the positive direction with respect to the upward unit normal $\vec{n}$ to S. Using Stokes's theorem, compute the line integral $\int_C \vec{F} \cdot d\vec{r}$. (You do not have to compute the integral two ways; only by the area integral in Stokes's theorem).

Answer: