MA 528 PRACTICE PROBLEMS

1. Find the directional derivative of $f(x,y) = 5 - 4x^2 - 3y$ at (x,y) towards the origin

   a. $-8x-3$ b. $\dfrac{-8x^2-3y}{\sqrt{x^2+y^2}}$ c. $\dfrac{-8x-3}{\sqrt{64x^2+9}}$ d. $8x^2+3y$ e. $\dfrac{8x^2+3y}{\sqrt{x^2+y^2}}$.

2. Find a vector pointing in the direction in which $f(x,y,z) = 3xy - 9xz^2 + y$ increases most rapidly at the point $(1,1,0)$.

   a. $3\mathbf{i}+4\mathbf{j}$ b. $\mathbf{i}+\mathbf{j}$ c. $4\mathbf{i}-3\mathbf{j}$ d. $2\mathbf{i}+\mathbf{k}$ e. $-\mathbf{i}+\mathbf{j}$.

3. Find a vector that is normal to the graph of the equation $2\cos(\pi xy) = 1$ at the point $(\frac{1}{6}, 2)$.

   a. $6\mathbf{i}+\mathbf{j}$ b. $-\sqrt{3}\,\mathbf{i}-\mathbf{j}$ c. $12\mathbf{i}+\mathbf{j}$ d. $\mathbf{j}$ e. $12\mathbf{i}-\mathbf{j}$.

4. Find an equation of the tangent plane to the surface $x^2+2y^2+3z^2 = 6$ at the point $(1,1,-1)$.

   a. $-x+2y+3z=2$ b. $2x+4y-6z=6$ c. $x-2y+3z=-4$
   d. $2x+4y-6z=6$ e. $x+2y-3z=6$.

5. Find an equation of the plane tangent to the graph of $z = \pi + \sin(\pi x^2 + 2y)$ when $(x,y) = (2,\pi)$.

   a. $4\pi x+2y-z=9\pi$ b. $4x+2\pi y-z=10\pi$ c. $4\pi x+2\pi y+z=10\pi$
   d. $4x+2\pi y-z=9\pi$ e. $4\pi x+2y+z=9\pi$.

6. Are the following statements true or false?
   1. The line integral $\int_C (x^3+2xy)dx + (x^2-y^2)dy$ is independent of path in the xy-plane.
   2. $\int_C (x^3+2xy)dx + (x^2-y^2)dy = 0$ for every closed oriented curve C in the xy-plane.
   3. There is a function $f(x,y)$ defined in the xy-plane, such that
      grad $f(x,y) = (x^3+2xy)\mathbf{i} + (x^2-y^2)\mathbf{j}$.

   a. all three are false b. 1 and 2 are false, 3 is true c. 1 and 2 are true, 3 is false
   d. 1 is true, 2 and 3 are false e. all three are true

7. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x, y) = (xy^2 - 1)\mathbf{i} + (x^2y - x)\mathbf{j}$ and C is the circle of radius 1 centered at $(1, 2)$ and oriented counterclockwise.

a. 2 b. π c. 0 d. $-\pi$ e. -2

8. If S is the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$, $\mathbf{n}$ is the unit normal vector on S directed upward, and $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, then $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA =$

a. 0 b. 8π c. 4π d. -4π e. -8π

9. Use Green's theorem to evaluate $\displaystyle\int_C \cos x \cos y dx + (2x - \sin x \sin y)dy$ where C is the circle $x^2 + y^2 = 9$ oriented counterclockwise.

a. 0
b. 18
c. 18π
d. 15π
e. 16π

10. Let S be the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 3$, and $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$. If $\mathbf{n}$ is the upward unit normal on S, $\iint_S (\text{curl}\, \mathbf{F}) \cdot \mathbf{n} dA =$

a. 0
b. -18π
c. 18π
d. $9\pi/\sqrt{2}$
e. $18\pi/\sqrt{2}$

11. Consider the following statements, where z and w are complex numbers, $w \neq 0$.
1. $\overline{z + w} = \overline{z} + \overline{w}$
2. $|z| = |\overline{z}|$
3. $|zw| = |z||w|$
4. $|z + w| = |z| + |w|$
5. $\dfrac{z}{w} = \dfrac{z\overline{w}}{|w|^2}$

Which of these statements is true for all possible values of z and $w \neq 0$?

a. 1, 2, and 3 only
b. 1, 2, 3, and 5 only
c. 1, 3, and 4 only
d. 2, 3, and 5 only
e. all of the statements

12. Evaluate $\int_C \frac{e^{2(z-2)}\cos \pi z}{z-2} dz$, where C is the circle with center i and radius 4.

a. 0
b. 1
c. $2\pi i$
d. e^{-4}
e. $2\pi i e^{-4}$

13. Suppose that f is analytic and nonzero in the unit disk, $|z| < 1$ and that $\text{Re}(f(z)) > 1$ for all z in the unit disk. Consider the following functions.

1. $f^2(z)$
2. $\overline{f(z)}$
3. $1/f(z)$
4. $\text{Log}(f(z))$
5. $|f(z)|$

Which of the following choices best describes which of these functions are analytic in the unit disk?

a. 1 and 3 only
b. 1, 3, 5 only
c. 1, 2, 3, only
d. 1, 3, 4, 5 only
e. Some other combination not listed above.

14. Suppose $u(x,y) = x^2 - y^2$ is the real part of an analytic function, f. Which of the following functions could be the imaginary part of f?

(a) $x^2 + y^2$
(b) $2xy + 2x$
(c) $2x - 2y$
(d) $2xy$
(e) None of the above.

15. Evaluate $\int_C \overline{z} dz$, where C is the upper half of the unit circle, traversed counterclockwise.

(a) 0
(b) π
(c) $-2i$
(d) -2
(e) πi

16. Evaluate $\int_C (\sin(z^2)/(z-5)^2) dz$, where C is the circle $|z - i| = 10$, traversed once counterclockwise.

(a) 0
(b) $\cos 25$
(c) $2\pi i \cos 25$
(d) $20\pi i \cos 25$
(e) None of the above.

17. Evaluate $\int_C (1/z^2)dz$, where C is the line from 1 to $1+5i$ followed by the line from $1+5i$ to $-1+5i$ followed by the line from $-1+5i$ to -1.

(a) 0

(b) 2

(c) -2

(d) πi

(e) None of the above.

18. Which of the following integrals is not equal to zero ?

a) $\int_{|z|=10} ze^{e^z}dz$

b) $\int_{|z-2|=\frac{1}{5}} (z-2)e^{1/z^2}dz$

c) $\displaystyle\int_{|z|=1} \frac{\cos z}{z^2}dz$

d) $\displaystyle\int_{|z|=1} \frac{\cos z}{z}dz$

e) $\displaystyle\int_{|z-1|=1} \frac{1}{(z-1)^3}dz$

19. Which of the numbers below is the value of $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3}dx$?

a) 0

b) $3\pi/4$

c) $3\pi/8$

d) $-3\pi/8$

e) $3\pi/4$

20. Which of the numbers below is the value of $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin x}{(x^2+1)(x^2+4)}dx$?

a) 0

b) $3\pi(e^2-1)$

c) $\dfrac{\pi}{3}(1-1/e)$

d) $\dfrac{\pi}{3e^2}(1-1/e)$

e) $\dfrac{\pi}{3e^2}(e-1)$

21. Suppose f has an isolated singularity at $z=0$ and $\lim_{z\to 0} z^3 f(z) = 0$. Which of the following statements **cannot** be true ?

a) f has a removable singularity at 0.

b) f is analytic at 0 and has a zero of order 2.

c) f has an essential singularity at 0.

d) f has a pole of order 2 at 0.

e) f has a pole of order 1 at 0.

22. Let $f(z) = z^2e^{1/z}$ and $g(z) = z^{-2}e^z$. Which of the following statements is true?

a) Both f and g have a removable singularity at ∞.

b) Both f and g have an essential singularity at ∞.

c) Both f and g have a pole at ∞.

d) f has a pole at ∞ and g has an essential singularity at ∞.

e) f has an essential singularity at ∞ and g has a pole at ∞.

23. The radius of convergence of the power series $\sum_{n=1}^{\infty} \sin(ni)\, z^n$ is

a) $1/e$
b) $2/(\pi e)$
c) π/e
d) $+\infty$
e) e

24. Let $f(z) = \sum_{n=-\infty}^{-1} z^n + \sum_{n=0}^{\infty} (\frac{z}{2})^n$. Then f converges precisely in which of the following regions.

a) $|z| > 2$
b) $|z| < \frac{1}{2}$
c) $1 < |z| < 2$
d) $\frac{1}{2} < |z| < 1$
e) $0 < |z| < 2$

25. Let $f(z) = \dfrac{e^z}{z^3 - z^2}$. Then the sum of the residues of f at its poles is equal to

a) 0
b) $e - 2$
c) $e + 2$
d) $(2e - 4)\pi i$
e) $(2e + 4)\pi i$

26. Let $f(z) = \sin z$. Then the image under f of a horizontal line $z = t + ic, \quad (c > 0,\ 0 \le t \le 2\pi)$ is

a) an ellipse
b) a circle
c) a line
d) a hyperbola
e) a parabola

27. If $T(z)$ is the linear fractional transformation such that $T(i) = 0,\ T(1) = 1,\ T(-1) = \infty$, then $T(-i) =$

a) $2/(1 - i)$
b) $2i/(1 - i)$
c) $(1 - i)$
d) 2
e) i

28. If $\mathbf{V} = x\mathbf{i} - y\mathbf{j}$ is the velocity field of an ideal fluid, then a complex potential is

a) z
b) $\overline{z}$
c) $z^2/2$
d) $\overline{z}^2/2$
e) $|z|$