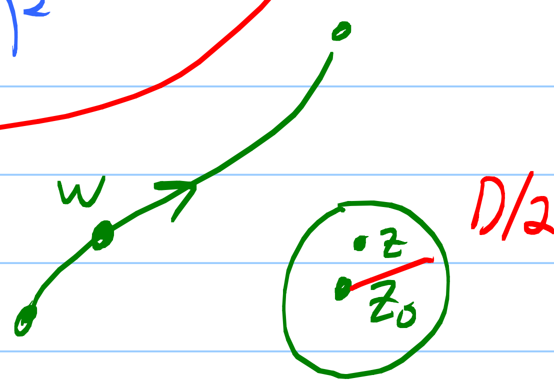
Hwk 2, #1. $f(z) = \int_\gamma \frac{\varphi(w)}{w-z}\,dw$

Show that $f'(z) = \int_\gamma \frac{\varphi(w)}{(w-z)^2}\,dw$.

$$\frac{f(z)-f(z_0)}{z-z_0} - \int_\gamma \frac{\varphi(w)}{(w-z_0)^2}\,dw = \int_\gamma \varphi(w)\left[\frac{1}{(w-z)(w-z_0)} - \frac{1}{(w-z_0)^2}\right]dw$$

$$= \int_\gamma \varphi(w)\left[\frac{(z-z_0)}{(w-z)(w-z_0)^2}\right]dw$$

$$= \underline{(z-z_0)}\underbrace{\int_\gamma \varphi(w)\,\frac{1}{(w-z)(w-z_0)^2}\,dw}_{E(z)}$$



$$D = dist(z_0, tr(\gamma))$$

$$= \min_{w \in tr(\gamma)} |z_0 - w|$$

$$|z-w| \geq D/2 \qquad |z_0 - w| \geq D/2$$

$$|E(z)| \leq |z-z_0|\, M\, \frac{1}{(D/2)(D/2)^2}\, Length(\gamma)$$ ✓