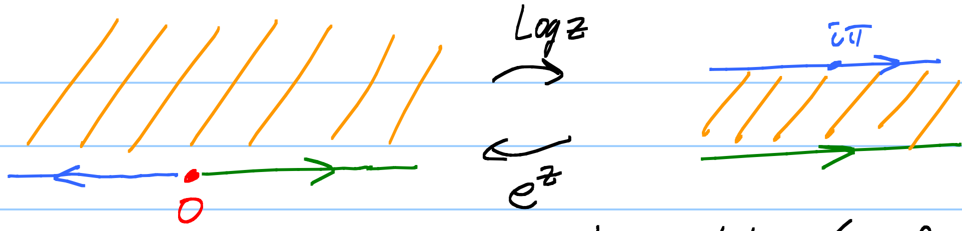
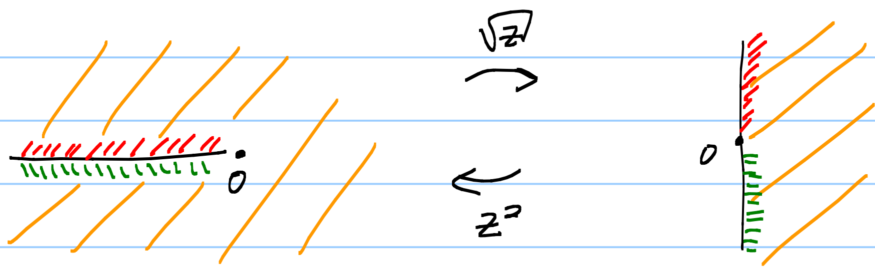
Lecture 25 Other conformal maps

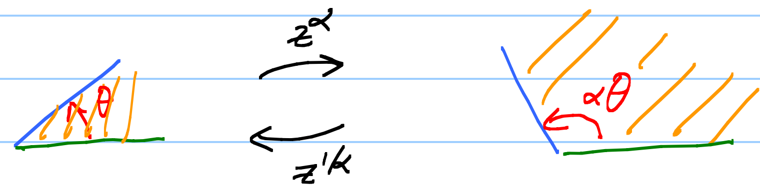
Exam 1
Ave = 65
Max 95



1-1 onto analytic (conformal)

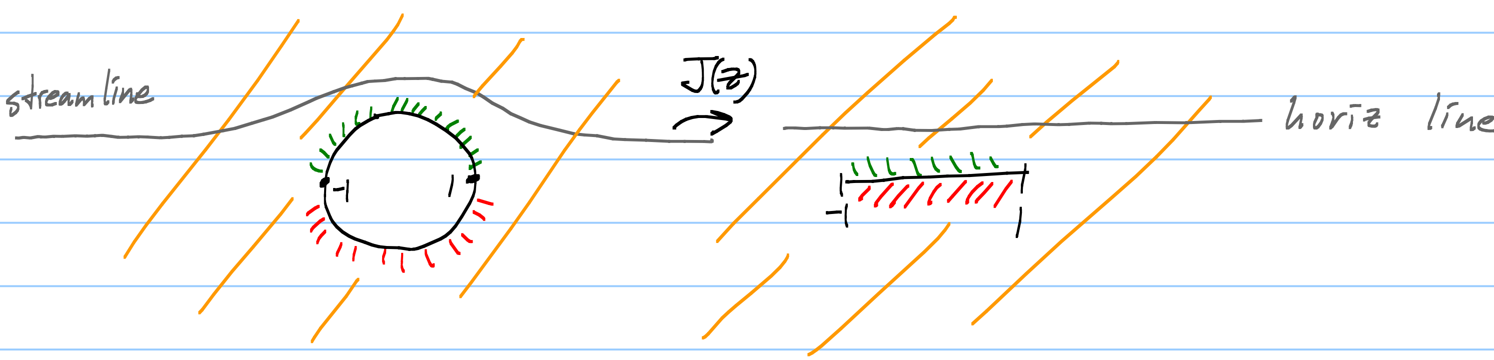


$\sqrt{z} = e^{\frac{1}{2}\operatorname{Log} z}$ ← Principal branch of $\sqrt{\ }$

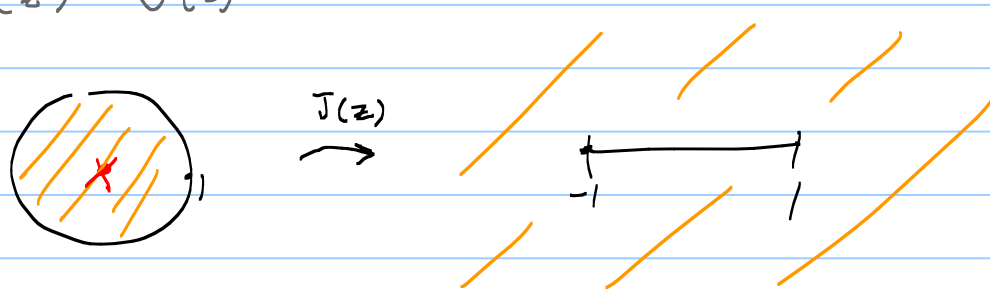


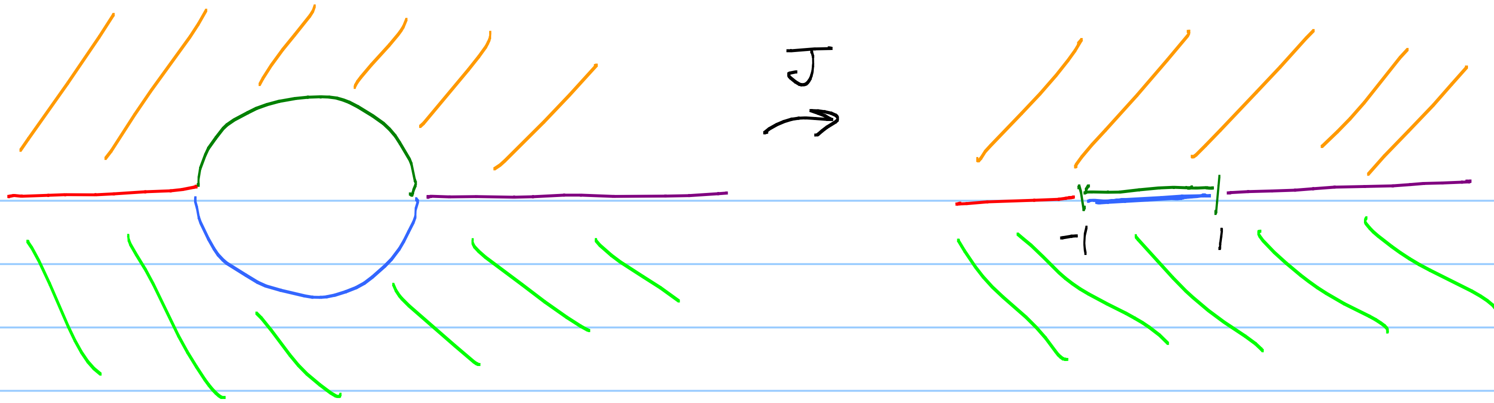
$z^\alpha = e^{\alpha \operatorname{Log} z}$ ← Principal branch

Jukovsky Map: $J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$

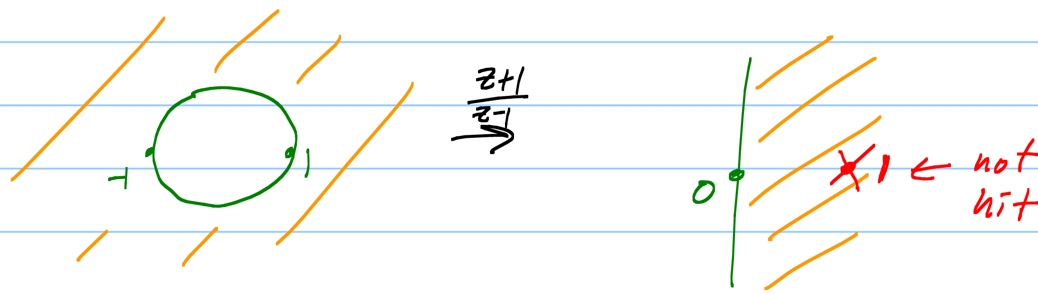


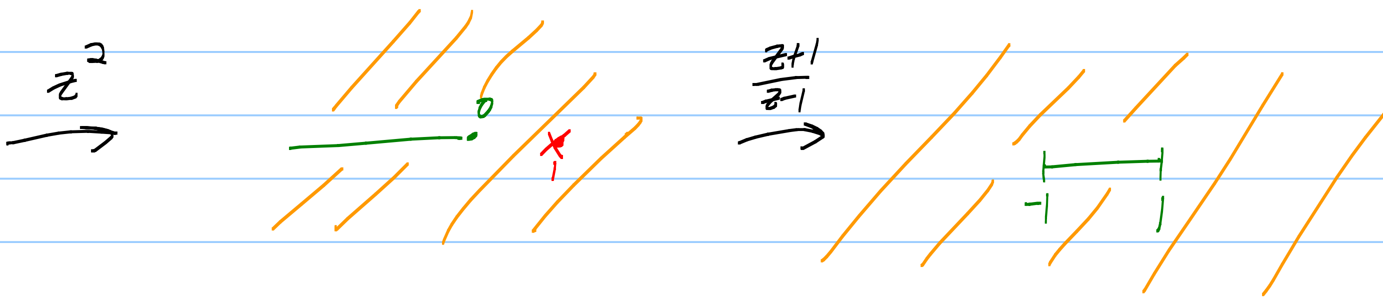
Hmmm. $J\left(\frac{1}{z}\right) = J(z)$



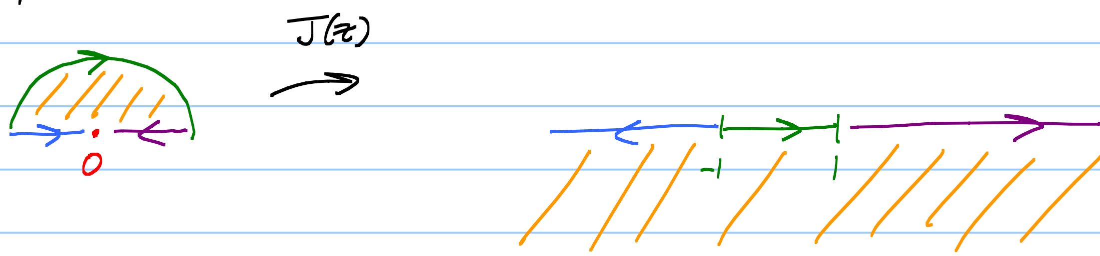


Cooking up $J(z)$:



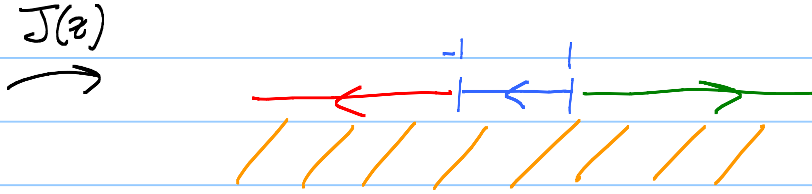


Composition $= J(z)$.



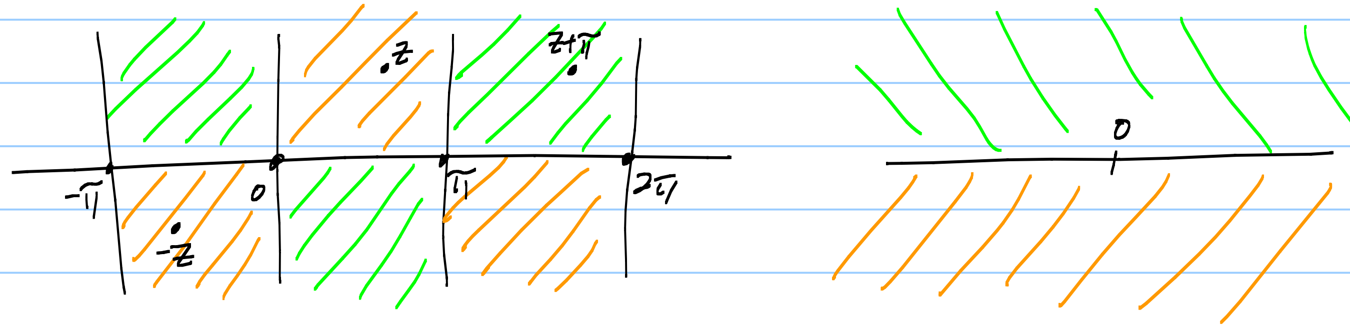
Complex cosine: $\cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right) = J(e^{iz})$

$e^{x+iy} = e^x e^{iy}$

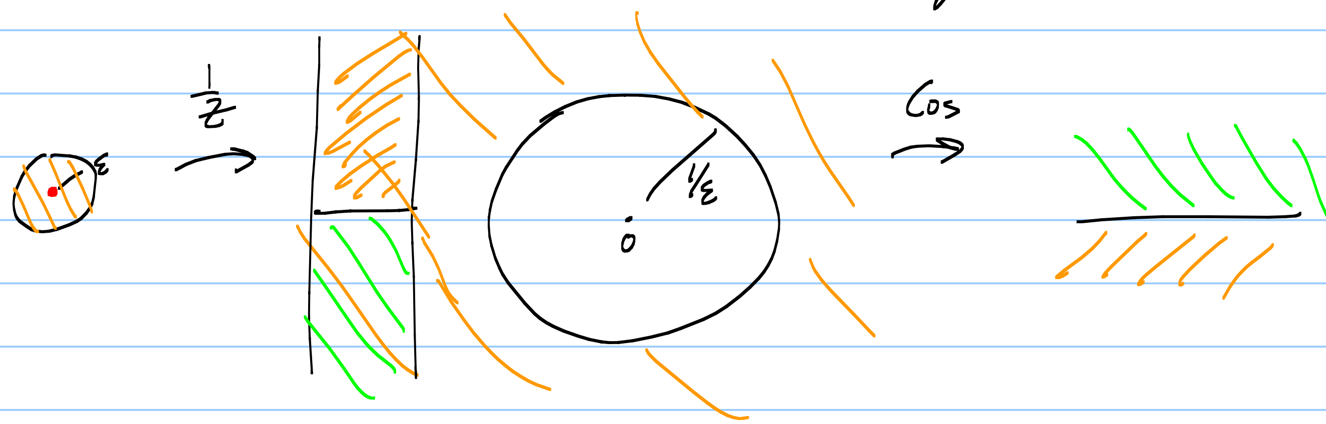


But $\cos(-z) = \cos z$

$\cos(z+\pi) = -\cos z$



Easy to see that $\cos\left(\frac{1}{z}\right)$ has an essential sing at $z=0$



Remark: Use big picture to cook $\cos^{-1} w$

$$w = \frac{1}{2}\left(e^{iz} + \frac{1}{e^{iz}}\right) \leftarrow \text{Mult by } e^{iz}$$

$$2we^{iz} = (e^{iz})^2 + 1$$

$$(e^{iz})^2 - 2we^{iz} + 1 = 0$$

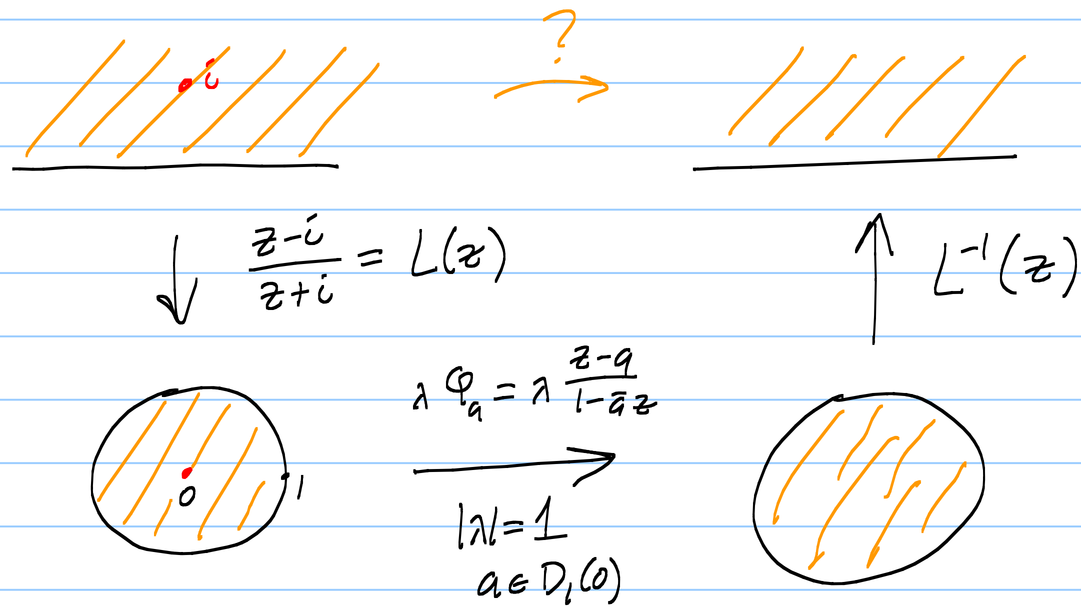
$$e^{iz} = \frac{2w + \sqrt{4w^2-4}}{2} = w + \sqrt{w^2-1}$$

$$z = \frac{1}{i}\log\left(w + \sqrt{w^2-1}\right)$$

Liouville! e^z and its inverse, plus algebraic fcns generate the "elementary fcns"

e^{x^2} does not have an elem. fcn for an antiderivative.

Application of LFTs:



Stein: Chapter 8

Get Aut(UHP).

Remark: $\sin(z) = \cos(z - \frac{\pi}{2})$