MA 530 Complex analysis

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MATH 750
494-1497

Stein, Shakarchi: Complex analysis, Princeton Press

<u>Office hours</u>: T, Th 2:00–3:00 pm in MATH 750 plus zoom

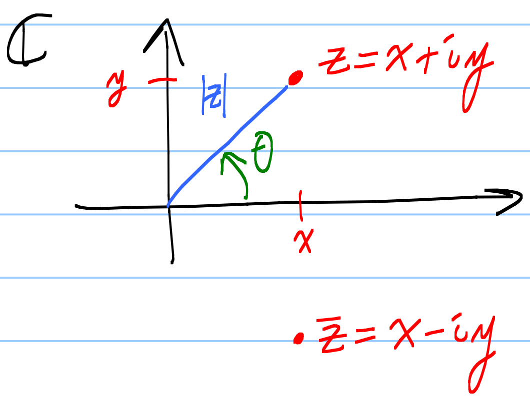
HWK	100 pts
ME	100 pts
FE	200 pts

Quadratic formula: Complete square:

$$a(x-r)^2 = c \leftarrow c<0!$$

Toss in i such that $i^2=-1$. Play games.

Algebra games, calculus games.



$$|z|^2 = x^2+y^2 \qquad \begin{cases} x = \operatorname{Re} z \\ y = \operatorname{Im} z \end{cases}$$

$$-\pi < \theta \le \pi \qquad \theta = \operatorname{Arg} z$$

Principal arg of z

$$\arg z = \{\operatorname{Arg} z + 2\pi n : n \in \mathbb{Z}\}$$

Polar form: $z = re^{i\theta}$, $r=|z|$, $\theta \in \arg z$

where $e^{i\theta} = \cos\theta + i\sin\theta$

Calculus game: $e^{(a+b)} = e^a e^b$

$$e^{x+iy} = e^x e^{iy}$$

$$= e^x\left(1 + (iy) + \frac{(iy)^2}{2!} + \cdots\right)$$

$$= e^x\left[\left(1-\frac{y^2}{2!}+\frac{y^4}{4!}+\cdots\right) + i\left(y - \frac{y^3}{3!}+\cdots\right)\right]$$

$$e^{x+iy} \underset{\text{def}^n}{=} e^x\left[\cos y + i \sin y\right]$$

<u>DeMoivre's formula</u>: $(e^{i\theta})^N = e^{iN\theta}$ $\quad N \in \mathbb{Z}$

Super fun thing: $\displaystyle\sum_{n=-N}^{N} e^{in\theta} = \frac{\sin(N+\frac{1}{2})\theta}{\sin\frac{\theta}{2}}$

$$\begin{cases} \sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i} \\ \cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2} \end{cases}$$

Algebra games:

$$\begin{cases} (x_1,y_1) + (x_2,y_2) \underset{\text{def}^n}{=} (x_1+x_2,\ y_1+y_2) \\ (x_1,y_1)\cdot(x_2,y_2) = (x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1) \\ 0 \sim (0,0),\quad 1 \sim (1,0),\quad i \sim (0,1) \end{cases}$$

Field! $\approx \mathbb{C}$ $\quad$ A <u>complete</u> field!

<u>Fund Thm Alg</u>: A poly $P(z) = a_N z^N + a_{N-1}z^{N-1} + \cdots + a_1 z + a_0$ of a deg $N \geq 1$ has a <u>root</u> r in $\mathbb{C}$. $\left[P(r)=0\right]$

Consequently, $P(z) = a_N \displaystyle\prod_{n=1}^{N}(z-r_n)$ might repeat

$$= a_N \prod_{k=1}^{M}(z-r_k)^{m_k} \qquad \sum_{k=1}^{N} m_k = N$$

Algebra game:

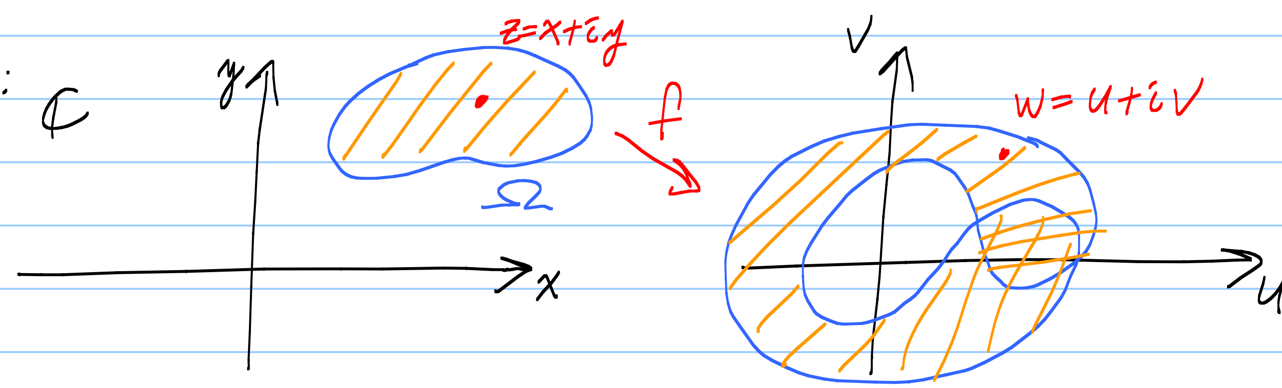
Frobenius: Every division ring gotten as a finite extension of $\mathbb{R}$ is either

1) $\approx \mathbb{R}$

2) $\approx \mathbb{C}$

3) $\approx \mathbb{H}$ ⟵ Quaternions

MA 530: $\mathbb{C}$



$f(z) = w$

$f(x+iy) = u + iv$

$u(x,y)$ $\quad$ $v(x,y)$

$\mathbb{C} \to \mathbb{C}$

$(x,y) \mapsto (u(x,y), v(x,y))$

$\mathbb{R}^2 \to \mathbb{R}^2$

Ex: $w = z^2 = (x+iy)^2 = \underbrace{(x^2-y^2)}_{u(x,y)} + i\,\underbrace{(2xy)}_{v(x,y)}$

$\mathbb{C} \approx \mathbb{R}^2$ Topology

Ω open set: If $a \in \Omega$, then $\exists\, r > 0$ such that $D_r(a) \subset \Omega$.

$$D_r(a) = \{z : \text{dist}(z,a) < r\} = \{z : |z-a| < r\}$$

Biggy: f is <u>complex diff'ble</u>:

$$\frac{f(z)-f(a)}{z-a} \longrightarrow L \quad \text{as} \quad z \to a$$

$L = f'(a)$

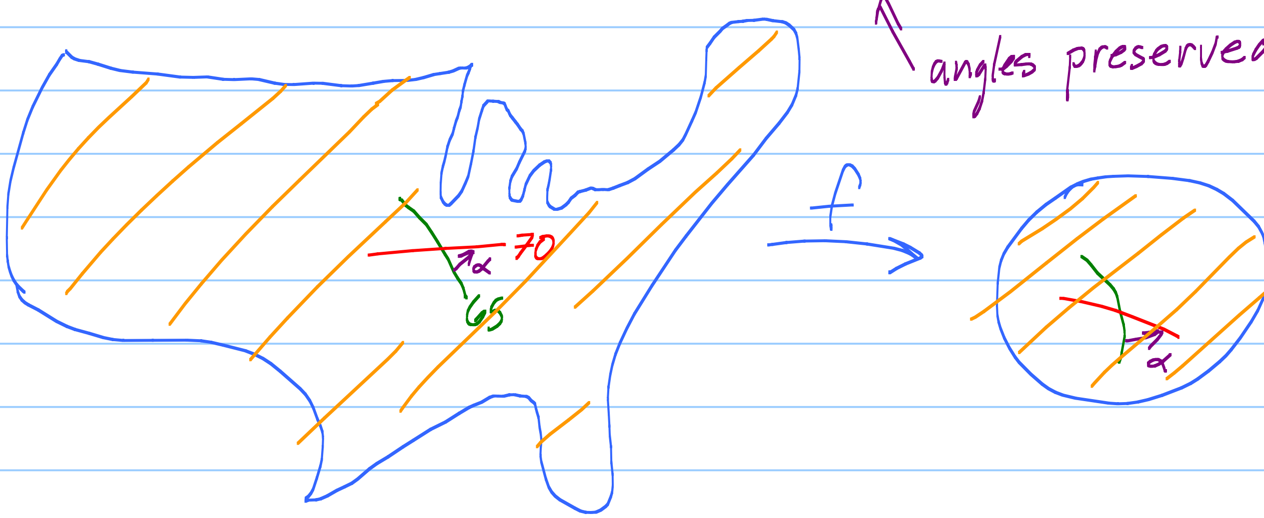
MA 530: Complex diff'ble fcns on an open set.

<u>Miracles:</u> 1) Fund Thm Alg

2) Riemann mapping theorem: Ω open connected set with no holes (simply connected), $\Omega \neq \mathbb{C}$. Then $\exists\ f : \Omega \to D_1(0)$ one-to-one, onto, complex diff'ble!

[f^{-1} $\mathbb{C}$-diff too, f' non-vanishing. <u>And</u> f is <u>conformal</u>!]

angles preserved



<u>Def'n</u>: f $\mathbb{C}$ valued fcn on $D_r(z_0) - \{z_0\}$.

$\lim_{z \to z_0} f(z) = L$ means, given $\varepsilon > 0$, $\exists\ \delta > 0$ such that $|f(z) - L| < \varepsilon$ when $|z - z_0| < \delta$, $z \neq z_0$.