Textbook Use of Connections to Children to Support Prospective Elementary Teachers' Geometric Understanding

By Brooke Max and Sydney Kohn

Researchers have suggested that one way to motivate and support prospective elementary teachers' (PTs) mathematical understanding is through the use of authentic examples of children's mathematical thinking (Philipp, 2008). Philipp noted that some PTs may care more about children as whole beings than they care about mathematics. Therefore, integrating how the mathematical content directly relates to the teaching and learning of children can offer a way to leverage the care PTs have for children to motivate PTs to care about the mathematics content in college courses.

Ball and her colleagues (2008) have identified the ability to analyze children's mathematical thinking as a valuable component of Specialized Content Knowledge (SCK), or knowledge unique to teachers of mathematics. However, research suggests that activities related to the Conference Board of Mathematical Sciences' content domains of Geometry and Measurement & Data provided fewer opportunities for PTs to develop their SCK (Max & Amstutz, 2019).

Therefore, the goal of this study is to investigate the intersection of the Geometry and Measurement & Data content domains with examples of connections to children in textbooks currently used in content courses for PTs. For this investigation, our analysis focused on the top three textbooks that US mathematics teacher educators recently reported using (Max & Newton, 2017): Beckmann (2018), Sowder et al. (2017), and Billstein et al. (2020). Findings provide examples of connection to children being utilized in the study of two-dimensional geometric concepts (e.g., shapes, polygons, angles) and measurement (e.g., length, angle size, area).

Theoretical Framework/Perspective

Mathematical knowledge for teaching (MKT) has been conceptualized to include two domains: subject matter knowledge and pedagogical content knowledge (Ball et al., 2008). Typically, subject matter knowledge is addressed more heavily in mathematical content courses specifically designed for elementary teachers and pedagogical content knowledge is addressed more heavily in mathematics methods courses designed for elementary teachers. This study focuses on the work done in mathematics content courses and, therefore, lies mostly in the subject matter knowledge domain. Within the subject matter knowledge domain are three sub-domains: common content knowledge (CCK), horizon content knowledge (HCK), and specialized content knowledge (SCK), with CCK constituting everyday mathematics used outside of the world of teaching, HCK constituting an awareness of mathematical topics over time, and SCK constituting knowledge unique to teachers of mathematics (Ball, et. al., 2008).

This perspective of PT knowledge domains is employed in this study in order to differentiate the ways in which connections to children is used to develop the domains in PTs' understanding.

Philipp's Circles of Caring model of growth (2008; See Figure 1) depicts PTs' relationship with caring about mathematics and caring about children. There are three concentric circles of PTs' caring, with the innermost circle being children, the next circle children's mathematical thinking, and the outermost circle including mathematics, suggesting that PTs care most about children and least about mathematics, but that children's mathematical thinking can be a way to motivate PTs' mathematical understanding. Phillip recommends interviewing children about mathematics, watching video clips of children thinking about mathematics, and referencing research about children's mathematical thinking, among other ways to encourage PTs to engage with the mathematical content. In the current study, the idea of children's mathematical thinking is extended to all ways the textbook authors may be engaging PTs to think about children within the context of mathematics and not solely through their mathematical thinking.

Methods

The goal of this study is to answer the question: In what ways do mathematics content for elementary teachers textbooks leverage children and children's thinking in the content areas of geometry and measurement to motivate PTs' mathematical understanding? To answer this question, the three most-used textbooks reported by instructors of mathematics for elementary

Table 1: Codes of Connections to Children in Geometry and Measurement Topics

CODE	DESCRIPTION	EXAMPLE
Referencing Children	Use of a name/explicitly stated child with direct connection to the school or classroom	Describe how four children could use 4 pieces of string to show a variety of different rhombuses. (Beckmann 2018, p. 487)
Referencing Classroom/Teacher	Mention of the teacher role, classroom, or how a teacher would respond to a student's thinking	Some teachers help their students learn about angles by making and using "angle explorers..." Discussing ways that teachers might help students learn about angle measurements in their classroom (Beckmann, 2018, p. 455)
Referencing Elementary Ideas/Standards	Mention of elementary ideas or standards	Describing measurement experiences/tools for separate grade bands of Pre-K-2, grades 3-5, and grades 6-8 (Sowder, 2017 p. 620)
Referencing a Name/ Person but not Identifying it as a Student	Use of a name without a direct connection to school or the classroom	Smallville is 7 miles south of Gotham. Will is 8 miles from Gotham and 6 miles from Smallville. Draw a map showing where Will could be. (Beckmann, 2018, p. 476)
Example of a Child's Assignment	Mention or image of assignment students could do in the classroom	Including a page from a 4th grade workbook on 2-D terms (e.g., point, line, ray) and types of angles (e.g., obtuse, straight, acute) (Billstein, 2020, p. 663)
Example of Assessment	Use of example questions from standardized tests or results from national tests	Providing questions that were used on a 2013 grade 4 and 2013 & 2017 grade 8 NAEP assessment about line segments and triangles (Billstein, 2020, p. 698)
Example of Children's Work/Actual Thinking	Use of work sample or direct quote from student	Including real sketches of rectangular prism nets drawn by children (Sowder, 2017, p. 438)

teachers' courses in the US (Max & Newton, 2017) were analyzed for connections to children. Because research suggests geometry and measurement were the least likely to include opportunities for PTs to develop SCK (Max & Amstutz, 2019), those topics were further investigated for evidence of attention. All three books contained two-dimensional geometric concepts (e.g., shapes, polygons, angles) and measurement (e.g., length, angle size, area) and were therefore the concepts included in the sample. Each textbook's geometry and measurement sections were examined, with instances noted of reference to children in any manner (e.g., noting a child, giving a name, referencing a standard, listing a grade level). The textbooks were then revisited to ensure no references to children in those topics were missed. Once the samples were identified, each sample was noted with how it made connections to children, leading to themes being developed. Once the themes were confirmed by all researchers, the samples were then coded to as many themes as applicable.

Codes were developed by several passes through the samples and included references to Common Core State Standards of Mathematics, names, and other connections outside of the pure mathematical content. For instance, in the Billstein textbook there were examples of NAEP questions regarding quadrilaterals for 4th grade. The textbook connected to children here by referencing a grade level with mathematical content, therefore this example was coded as "Example of Assessment." Another code of "Referencing Children" was recorded when the textbook mentioned things children think about (e.g., how children determine shapes and their properties (Beckmann, 2018) or of asking young children about squares (Sowder, 2017). Some samples mentioned names, but the name was not referenced specifically as a child. For example, "Carmina and Antone measure that the distance between the spots where they are standing is 10 feet..." (Beckmann, p. 576). This was coded as "Referencing a name/person but not identifying it as a student." These were then sorted by similarities and codes were developed from there.

Table 2: Textbook Instances of Attention to Children in Geometry and Measurement

CODE	BECKMANN (S = 40)	BILLSTEIN (S = 25)	SOWDER (S = 37)	TOTAL CODES (N = 149)	% OF TOTAL CODES
Referencing Children	17	6	19	42	28
Referencing Classroom/Teacher	8	6	15	29	20
Referencing Elementary Ideas/ Standards	16	6	6	28	19
Referencing a Name/Person but not Identifying it as a Student	20	0	2	22	15
Example of a Child's Assignment	0	7	4	11	7
Example of Assessment	0	6	3	9	6
Example of Children's Work/Actual Thinking	1	0	7	8	5

Seven codes emerged from the data, which can be found in Table 1 along with a description of the code and an example from the samples.

Six of these seven codes consistently attended to the SCK domain of MKT, accessing knowledge unique to teachers of mathematics. The code "Referencing a name but not identifying it as a student" code included two distinct types of references in terms of MKT within those samples: SCK or CCK. With the realization that the mention of a name may cause the reader to assume the name was that of a child, this code was investigated further to determine if more information could be gleaned from those samples. Therefore, those samples were further categorized by the MKT domain it attended to. For example, Beckmann (2018, p. 463) included a problem that stated "Tiffany says that the angle at A in figure 10.29 is bigger than the angle at B. Why might she think this? How might you discuss angles with Tiffany?" was coded as SCK while "Give instructions to Robot Rob how to move and turn so that his path is a regular pentagon that has side lengths of 2 meters. Explain how to determine his instructions." was coded as CCK.

Findings

One hundred forty-nine instances of codes emerged within the 102 samples of connections to children in the content areas of Geometry and Measurement, meaning a sample could be recorded as more than one code. Table 2 reports findings of codes within each textbook and across all codes. Notice the number of samples from each textbook is indicated with an *s*, but because each sample could be recoded as more than one code, the total number of codes is indicated with *n*.

As seen in Table 2, nearly half of the connections to children occurred through referencing children or referencing a classroom/ teacher, and these codes were found in each book. Present mostly in Beckmann (2018) and some in Sowder (2017) was the code "Referencing a Name/Person but not Identifying it as a Student." Of these 22 instances, 12 were identified as developing SCK and 9 were identified as developing CCK. One instance was coded with both SCK and CCK.

Discussion

This study examined geometry and measurement content areas because of their lack of attention to develop SCK previously reported (Max & Amstutz, 2019). Data in this study suggest that mathematics content textbooks specifically designed for PTs are using connections to children to motivate the PTs to learn the mathematics needed for teaching in the content areas of geometry and measurement, therefore providing PTs opportunities to develop SCK. While some samples included actual examples of children's work, a strategy Philipp (2008) suggested using to engage PTs with mathematics, this study highlights other methods of engaging PTs with mathematics through connections to children that can also develop their MKT. Referencing children or classroom/teacher occurred in nearly

half of the samples, indicating that the authors of these textbooks are actively attempting to connect the work in the mathematics content courses with the future work of PTs. More specifically, and mathematically, the presentation of questions from national tests (e.g., NAEP), referencing elementary ideas and standards, referencing student work pages, and providing examples of children's work intertwine the mathematics, children, and teaching of mathematics. These examples of motivating PTs to learn mathematics, while possibly not unique to the content areas of geometry and measurement, have potential to be productive ways to engage PTs with mathematical content.

This study aims to support mathematics teacher educators in creating classroom cultures that can leverage the care PTs have for children by revealing and highlighting the integration of connections to the mathematics teaching and learning of children. This work in content courses designed for PTs can motivate PTs to deepen their mathematical understanding in geometry and measurement in ways that will support the learning of their future students.

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