MA16010 : Applied Calculus I (Traditional)
Fall 2017

COURSE WEB PAGE: http://www.math.purdue.edu/ma16010

PREREQUISITE: MA 15400 C- or better, MA 15800 C- or better, ALEKS score of 75% or above, SAT math score of 600 or above, or ACT math score of 26 or above

TEXTBOOK: No textbooks required to purchase. Course contents will be provided to students online through LON-CAPA, free of charge.

HOMEWORK ACCESS: Online homework access through LON-CAPA will be provided to students, free of charge.

Discussion Board: We will use Piazza for online discussion. A graduate assistant will moderate the discussion board on a regular basis.

Calculator: A scientific calculator with a one-line display is required. ONLY THIS TYPE OF CALCULATORS WILL BE ALLOWED. NO EXCEPTIONS. Recommended is the TI-30Xa. If in doubt, please double check with your instructor. You are allowed to use but NOT to share the approved calculators on quizzes (no calculators on the first or the last quiz) or exams.

Homework: Homework assignments will be assigned regularly. Each assignment is due at **10:00pm Eastern time** the following meeting day. There are 36 homework assignments with 2 points each. The **lowest** homework score will be dropped. If you are not able to complete your homework on time and have extenuating circumstances with valid supporting documents, please talk to your instructor.

Quizzes: There will be **15 quizzes total with 5 points each**. The **lowest** quiz score will be dropped. There will be no make-up quizzes. In extenuating circumstances, your instructor may choose to excuse you from a quiz.

Exams: There will be three midterm exams and a final exam. All exams are course-wide, multiple-choice, machine-graded exams. The three midterms are all evening exams. The time and date of the midterms can be found in the calendar. The final exam information will be given later in the semester. **The semester does not end until Saturday, Dec 16 at 9:00 pm. Individuals wanting to leave campus early will not be granted early final exams to accommodate travel plans.**

If you miss an exam for any reason, please contact your instructor immediately and explain why you missed the exam. You should be prepared to present documentation to your instructor that supports the reason for your absence. If you contact your instructor within 24 hours from the scheduled exam, your instructor will allow you to take an alternate exam either with no penalty OR with a 20 point deduction, depending on the reason for your absence. If you miss an exam with no valid reasons and you do not contact your instructor within 24 hours from the scheduled exam, you will not be allowed for an alternate exam. Not knowing the right time, date or location of an exam is not a valid reason for missing it.

Warning: If there are any special circumstances that may affect your ability to successfully complete an exam (illness, family emergency, etc.), you must discuss the situation with your instructor before taking the exam, even if you must do so right before the exam. Your instructor will then be able to advise you on your options. Do not wait until after you take the exam to mention a situation to your instructor.

OFFICE HOURS: Most instructors hold office hours in the Math Help Rooms, MATH 205 or MATH 211. In addition to instructors from your course, instructors from other courses in the help rooms can also help you. The office hour schedule can be found on the course website. The schedule will be finalized after the first week of classes. You are strongly urged to go to office hours if you have questions. It is the best way to get individual help.

GRADES: The course grade will be based on a total of 640 points.

Homework	70
Quizzes	70
Exam 1	100
Exam 2	100
Exam 3	100
Final	200
Total	640

Final letter grades will be determined using the following grading scale.

total points > 615: A+
$563 <$ total points ≤ 615: A
$545 <$ total points ≤ 563: A-
$527 <$ total points ≤ 545: B+
$480 <$ total points ≤ 527: B
$462 <$ total points ≤ 480: B-
$444 <$ total points ≤ 462: C+
$389 <$ total points ≤ 444: C
$371 <$ total points ≤ 389: C-
$353 <$ total points ≤ 371: D+
$306 <$ total points ≤ 353: D
$288 <$ total points ≤ 306: D-
total points ≤ 288: F

ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES: If you have been certified by the Disability Resource Center (DRC) as eligible for academic adjustments on exams or quizzes, please see http://www.math.purdue.edu/ada for exam and quiz procedures for your mathematics course or go to MATH 202 for paper copies.

In the event that you are waiting to be certified by the Disability Resource Center we encourage you to review our procedures prior to being certified. For all in-class accommodations, please see your instructors outside class hours-before or after class or during office hours-to share your Accommodation Memorandum for the current semester and discuss your accommodations as soon as possible.

CAMPUS EMERGENCY PROCEDURE: In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control.

Announcements regarding campus emergencies will be sent via course-wide emails and be posted on the course web page.

ACADEMIC DISHONESTY: The Mathematics Department will not tolerate academic dishonesty of any sort. If academic dishonesty occurs, then grade penalties will be imposed, possibly to the extent of an "F" in the course. Additionally, all cases of academic dishonesty will be reported to the Office of the Dean of Students for disciplinary action (which may include probation, suspension, or expulsion). If you would like to report issues of academic integrity, you can report to the Office of the Dean of Students (purdue.edu/odos), call 765-494-8778 or email integrity@purdue.edu.

SECTION CHANGES AND DROPS: During the first week of classes, you can make section changes via Banner within myPurdue, and no signatures are required. From the second through ninth weeks of the semester, see the instructor of the section you want to enter for the required signature. If you want to drop this course during the first nine weeks of the semester, your instructor can sign your drop form. If your instructor is not available, go to MATH 835.

LAST ADD DATE: The last day you can add this course is Tues, Sep 26. Students adding at this time must take an alternate Exam 1. Students are expected to keep up with the current material while studying for the alternate Exam 1.

COURSE EVALUATIONS: On Monday of the fifteenth week of classes, you will receive an official email from evaluation administrators with a link to online course evaluations. You will have two weeks to complete this evaluation. You are strongly encouraged to participate. Your feedback is vital to maintaining and improving the quality of education at Purdue University.

EMERGENCY PREPAREDNESS SUMMARY: A document about emergency preparedness can be found on the course web page under syllabus. Here is a summary.

If an alarm is heard inside a building, immediately evaluate the building. Get a safe distance from the building. Remain outside the building until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave or return to the building.

If an alarm is heard outside a building, immediately seek shelter in a safe location within the closest building. These types of alarms may indicate a tornado, a civil disturbance, or release of hazardous materials in the outside air. Remain inside the building until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

In both cases above, you should seek additional clarifying information by all means possible such as Purdue University home page, email alert, TV, radio, etc.

MA 16010 Applied Calculus I

Calendar (Traditional and Distance), Fall 2017

Exam 1: Lesson 2-10 Exam 2: Lesson 11-18 Exam 3: Lesson 19-28

Date	Lesson	Topics
8/21 Mon	1	Course Information; CCI (no calculators)
8/23 Wed	2	Finding Limits Numerically; One-sided Limits
8/25 Fri	3	Finding Limits Graphically
8/28 Mon	4	Finding Limits Analytically
8/30 Wed	5	Continuity
9/1 Fri	6	The Derivative
9/4 Mon		LABOR DAY (NO CLASSES)
9/6 Wed	7	Basic Rules of Differentiation; Derivatives of the Sine and Cosine Functions; Derivative of the Natural Exponential Function
9/8 Fri	8	Instantaneous Rates of Change
9/11 Mon	9	The Product Rule
9/13 Wed	10	The Quotient Rule; Derivatives of the Other Trigonometric Functions
9/15 Fri	11	The Chain Rule
9/18 Mon	12	The Chain Rule; Derivative of the Natural Logarithmic Function
9/20 Wed	13	Higher Order Derivatives
9/22 Fri		REVIEW FOR EXAM 1
9/25 Mon		**NO CLASSES**
9/25 Mon		**EXAM 1 Time: 6:30-7:30pm Location: ELLT 116**
9/27 Wed	14	Implicit Differentiation
9/29 Fri	15	Related Rates
10/2 Mon	16	Related Rates
10/4 Wed	17	Relative Extrema and Critical Numbers
10/6 Fri	18	Increasing and Decreasing Functions and the First Derivative Test
10/9 Mon		**FALL BREAK (NO CLASSES)**
10/11 Wed		REVIEW FOR EXAM 2
10/11 Wed		**EXAM 2 Time: 8:00-9:00pm Location: ELLT 116**
10/13 Fri	19	Concavity, Inflection Points and the Second Derivative Test
10/16 Mon	20	Absolute Extrema on an Interval
10/18 Wed	21 Q	Graphical Interpretation of Derivatives
10/20 Fri	22 Q	Limits at Infinity
10/23 Mon	23	A Summary of Curve Sketching
10/25 Wed	24 Q	Optimization
10/27 Fri	25	Optimization
10/30 Mon	26 Q	Optimization
11/1 Wed	27	Antiderivatives and Indefinite Integration
11/3 Fri	28 Q	Antiderivatives and Indefinite Integration

MA 16010 Applied Calculus I

Calendar (Traditional and Distance), Fall 2017

Exam 1: Lesson 2-10 Exam 2: Lesson 11-18 Exam 3: Lesson 19-28

Date	Lesson	Topics
11/6 Mon	29 Q	Area and Riemann Sums
11/8 Wed	30 Q	Definite Integrals
11/10 Fri		REVIEW FOR EXAM 3
11/13 Mon		**NO CLASSES**
11/13 Mon		**EXAM 3 Time: 6:30-7:30pm Location: ELLT 116**
11/15 Wed	31	The Fundamental Theorem of Calculus
11/17 Fri	32	The Fundamental Theorem of Calculus
11/20 Mon		**NO CLASSES**
11/22 Wed		**THANKSGIVING VACATION (NO CLASSES)**
11/24 Fri		**THANKSGIVING VACATION (NO CLASSES)**
11/27 Mon	33	Numerical Integration
11/29 Wed	34 Q	Exponential Growth
12/1 Fri	35 Q	CCI (no calculators)
12/4 Mon	36	Exponential Decay
12/6 Wed		REVIEW FOR FINAL EXAM
12/8 Fri		REVIEW FOR FINAL EXAM
12/11-12/16		WEEK OF FINAL EXAMS

12/11 Final: 7:00–9:00 pm

MA 16010 - 120/110

Alden Bradford
bradfoa@purdue.edu
Math 605

math.purdue.edu/courses/MA16010
math.purdue.edu/academic/courses/ada

- Full Name
- Prefer to be called
- year
- Major or Area of interest
- $\sqrt{x+y} - \ln(2e^{17}) + \sin(u)$

Agenda

- Attendance / cards
- course structure:
  - 1 quiz per week (about), 15 total, 1 dropped, first friday
  - you need a calculator
  - we use LON-CAPA
  - review materials available
  - office hours to be announced by weds., or by appointment tues/thurs
- CCI (calculus concept inventory)
  - worth 2 bonus points
  - may write an essay instead

Finding limits numerically

Math Help Room hours:
Mon-Thurs 10:30-5:30
Friday 10:30-2:30
My hours:
Thursday 12:30-2:30

In[7]:=
```
f[x_] := 3 x + 4;
maketable[f, 2]
```

Out[8]=

x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
4 + 3 x	9.7	9.97	9.997	9.9997	10	10.0003	10.003	10.03	10.3

In[11]:= $f[x\_] := \frac{x^2 + 2x - 3}{2x - 2};$
maketable[f, 1]

Out[12]=

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
$\frac{-3+2x+x^2}{-2+2x}$	1.95	1.995	1.9995	1.99995	Indeterminate	2.00005	2.0005	2.005	2.05

In[13]:= $f[x\_] := \frac{\mathrm{Tan}[x]}{x};$
maketable[f, 0]

Out[14]=

x	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1
$\frac{\mathrm{Tan}[x]}{x}$	1.00335	1.00003	1.	1.	Indeterminate	1.	1.	1.00003	1.00335

In[15]:= $f[x\_] := \frac{1}{x^2};$
maketable[f, 0]

Out[16]=

x	-0.1	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01	0.1
$\frac{1}{x^2}$	100.	10000.	$1. \times 10^6$	$1. \times 10^8$	ComplexInfinity	$1. \times 10^8$	$1. \times 10^6$	10000.	100.

In[17]:= $f[x\_] := \frac{x+1}{x-1};$
maketable[f, 1]

Out[18]=

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
$\frac{1+x}{-1+x}$	-19.	-199.	-1999.	-19999.	ComplexInfinity	20001.	2001.	201.	21.

In[21]:= $f[x\_] := \sqrt{1 - 3x};$
maketable[f, -1]

Out[22]=

x	-1.1	-1.01	-1.001	-1.0001	-1	-0.9999	-0.999	-0.99	-0.9
$\sqrt{(1 - 3x)}$	2.07364	2.00749	2.00075	2.00007	2	1.99992	1.99925	1.99249	1.92354

In[31]:=
```
f[x_] := 1/(4 x + 16);
maketable[f, -4]
```

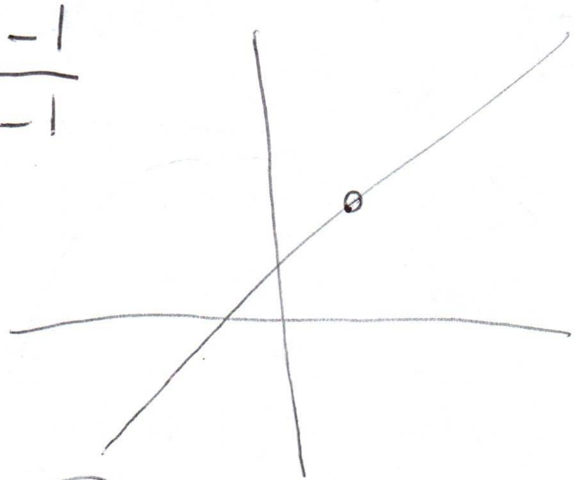
$$f[x\_] := \frac{1}{4\,x + 16};$$

Out[32]=

x	-4.1	-4.01	-4.001	-4.0001	-4	-3.9999	-3.999	-3.99	-3.9
$\frac{1}{16+4\,x}$	-2.5	-25.	-250.	-2500.	ComplexInfinity	2500.	250.	25.	2.5

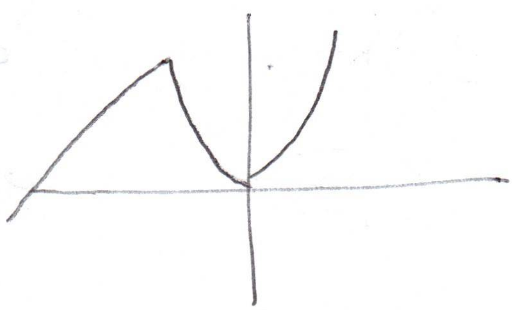
finding limits graphically (Aug 25)

$f(x)\,\cancel{y} = \dfrac{x^2-1}{x-1}$



$\lim\limits_{x\to 1} f(x) = ?$

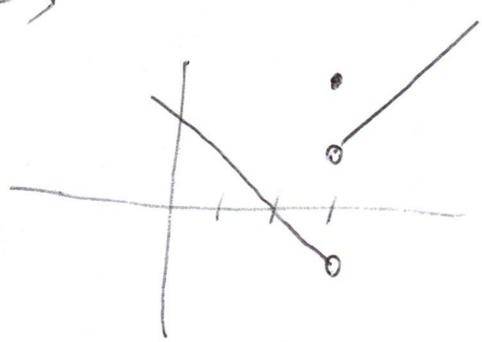
$$f(x) = \begin{cases} x^2 & \text{if } x \geq -2 \\ x+6 & \text{if } x < -2 \end{cases}$$



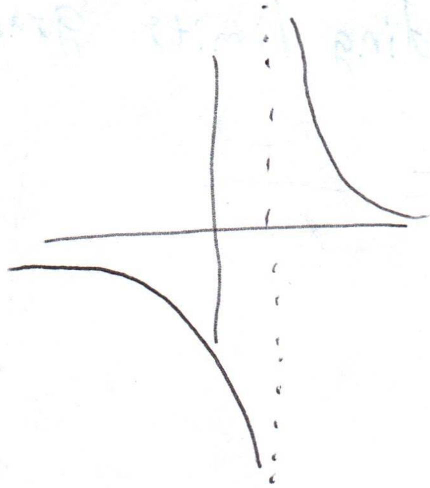
$\lim\limits_{x\to -2} f(x) = ?$

$$f(x) = \begin{cases} 2-x & \text{if } x < 3 \\ x-2 & \text{if } x > 3 \\ 2 & \text{if } x = 3 \end{cases}$$

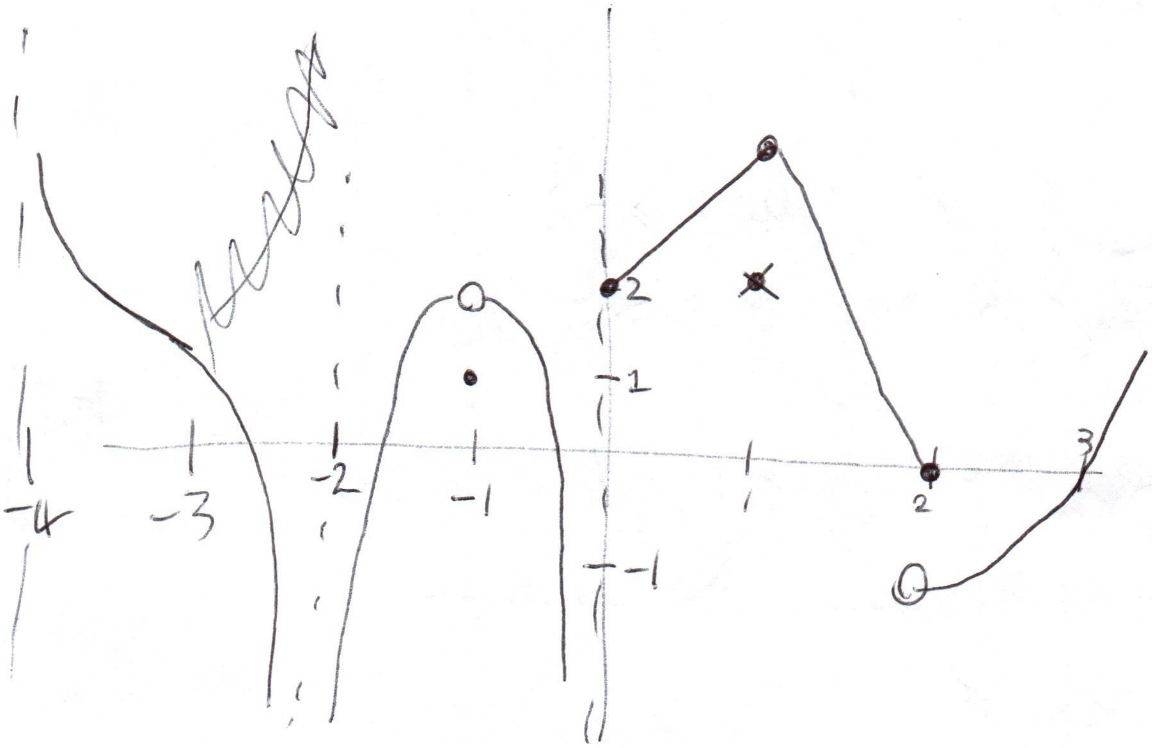
$\lim\limits_{x\to 3} f(x) = ?$



$$f(x) = \frac{1}{x-1}$$



$$\lim_{x \to 1} f(x) = ?$$



Quiz 1 — MA16010 — August 24, 2017

All quizzes are scored out of 5 points. Show your work on all problems to receive full credit. Partial credit will be awarded where progress is shown. 5 points also earns you a sticker.

1. (1 point) What is your instructor's name?

2. (1 point) What is your section number?

3. (1 point) What does "calculus" mean?

4. (1 point) $\cos\theta = 1/10$ and θ is in the fourth quadrant. Find $\csc\theta$.

5. (1 point) Simplify $\log\left(\sqrt{\frac{x^{10}y^4}{w^2}}\right)$ completely.

Quiz 1 Key — MA16010 — August 24, 2017

Alden Bradford

Min	Mean	Max
1.5	4.1	5

1. (1 point) What is your instructor's name?

   Alden Bradford (Alden is fine)

2. (1 point) What is your section number?

   If you start at 9:30, your section number is 120.
   If you start at 10:30, your section number is 110.

3. (1 point) What does "calculus" mean?

   Tiny rocks

4. (1 point) $\cos\theta = 1/10$ and θ is in the fourth quadrant. Find $\csc\theta$.

   $-10/(3\sqrt{11})$

5. (1 point) Simplify $\log\left(\sqrt{\frac{x^{10}y^4}{w^2}}\right)$ completely.

   $5\log(x) + 2\log(y) - \log(w)$

Finding Limits Analytically

Quiz wednesday, bring a calculator!

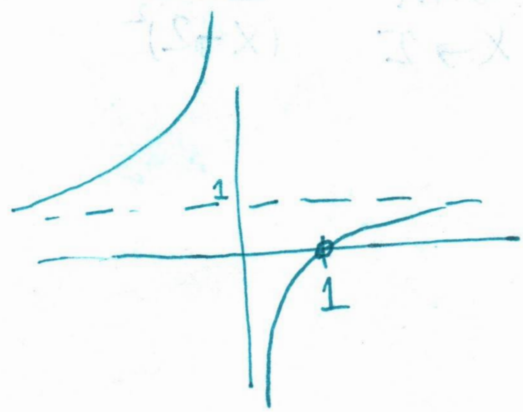
$$\lim_{x\to 5} x^2+3x+10 = 25+15+10 = 50$$

$$\lim_{x\to -1} \frac{x+1}{x-1} = \frac{-1+1}{-1-1} = \frac{0}{-2} = 0$$

$$\lim_{x\to 1} \frac{x^2-2x+1}{x^2-x} = \frac{1-2+1}{1-1} = \frac{0}{0}$$

$$\frac{x^2-2x+1}{x^2-x} = \frac{(x-1)(x-1)}{x(x-1)} \overset{x\neq 1}{=} \frac{x-1}{x}$$

$$\lim_{x\to 1} \frac{x-1}{x} = \frac{1-1}{1} = \frac{0}{1}$$



$$\cancel{\lim \frac{(x+6)(x^2-3)(x+2)}{(x+2)(x+3)(x+6)}}$$

$$\lim_{x\to -2} \frac{(x-3)(x+2)}{(x+2)(x+1)^2} = \lim_{x\to -2} \frac{x-3}{(x+1)^2} = \frac{-2-3}{(-2+1)^2} = \frac{-5}{(-1)^2} = -5$$

$$\lim_{x\to 3} \frac{(x-3)(x-4)}{x^3-3x^2} = \frac{x^2-7x+12}{x^3-3x^2} = \frac{x-4}{x^2}$$

$$\lim_{x\to 3} \frac{x-4}{x^2} = \frac{3-4}{3^2} = -\frac{1}{9}$$

we have discussed:

$$\lim_{x \to c} f(x) = \text{number}$$

$$\lim_{x \to c} f(x) = \frac{0}{0}$$

what about $\lim_{x \to c} f(x) = \dfrac{0}{\text{number}}$?

what about $\lim_{x \to c} f(x) = \dfrac{\text{number}}{0}$?

$$\lim_{x \to 2^-} \frac{x-5}{(x-2)^2}$$

$x-2$ negative

$(x-2)^2$ positive

$\dfrac{1}{(x-2)^2}$ positive

$x-5$ negative

$\dfrac{x-5}{(x-2)^2}$ ~~positive~~ negative

$$\lim_{x \to 2^-} \frac{x-5}{(x-2)^2} = -\infty$$

$$\lim_{x \to 2^+} \frac{x-5}{(x-2)^2}$$

$x-2$ positive

$(x-2)^2$ positive

$x-5$ negative

$\dfrac{x-5}{(x-2)^2}$ negative

$$\lim_{x \to 2^+} \frac{x-5}{(x-2)^2} = -\infty = \lim_{x \to 2} \frac{x-5}{(x-2)^2}$$

$$\lim_{x\to 3} \frac{(x-3)(x+5)}{(x-3)^2(x+2)} = \lim_{x\to 3} \frac{x+5}{(x-3)(x+2)}$$

$x+5$ positive

$x+2$ positive

$\lim_{x\to 3^-}$ $x-3$ negative $\lim_{x\to 3^-} \frac{x+5}{(x-3)(x+2)} = -\infty$

$\lim_{x\to 3^+}$ $x-3$ positive $\lim_{x\to 3^+} \frac{x+5}{(x-3)(x+2)} = \infty$

lim DNE

$\lim_{x\to 3} \frac{(x-3)(x+5)}{(x-3)^2(x+2)}$ does not exist.

$\lim_{x\to 2} f(x)$ $\quad f(x) = \begin{cases} 3-x & \text{if } x \geq 2 \\ \frac{1+2x}{5} & \text{if } x < 2 \end{cases}$

$$\lim_{x\to 2^-} f(x) = \frac{1+2(2)}{5} = \frac{5}{5} = 1$$

$$\lim_{x\to 2^+} f(x) = 3-(2) = 1$$

$$\lim_{x\to 2} f(x) = 1$$

$$\lim_{x \to \pi} f(x) \qquad f(x) = \begin{cases} 2\cos x & \text{if } x < \pi \\ 3 & \text{if } x = \pi \\ \frac{x}{\pi} - \cancel{3}4 & \text{if } x > \pi \end{cases}$$

$$\lim_{x \to \pi^-} f(x) = 2\cos\pi = -2$$

$$\lim_{x \to \pi^+} f(x) = \frac{\pi}{\pi} - 4 = -3$$

$$\lim_{x \to \pi} f(x) \text{ DNE}$$

$$\lim_{x \to 1} f(x) \qquad f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ 3x - 1 & \text{if } x \geq 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{(x+1)(x-1)}{(x-1)} = \lim_{x \to 1^-} x+1 = 1+1 = 2$$

$$\lim_{x \to 1^+} f(x) = 3(1) - 1 = 2$$

$$\lim_{x \to 1} f(x) = 2$$

Quiz 2 — MA16010 — August 30, 2017

Alden Bradford

1. (2 points) Find $\lim_{x\to -1} \dfrac{x^2+x}{x^2-3x-4}$ analytically.

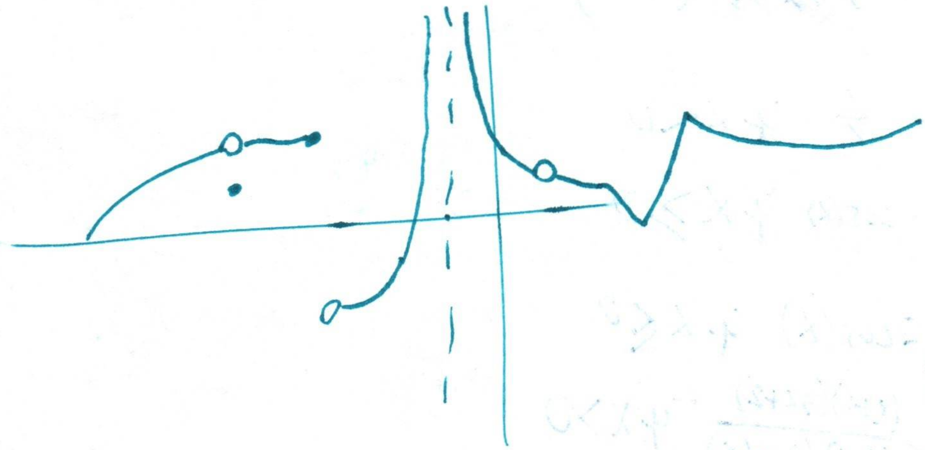
2. (3 points) Let $f(x) = \dfrac{\sin x}{e^{1/x}}$.

   (a) Copy and fill in the following table. Record 6 decimal places on every number you write in the table. Be sure your calculator is in radians mode.

x	0	0.1	0.5	1
$f(x)$	—			

   (b) Use your table from part (a) to find $\lim_{x\to 0^+} f(x)$.

Continuity   Aug 30



f is continuous at c if all 3 of these are true:

$$f(c) \text{ exists}$$

$$\lim_{x\to c} f(x) \text{ exists}$$

$$f(c) = \lim_{x\to c} f(x)$$

Is $f(x)$ continuous at $x=4$?

$$f(x) = \begin{cases} \dfrac{(x-4)(x+1)}{(x-4)} & \text{if } x \neq 4 \\ 5 & \text{if } x = 4 \end{cases}$$

$$f(x) = \begin{cases} 6x-2 & \text{if } x < 1 \\ 3x+1 & \text{if } x > 1 \\ 2 & \text{if } x = 1 \end{cases}$$

$$f(x) = \frac{(3x+1)(x-5)}{(2x+1)\cancel{(x+3)}(x-5)}$$

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \cos(x) & \text{if } x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} -\cos(x) & \text{if } x \leq 0 \\ \frac{(x+1)(3x+2)}{(x-1)(2-6x)} & \text{if } x > 0 \end{cases}$$

COMMENTS (Written comments and constructive suggestions are most helpful to instructors)

Arrived early. Good.

Good: asked why student ~~a~~ had trouble w/ HW problem (limit).

Volume is good, speed is good. Sometimes your volume drops near the end of a sentence.

You explained, by saying, what it means for f(x) to be discontinuous at x=a, but didn't write it down. This is something important enough to write down.

Good diagrams. Give them a few moments to sketch before doing anything with them. Same w/ examples.

~~You explained, by referencing the diagram, what f(c) exists and~~

Good job explaining what the requirements for continuity means by referencing the diagram you drew earlier.

Writing is generally good, a little sloppy once in a while.

... "any question?" more often. Not all students ~~fe~~ feel comfortable asking w/o being prompted.

Good: asked questions. Good: followed up w/ "why" after getting an answer.

Good: asked for volunteers to start an example.

- be sure to explain thoroughly the answers you got from volunteers.

In all your examples discontinuities can only happen at where the piecewise function splits. Is this always true in this course? (ok, you had one such example)

	until	workshop suggestions
...eturning papers	—	
...ver quiz	—	
...swer HW questions	9:35	
...over new material	10:02	
...ve quiz	end	

CLASSROOM VISIT REPORT

TA name : Alden Bradford

Date of visit : 8/30

Evaluator : Joe Chen

Time of visit : 9:30

Course : 16010 Topic of class : Continuity

VG = very good ; G = good ; AD = adequate ; P = poor ; VP = very poor

1. Content					
correctness	VG	G	AD	P	VP
preparedness	VG	G	AD	P	VP
consistency with textbook	VG	G	AD	P	VP
use of diagrams	VG	G	AD	P	VP
presentation : clarity	VG	G	AD	P	VP
presentation : level	VG	G	AD	P	VP
presentation : pace	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
2. Use of the blackboard					
legibility	VG	G	AD	P	VP
use of space/timely erasing	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
3. Communication					
facing the class/eye contact	VG	G	AD	P	VP
speech : loudness	VG	G	AD	P	VP
speech : clarity	VG	G	AD	P	VP
speech : speed	VG	G	AD	P	VP
speech : mannerism	VG	G	AD	P	VP
gives opportunities for questions	VG	G	AD	P	VP
asks questions	VG	G	~~AD~~	P	VP
understands questions	VG	G	AD	P	VP
answers questions	VG	G	AD	P	VP
can rephrase explanations	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP
4. Atmosphere					
motivating students	VG	G	AD	P	VP
student involvement	VG	G	AD	P	VP
student attention	VG	G	AD	P	VP
other : ______	VG	G	AD	P	VP

Is a follow-up visit needed ? : no ✓ maybe ___ yes ___

Ready to teach his/her own Algebra/Trig class ? (circle one) yes maybe no unknown

PLEASE USE OTHER SIDE FOR COMMENTS

(Written comments and constructive suggestions are most helpful to instructors)

Quiz 2 Key — MA16010 — August 30, 2017

Alden Bradford

Min	Mean	Max
1	3.8	5

1. (2 points) Find $\lim_{x \to -1} \dfrac{x^2 + x}{x^2 - 3x - 4}$ analytically.

1/5

2. (3 points) Let $f(x) = \dfrac{\sin x}{e^{1/x}}$.

   (a) Copy and fill in the following table. Record 6 decimal places on every number you write in the table. Be sure your calculator is in radians mode.

x	0	0.1	0.5	1
$f(x)$	—			

   (b) Use your table from part (a) to find $\lim_{x \to 0^+} f(x)$.

(a)

x	0	0.1	0.5	1
$f(x)$	—	0.000005	0.064883	0.309560

(b) $\lim_{x \to 0^+} f(x) = 0$

The Derivative

equations for lines

NO Class Mon

Slope-intercept form:

$$y = mx + b$$

point-slope form:

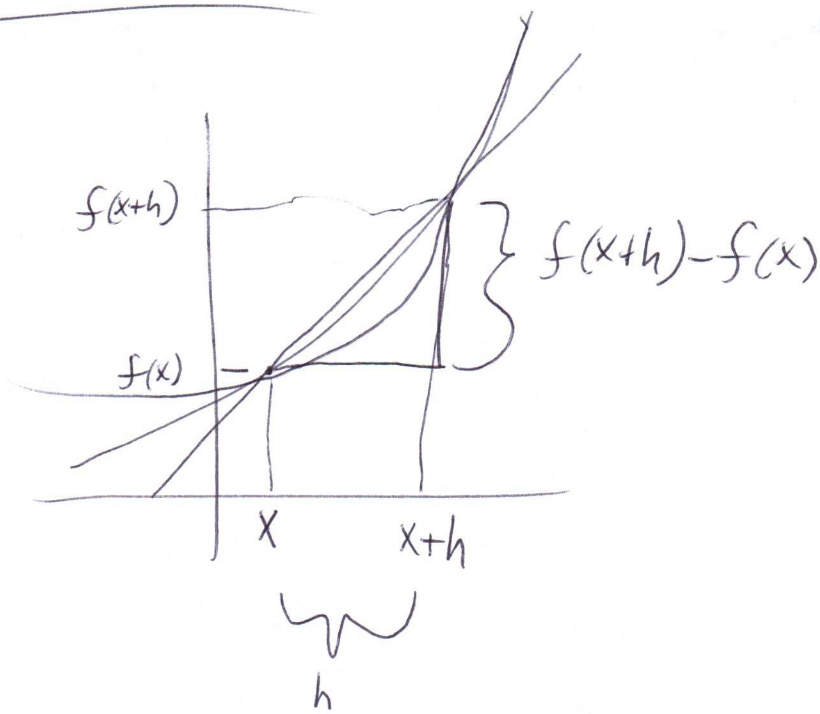
$$y - y_1 = m(x - x_1)$$

$$y = m(x - x_1) + y_1$$

Slope of a line joining two points:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

tangent lines



Slope of tangent line: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{df(x)}{dx} = y' = \frac{dy}{dx}$

$y = f(x)$

ex: $f(x) = 3x^2+5$

- find the slope of the tangent line at a point x
- find the tangent line when $x=1$

ex $f(x) = \frac{1}{2}x - 3$

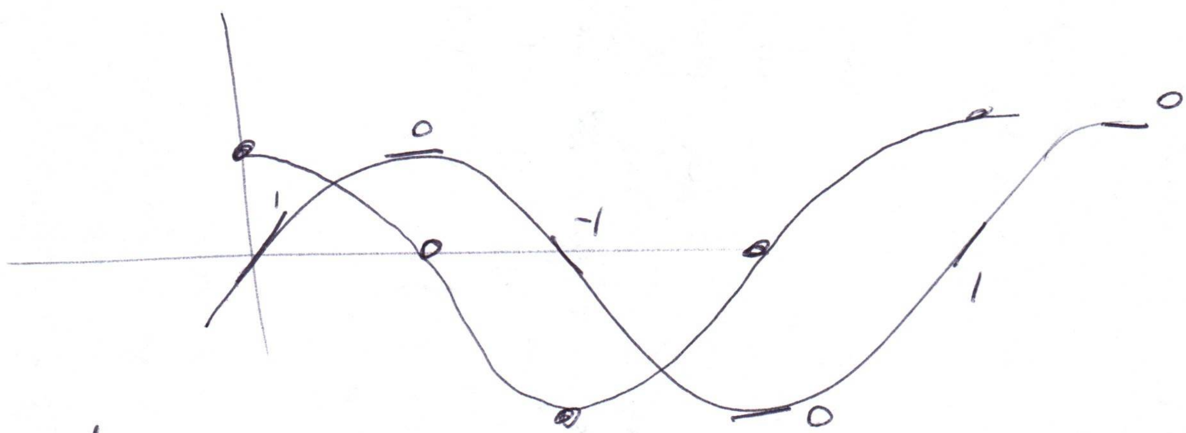
ex $f(x) = \frac{1}{x^2}$

ex $f(x) = 10$

ex $f(x) = \dfrac{4}{3x-2}$

ex

derivatives of sine, cosine, e^x



$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(-\sin(x)) = -\cos(x)$$

$$\frac{d}{dx}(-\cos(x)) = \sin(x)$$

$$\frac{d}{dx} e^x = e^x$$

examples

$$\frac{d}{dx}\left(\sin(x) + 3\cos(x) + 4e^x \right)$$

$$\cos(x) - 3\sin(x) + 4e^x$$

if $f(x) = \sqrt{x} \cancel{+} -\sin(x)$

find $f'(\pi)$.

find the x value where y' is 2 if

$$y = 3x^{5/2}$$

$$y' = 3\left(\frac{5}{2}x^{3/2}\right) = 2$$

$$15x^{3/2} = 4$$

$$x^{3/2} = \frac{4}{15}$$

$$x^3 = \left(\frac{4}{15}\right)^2$$

$$x = \left(\frac{4}{15}\right)^{2/3}$$

The power rule

notes:

Quiz Monday

Note taker needed (\$110)

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx} x^4 = 4x^3$$

$$\frac{d}{dx} x^5 = \_\_\_\_$$

$$\frac{d}{dx} x^{-2} = \frac{d}{dx}\frac{1}{x^2} = -\frac{2}{x^3} = -2x^{-3}$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

examples:

$$\frac{d}{dx} x^{100} = 100x^{99}$$

$$\frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{\frac{1}{2}}{\sqrt{x}}$$

$$\frac{d}{dx}\sqrt[10]{x^3} = \frac{d}{dx} x^{\frac{3}{10}} = \frac{3}{10}x^{-\frac{7}{10}} = \frac{3}{10\sqrt[10]{x^7}}$$

$$\frac{d}{dx} x^3\Big|_{x=2} = 3x^2\Big|_{x=2} = 12$$

Question: $\frac{d}{dx} x^0 = ?$

constant rule

$$\frac{d}{dx} c = 0$$

constant multiple rule: $\frac{d}{dx} c f(x) = c\frac{d}{dx} f(x) = c\frac{df(x)}{dx}$

sum rule: $\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$

difference rule: $\frac{d}{dx}\left(f(x)-g(x)\right) = \frac{df(x)}{dx} - \frac{dg(x)}{dx}$

ex $\frac{d}{dx}\, 3x^2+5 = 3\frac{d}{dx}x^2 + \frac{d}{dx}5$

$= 3(2x) + 0$

$= 6x$

ex $\frac{d}{dx}\left(5x^3 - \sqrt{x^3}\right)$

$5\frac{d}{dx}x^3 - \frac{d}{dx}\sqrt{x^3}$

$5(3x^2) - \frac{d}{dx}\, x^{\frac{3}{2}}$

$5(3x^2) - \frac{3}{2}\, x^{\frac{1}{2}}$

$15x^2 - \frac{3}{2}\sqrt{x}$

ex $\frac{d}{dx}\left(\frac{x^{2.3} - x^{1/2}}{x}\right)$

$\frac{d}{dx}\left(x^{1.3} - x^{-1/2}\right)$

$1.3x^{0.3} - \left(-\frac{1}{2}\right)x^{-3/2}$

Instantaneous Rate of Change

The position of an object is given by

$$s(t) = 5t^2 - 3t + 6$$

Find the position and velocity when $t = 3$.

$$s(3) = 5(3)^2 - 3(3) + 6$$
$$= 5 \cdot 9 - 9 + 6$$
$$= 45 - 9 + 6$$
$$= 42$$

$$v(t) = s'(t) = 10t - 3$$
$$s'(3) = 10(3) - 3$$
$$= 27$$

The number of rats in the hold of a pirate ship is given by

$$R(t) = 2e^{t} + 3t$$

where t is in weeks. How many rats will there be by the end of their six week voyage? How long until they are adding 100 rats/week?

The amount of money in my savings account is described by $M(t) = 15t^2 - \frac{10}{3}t^3 + 30$. ~~How fast is~~ where M is in dollars and t is in months since school started. How fast am I adding money one month in? four months in?

I have determined that my happiness is related to the amount of chocolate I eat by the formula

$$H(c) = 3\sin(c) - 0.7$$

where H is in smiles and c is in bars.

Find $\frac{dH}{dc}$ when I have had 1, 2, and 3 bars of chocolate. Use that info to tell me when to stop.

~~The temperature of my tea is given by~~

The population of the Hornburg Fortress during the battle ~~for~~ of Helm's Deep is given by $P(t) = \cancel{3800}\ 3000 - \frac{\cancel{1}}{20}t^2$

where t is in hours since midnight. How fast are the Rohirrim getting slaughtered by the uruk-hai at 6:00 AM, just before Gandalf arrives with reinforcements?

Quiz 3 — MA16010 — September 11, 2017
Alden Bradford

1. (1 point) What is the definition of $f'(x)$,
   the derivative of $f(x)$?

2. (2 points) What is $\dfrac{dy}{dx}$ when $y = 3e^x - \cos(x) + \sqrt{x}$?

3. (2 points) Find $f'(x)$ using the limit definition when

$$f(x) = \frac{2}{3 - 4x}.$$

Sept 11

Two ways to differentiate:

$$\frac{d}{dx}(x^2+4)(5x^4+3)$$

first way: multiply first, then differentiate

$$\frac{d}{dx}\left(5x^6+20x^4+3x^2+12\right)$$

$$30x^5+80x^3+6x$$

second way: Product rule!

$$\frac{d}{dx} f(x)g(x) = f'(x)\,g(x)+f(x)\,g'(x)$$

in this example:

$$f(x)=x^2+4 \qquad g(x)=5x^4+3$$

$$f'(x)=2x \qquad g'(x)=20x^3$$

$$\frac{d}{dx}(x^2+4)(5x^4+3)=2x(5x^4+3)+20x^3(x^2+4)$$

$$=10x^5+6x+20x^5+80x^3$$

$$=\underline{30x^5+80x^3+6x}$$

<u>another example</u>

$$\frac{d}{dx}x^{5/7}(x+1)=\left(\frac{d}{dx}x^{5/7}\right)(x+1)+\left(x^{5/7}\right)\left(\frac{d}{dx}(x+1)\right)$$

$$=\frac{5}{7}x^{-2/7}(x+1)+x^{5/7}(1)$$

$$=\frac{5}{7}x^{5/7}+\frac{5}{7}x^{-2/7}+x^{5/7}$$

More examples

$$\frac{d}{dx}\, 5x\cos(x) = \left(\frac{d}{dx}5x\right)\cos x + 5x\,\frac{d}{dx}\cos x$$
$$= 5\cos x + 5x(-\sin x)$$
$$= 5\cos x - 5x\sin x$$

$$\frac{d}{dx}\, e^x x^3 = \left(\frac{d}{dx}e^x\right)x^3 + e^x\frac{d}{dx}x^3$$
$$= e^x x^3 + e^x\, 3x^2$$
$$= (x^3+3x^2)e^x$$

$$\frac{d}{dx}\cos(x)\sin(x) = \left(\frac{d}{dx}\cos x\right)\sin x + \cos x\left(\frac{d}{dx}\sin x\right)$$
$$= -\sin x\,\sin x + \cos x\,\cos x$$
$$= \cos^2 x - \sin^2 x$$

$$\frac{d}{dx}\sin x\, e^x = \cos x\, e^x + \sin x\, e^x$$
$$= (\cos x + \sin x)e^x$$

$$\frac{d}{dx}\sqrt{x}\,\sin x = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$$

$$\frac{d}{dx}\left((8x+3)e^x + 2x^2\cos x\right)$$

$$\frac{d}{dx}\left(5e^x\sin x + x^{3/2}e^x\right)$$ (factor first)

Find the x values where the graph

$$y = x^3 e^x$$

has a horizontal tangent

Quiz 3 Key — MA16010 — September 11, 2017

Alden Bradford

Min	Mean	Max
1	3.4	5

1. (1 point) What is the definition of $f'(x)$, the derivative of $f(x)$?

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

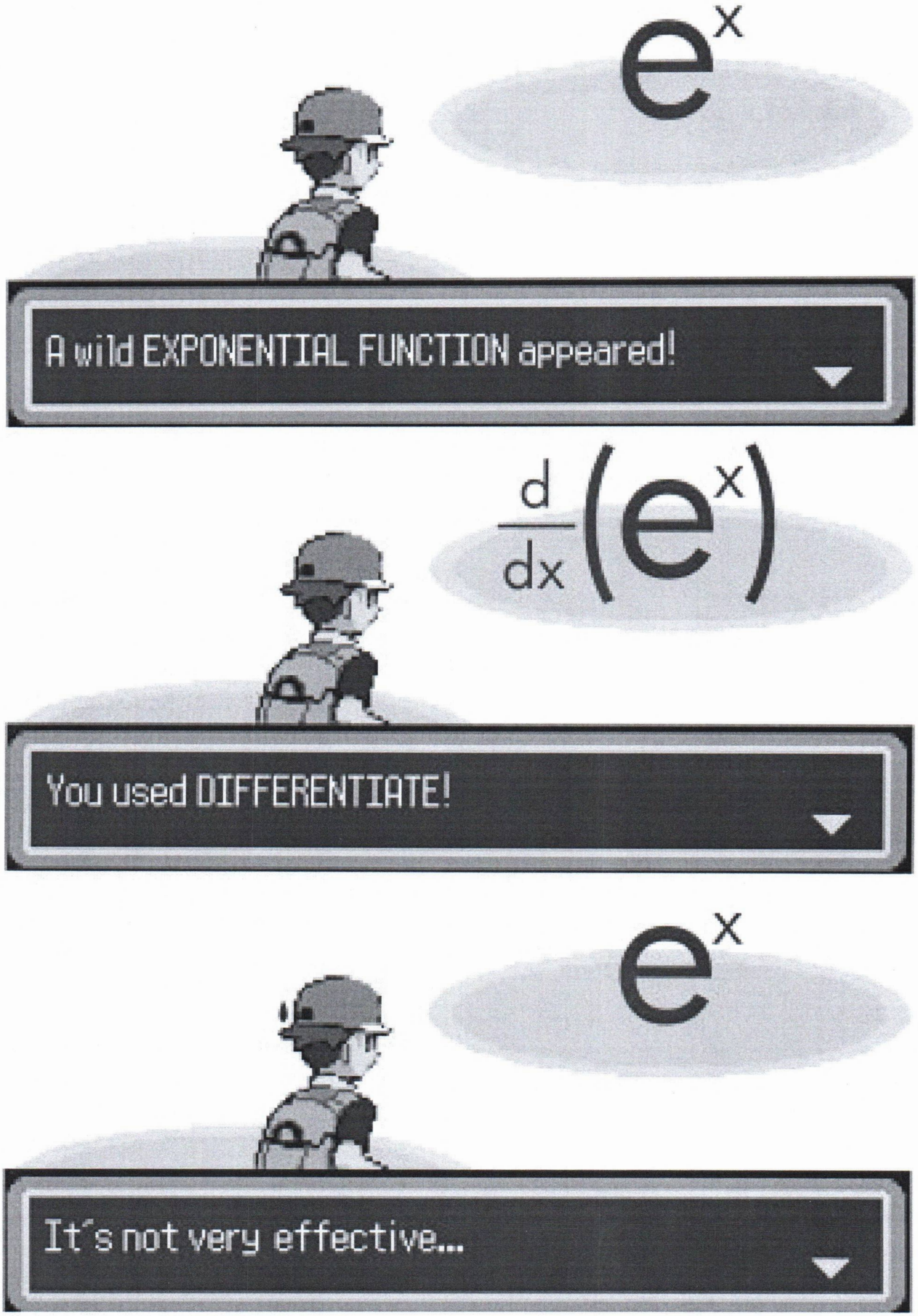
2. (2 points) What is $\frac{dy}{dx}$ when $y = 3e^x - \cos(x) + \sqrt{x}$?

$$\boxed{\frac{dy}{dx} = 3e^x + \sin(x) + \frac{1}{2\sqrt{x}}}$$

3. (2 points) Find $f'(x)$ using the limit definition when

$$f(x) = \frac{2}{3-4x}.$$

$$\begin{aligned}
f(x+h) &= \frac{2}{3-4x-4h} \\
f(x+h) - f(x) &= \frac{2}{3-4x-4h} - \frac{2}{3-4x} \\
&= \frac{2(3-4x) - 2(3-4x-4h)}{(3-4x-4h)(3-4x)} \\
&= \frac{8h}{(3-4x-4h)(3-4x)} \\
\frac{f(x+h)-f(x)}{h} &= \frac{8}{(3-4x-4h)(3-4x)} \\
f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \frac{8}{(3-4x)^2}
\end{aligned}$$


e^x
A wild EXPONENTIAL FUNCTION appeared!
$\frac{d}{dx}(e^x)$
You used DIFFERENTIATE!
e^x
It's not very effective...

Derivatives of Quotients

$$\frac{d}{dx} \frac{8x^5+1}{x^2+3} \; ?$$

Quotient rule

$$\boxed{\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}}$$

$f(x) = 8x^5+1 \qquad g(x) = x^2+3$

$f'(x) = 40x^4 \qquad g'(x) = 2x$

$$\frac{d}{dx} \frac{8x^5+1}{x^2+3} = \frac{40x^4(x^2+3) - (8x^5+1)2x}{(2x)^2}$$

examples

$$\frac{d}{dx} \frac{3x^3}{x^{10}-5}$$

$$\frac{d}{dx} \frac{\sqrt[3]{x}}{1-x^2}$$

$$\frac{d}{dx} \frac{2}{3-4x}$$

$$\frac{d}{dx} \frac{2b+x}{x+x^3}, \; b \text{ const.}$$

$$\frac{d}{dx} \frac{\sin x}{x^2+1}$$

$$\frac{d}{dx} \frac{\sin x + \cos x}{\sin x - \cos x}$$

$$\frac{d}{dx} \frac{e^x}{\cos x}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\frac{d}{dx}\ \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta\cos\theta - (-\sin\theta\sin\theta)}{(\cos\theta)^2} = \frac{1}{(\cos\theta)^2} = \sec^2\theta$$

$$\frac{d}{dx}\ \frac{\cos\theta}{\sin\theta} = \frac{-\cos\theta(\cos\theta) + (-\sin\theta\sin\theta)}{(\sin\theta)^2} = -\frac{1}{\sin^2\theta} = -\csc^2\theta$$

$$\cancel{\frac{d}{dx}\ \frac{1}{\cos\theta} = \frac{\cos\theta\left(\frac{d}{d\theta}1\right) + 1(-\sin\theta)}{\cos\theta}}$$

$$\frac{d}{dx}\ \sec\theta = \sec\theta\tan\theta$$

$$\frac{d}{dx}\ \csc\theta = -\csc\theta\cot\theta$$

---

examples

$$\frac{d}{dx}\ 8\sin x\tan x$$

$$\frac{d}{dx}\ \frac{\sin x}{x}$$

$$\frac{d}{dx}\ e^x\sec x$$

$$\frac{d}{dx}\ \frac{x^2}{\tan x + \cos x}$$

$$\frac{d}{dx}\ x\tan x$$

1

Due date: Fri Sep 15 10:00:00 pm 2017 (EDT)

Find the derivative of $y = \frac{x^7}{x^5+6x+8}$.

$y' =$

Tries 0/99

Answer for Part: 0	• (2*x^11+36*x^7+56*x^6)/(x^5+6*x+8)^2

2 Find the derivative of $y = \frac{3\sqrt[3]{x}}{x^2+1}$.

$y' =$

Tries 0/99

Answer for Part: 0	• ((x^2+1)/x^(2/3)-6*x^(4/3))/(x^2+1)^2

3 Given $f(x) = \frac{\sin x}{3x+1}$. Find $f'(\pi)$.

$f'(\pi) =$

Tries 0/99

Answer for Part: 0	• -1/(3*pi+1)

4 Given $g(x) = \frac{4\sin x - 4\cos x}{\sin x + \cos x}$. Find $g'(x)$.

$g'(x) =$

Tries 0/99

Answer for Part: 0	• 8/(sin(x)+cos(x))^2

5 Find the derivative of $y = \frac{x-6}{7\sqrt{x}+7}$ at $x = 36$.

$y'(36) =$

Tries 0/99

Answer for Part: 0	• 9/686

6 Given $f(t) = \frac{e^t}{1-e^t}$. Find $f'(2)$.

$f'(2) =$

Tries 0/99

Answer for Part: 0	• e^2/(1-e^2)^2

7 Given $y = \frac{5(a^2-x^2)}{a^2+x^2}$, where a is a constant. Find y'.

$y' =$

Tries 0/99

Answer for Part: 0	• -20*x*a^2/(a^2+x^2)^2

8 Find the equation for the tangent line to the graph of $f(x) = \frac{x^2-4}{5-x}$ at $x = 2$.

$y =$

Tries 0/99

Answer for Part: 0	• 4*x/3-8/3

9 Find the equation of the tangent line to the graph of $f(x) = \frac{x-2}{x^3+6x-6}$ at $x = 2$.

$y =$

Tries 0/99

Answer for Part: 0	• x/14-1/7

10 The curve $y = \frac{7}{6+x^2}$ is an example of a class of curves each of which is called a witch of Agnesi. Find the equation of the tangent line to the curve at $x = 1$.

$y =$

Tries 0/99

Answer for Part: 0	• 9/7-2*x/7

11 Given $y = 7\sin x \tan x$, find $y'\left(\frac{\pi}{3}\right)$.

$y'\left(\frac{\pi}{3}\right)$

Tries 0/99

Answer for Part: 0	• 35*sqrt(3)/2

12 Find the derivative of $y = 9e^x \csc x$.

$y' =$

Tries 0/99

Answer for Part: 0	• 9*e^x*(csc(x)-cot(x)*csc(x))

13 Differentiate $y = 7x^{10} \csc x$.

$y' =$ [ ]

Tries 0/99

Answer for Part: 0	• 70*x^9*csc(x)-7*x^10*cot(x)*csc(x)

14 Find the derivative of $y = 10 \tan x \sec x$.

$y' =$ [ ]

Tries 0/99

Answer for Part: 0	• 1_5_1_1 • 10*sec(x)*tan(x)^2+10*sec(x)^3 • 1_5_1_2 • 10*(sin(x)^2+1)/cos(x)^3

15 Find the derivative of $g(x) = 7x^3 \cot x$ at $x = \frac{\pi}{2}$.

$g'\left(\frac{\pi}{2}\right) =$ [ ]

Tries 0/99

Answer for Part: 0	• -7*pi^3/8

16 Given $f(x) = \frac{10 \cot x}{8+6 \cos x}$.

Find $f'(\frac{\pi}{2})$.

[ ]

Tries 0/99

Answer for Part: 0	• -5/4

17 Find the equation of the tangent line to the graph of $y = 9x^9 \sec x$ at $x = \pi$.

$y =$ [ ]

Tries 0/99

Answer for Part: 0	• 72*pi^9-81*pi^8*x

The Chain Rule

Lucy grows twice as fast as Ben

Ben grows thee times as fast as Cosmo.

How fast does Lucy grow, compared to Cosmo?

answer: $2 \times 3 = 6$ times as fast.

mathematically: $\frac{dL}{dB} = 2$, $\frac{dB}{dC} = 3$, $\frac{dL}{dC} = \frac{dL}{dB} \times \frac{dB}{dC} = 2 \times 3 = 6$

The chain rule:

$$\frac{df}{dx} = \frac{df}{dg} \times \frac{dg}{dx}$$

$$\frac{d}{dx} f(g(x)) = f'(g)\,g'(x)$$
$$= f'(g(x))\,g'(x)$$

---

examples

$$\frac{d}{dx}\left(x^2+a^2\right)^{3/2}$$, a is a constant

$f(g) = (g)^{3/2}$ $\qquad$ $g(x) = x^2+a^2$

$f'(g) = \frac{3}{2} g^{\frac{1}{2}}$ $\qquad$ $g'(x) = 2x$

$$\frac{d}{dx}(x^2+a^2)^{3/2} = \left(\frac{3}{2} g^{\frac{1}{2}}\right)(2x)$$
$$= \frac{3}{2}(x^2+a^2)^{\frac{1}{2}}\, 2x$$

$$\frac{d}{dx}\left(x^3-6x^2-2\right)^4$$

$f(g) = (g)^4$ $\qquad$ $g(x) = x^3-6x^2-2$

$f'(g) = 4g^3$ $\qquad$ $g'(x) = 3x^2-12x$

$$\frac{d}{dx}(x^3-6x^2-2)^4 = 4g^3(3x^2-12x) = 4(x^3-6x^2-2)^3(3x^2-12x)$$

$$\frac{d}{dx}\left(\sin(x)\right)^2 = 2\sin x \cos x$$

$$\frac{d}{dx} e^{3x} = e^{3x}\, 3 = 3e^{3x}$$

$$\frac{d}{dx} e^{5x^2+1} = e^{5x^2+1}\frac{d}{dx}(5x^2+1)$$

$$= e^{5x^2+1}\,(10x)$$

$$= 10x e^{5x^2+1}$$

$$\frac{d}{dx}\sin(5e^{x^3}) = \cos(5e^{x^3})\frac{d}{dx}(5e^{x^3})$$

$$= \cos(5e^{x^3})\, 5e^{x^3}\frac{d}{dx}x^3$$

$$= \cos(5e^{x^3})\, 5e^{x^3}\, 3x^2$$

$$\frac{d}{dx}\sqrt{\frac{x+1}{x-1}} = \frac{1}{2\sqrt{\frac{x+1}{x-1}}}\,\frac{d}{dx}\frac{x+1}{x-1}$$

$$= \frac{1}{2\sqrt{\frac{x+1}{x-1}}}\,\frac{(x-1)-(x+1)}{(x-1)^2}$$

$$= \frac{\sqrt{x-1}}{2\sqrt{x+1}}\,\frac{-2}{(x-1)^2}$$

$$= -\frac{1}{\sqrt{x+1}\,(x-1)^{3/2}}$$

$$\frac{d}{dx}\frac{1}{\sqrt{x^3+1}} \qquad f(g)=\frac{1}{\sqrt{g}}=g^{-\frac{1}{2}} \qquad g(x)=x^3+1$$

$$f'(g)=-\frac{1}{2}g^{-\frac{3}{2}} \qquad g'(x)=3x^2$$

$$\frac{d}{dx}\frac{1}{\sqrt{x^3+1}} = -\frac{1}{2}g^{-\frac{3}{2}}\,3x^2$$

$$= -\frac{3x^2}{(x^3+1)^{3/2}}$$

$$\frac{d}{dx}\ 3\sin\left(2\pi x+\frac{\pi}{2}\right)$$

$$f(g)=3\sin(g) \qquad g(x)=2\pi x+\frac{\pi}{2}$$

$$f'(g)=3\cancel{\sin}\cos(g) \qquad g'(x)=2\pi$$

$$\frac{d}{dx}3\sin\left(2\pi x+\frac{\pi}{2}\right)=3\cos(g)\,2\pi$$

$$=3\cos\left(2\pi x+\frac{\pi}{2}\right)2\pi$$

Skip

$$\frac{d}{dx}\frac{1}{\sqrt{2\pi}\,\sigma}e^{-x^2/2\sigma^2}, \quad \sigma \text{ is a constant}$$

$$f(g)=\frac{1}{\sqrt{2\pi}\,\sigma}e^{g} \qquad g(x)=-\frac{x^2}{2\sigma^2}$$

$$f'(g)=\frac{1}{\sqrt{2\pi}\,\sigma}e^{g} \qquad g'(x)=-\frac{x}{\sigma^2}$$

$$\frac{d}{dx}\frac{1}{\sqrt{2\pi}\,\sigma}e^{-x^2/2\sigma^2}=\frac{1}{\sqrt{2\pi}\,\sigma}e^{g}\left(-\frac{x}{\sigma^2}\right)$$

$$=-\frac{x}{\sqrt{2\pi}\,\sigma^3}e^{-x^2/2\sigma^2}$$

Due date: Mon Sep 18 10:00:00 pm 2017 (EDT)

1 Find the derivative of $y = (x^5 + 1)^7$.

$y' =$ [ ]
Tries 0/99

---

2 Find the derivative of $y = (8 - 3x^3)^3$ at $x = 2$.

$y'(2) =$ [ ]
Tries 0/99

---

3 Find the derivative of $y = 2(10x^2 - 3x + 1)^{-8}$.

$y' =$ [ ]
Tries 0/99

---

4 Find the derivative of $f(x) = \sqrt[3]{6 + 9x^8}$.

$f'(x) =$ [ ]
Tries 0/99

Find the derivative of $h(x) = \sqrt[3]{8x^8 + 4x^4}$.

5 $h'(x) =$ [ ]
Tries 0/99

---

6 Find the derivative of $y = \sqrt{3x^3 - 5x^2 - \frac{5}{x}}$.

$y' =$ [ ]
Tries 0/99

---

7 Find the derivative of $y = \frac{10}{(10-x^8)^{\frac{3}{2}}}$.

$y' =$ [ ]
Tries 0/99

---

8 Find the derivative of $g(x) = (\frac{3x^2}{2x+4})^3$ at $x = 2$. Keep your answer exact.

$g'(2) =$ [ ]
Tries 0/99

Find the derivative of $y = \sqrt{r^2 - 2x^2}$, where r is a constant.

9 $y' =$ [ ]
Tries 0/99

---

10 Find the derivative of $y = 20(\sin x + 4)^5$.

$y' =$

Tries 0/99

---

11 Find the derivative of $y = 6(3e^x - 24)^9$.

$y' =$

Tries 0/99

The Logarithm

$$\ln(AB) = \ln(A) + \ln(B)$$
$$\ln(A^B) = B\ln(A)$$
$$\ln(A/B) = \ln(A) - \ln(B)$$
$$\ln(1) = 0$$
$$\ln(e) = 1$$
$$\ln(e^x) = x$$
$$\cancel{\ln}\ e^{\ln(x)} = x$$
$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln(5x)$$

$$\frac{d}{dx}\cos(3x) + 5\ln(x)$$

$$\frac{d}{dx}\ln(5x^2+2x+1)\tan(\pi x)$$

$$\frac{d}{dx}\ln\left(\sqrt{\frac{(x+5)^4}{2x^3}}\right)$$

"Lumberjacks make good musicians because of their natural logarithms"

$$\frac{d}{dx} \ln\left(\frac{\sqrt{x+1}}{x-1}\right)$$

$$\frac{d}{dx} \csc x \ln(x^2+1)$$

Quiz 4 — MA16010 — September 18, 2017

Alden Bradford

1. (3 points) Find $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify your answer.

   (a) $y = (2x^2 + 5x)e^x$

   (b) $y = \dfrac{5x + 3}{2\cos(x) - 1}$

   (c) $y = \cos(x)\sin(x)$

2. (2 points) For a certain function $f(x)$ (which you do not know), we do know that $f(3) = 2$ and $f'(3) = -4$. Write an equation for the line tangent to $f(x)$ at $x = 3$.

Higher-Order Derivatives

given $f(x) = x^3 + 2x^2$

find $\frac{d}{dx} f'(x)$.

$$f'(x) = 3x^2 + 4x$$

$$\frac{d}{dx} f'(x) = \frac{d}{dx}(3x^2 + 4x)$$

$$f''(x) = 6x + 4$$

find $f'''(x)$: $f'''(x) = \frac{d}{dx} f''(x)$

$$= \frac{d}{dx}(6x + 4)$$

$$f^{(3)}(x) = 6$$

find $f^{(4)}(x)$: $f^{(4)}(x) = \frac{d}{dx} f^{(3)}(x)$

$$= \frac{d}{dx} 6$$

$$= 0$$

---

given $f(x) = e^x \cos x$

find $f''(0)$.

---

given $f(x) = 3\sqrt{x}$

find $f'''(9)$

---

given $f^{(6)}(x) = 24x \sin x$

find $f^{(7)}(x)$

---

position of a particle is given by

$$S(t) = \frac{3t^2}{2t+1}$$

find $V(t)$ and $a(t)$

---

velocity of a particle is given by

$$V(t) = \frac{\sin(x)}{x}$$

find $a(t)$

Due date: Wed Sep 20 10:00:00 pm 2017 (EDT)

1 Find the derivative of $y = (5x-1)^2(x^4+2)^2$ at $x = 1$.

$y' =$ ______

Tries 0/99

---

2 Find the derivative of $g(x) = 5x^3\sqrt{25-x^2}$ at $x = 4$.

$g'(4) =$ ______

Tries 0/99

---

3 Find the derivative of $g(x) = \frac{\sqrt{25-x^2}}{5x}$ at $x = 2$.

$g'(2) =$ ______

Tries 0/99

---

4 Differentiate $y = 7\csc(6x^3 - 3x + 7)$.

$y' =$ ______

Tries 0/99

---

5 Find the derivative of $y = 2\tan^2(6x)$ at $x = \frac{\pi}{18}$.

$y'\left(\frac{\pi}{18}\right) =$ ______

Tries 0/99

---

6 Find the derivative of $f(x) = e^{8x}\cos(3x)$.

$f'(x) =$ ______

Tries 0/99

---

7 Find the equation for the tangent line to $y = (x^2 - 10x + 4)\sqrt{25-x^2}$ at $x = 3$.

$y =$ ______

Tries 0/99

---

8 At sea level, air pressure is 30 inches of mercury. At an altitude of h feet above the sea level, the air Pressure, P, in inches of mercury, is given by the function

$$P = 30e^{-0.0000323h}.$$

Determine the rate of change of the air pressure, in inches of mercury per feet, for a pilot of a small plane passing through an altitude of 1800 feet. Round your answer to five decimal places.

______

Tries 0/99

---

9 The position, in meters, of a particle moving on a straight line is given by

$$s(t) = \frac{4400t}{(t^2+5)^3},$$

where t is time in seconds. What is the particle's velocity, in meters per second, at $t = 10$ seconds? Round your answer to two decimal places.

______

Tries 0/99

---

10 Find the derivative of $f(x) = \sqrt{9x}\ln(11x)$.

$f'(x) =$ ______

Tries 0/99

---

11 Find the derivative of $y = \ln(9x^7 + 4x - 8)$.

$y' =$ ______

Tries 0/99

---

12 Find the derivative of $g(x) = \frac{9\ln x}{3x+5}$ at $x = e$.

$g'(e) =$ ______

Tries 0/99

---

13 Find the derivative of $y = \ln\sqrt{\frac{2x+1}{x^2-3}}$ at $x = 4$.

$y'(4) =$ ______

Tries 0/99

Quiz 4 Key — MA16010 — September 18, 2017

Alden Bradford

Min	Mean	Max
1	3.9	5

1. (3 points) Find $\frac{dy}{dx}$ for each of the following functions. You do not need to simplify your answer.

   (a) $y = (2x^2 + 5x)e^x$

   (b) $y = \dfrac{5x+3}{2\cos(x) - 1}$

   (c) $y = \cos(x)\sin(x)$

   (a) $\dfrac{dy}{dx} = (2x^2 + 5x)e^x + (4x+5)e^x$

   (b) $\dfrac{dy}{dx} = \dfrac{5(2\cos(x) - 1) - (5x+3)(-2\sin(x))}{(2\cos(x) - 1)^2}$

   (c) $\dfrac{dy}{dx} = \cos^2 x - \sin^2 x$

2. (2 points) For a certain function $f(x)$ (which you do not know), we do know that $f(3) = 2$ and $f'(3) = -4$. Write an equation for the line tangent to $f(x)$ at $x = 3$.

   $y - 2 = -4(x - 3)$

Implicit Differentiation

Friday: Quiz on chain rule, derivative of log(x).
Next week, exams returned, applications begin
Today, implicit differentiation!

$y-5=3(x+2)$ ← differentiate implicitly to save work

$(x-y)^2=2x^3$ ← find $\frac{dy}{dx}$

$5\cancel{8}xy=y^2+4$ ← find $\frac{dy}{dx}$ at the point $(\cancel{8}1,4)$

$x^2+4y^2=\cancel{9}25$ ← find equation of a tangent line at $(3,2)$
and then at $(3,-2)$

---

find $\frac{dy}{dx}$ if

$\sin x \tan y = \ln x$

$\cos(x+3y)=8y$

$e^{5xy}=2x^2$

Due date: Wed Sep 27 10:00:00 pm 2017 (EDT)

Given $f(x) = 5\sqrt{x} - \frac{6}{x}$, answer the following questions:

(a) $f''(x) =$ ______

Tries 0/99

(b) $f''(4) =$ ______

Tries 0/99

Find the second derivative of $g(x) = 10e^{4x}\cos(10x)$.

$g''(x) =$ ______

Tries 0/99

Find the second derivative of $h(x) = 6x^5\ln(5x)$.

$h''(x) =$ ______

Tries 0/99

Given the fourth derivative of $f(x)$, find the fifth derivative of $f(x)$.

$$f^{(4)}(x) = 7\csc(2x - 9)$$

$f^{(5)}(x) =$ ______

Tries 0/99

A particle is traveling on a straight line with a velocity function of

$$v(t) = \frac{2t^2 + 2}{4t + 5},$$

where t is time in seconds and $v(t)$ is velocity in m/sec. Answer the following questions:

(a) What is the particle's acceleration?

$a(t) =$ ______ $\mathrm{m/sec^2}$

Tries 0/99

(b) What is the acceleration at $t = 2$ seconds?

______ $\mathrm{m/sec^2}$

Tries 0/99

A particle is traveling on a straight line with a position function of

$$s(t) = \frac{2}{3}t^3 + 6t^2,$$

where t is time in seconds and $s(t)$ is position in feet. Answer the following questions:

(a) What is the particle's acceleration?

$a(t) =$ ______ $\mathrm{ft/sec^2}$

Tries 0/99

(b) What is the acceleration when the velocity of the particle is 54 ft/sec?

______ $\mathrm{ft/sec^2}$

Tries 0/99

Due date: Fri Sep 29 10:00:00 pm 2017 (EDT)

Use implicit differentiation to find $\frac{dy}{dx}$ given $13y^2 = 15+12x^2$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $10x^2 + 3xy + 7y^2 = 19$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $8x^3 + 6xy^2 = 4y^3 + 6yx^2$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Find the equation of the tangent line to $6x^4 = 4y^2 + 3x^2$ at $(2, \sqrt{21})$.

$y =$ [ ]
Tries 0/99

Use implicit differentiation to find the slope of the tangent line to the graph of $\frac{3}{x} + \frac{1}{3y} = 7$ at $(1, \frac{1}{12})$.

$\left.\frac{dy}{dx}\right|_{(1,\frac{1}{12})} =$ [ ]
Tries 0/99

Use implicit differentiation to find the slope of the tangent line to the graph of $18\sqrt{x} + 3\sqrt{y} = 11$ at $(\frac{64}{324}, 1)$.

$\left.\frac{dy}{dx}\right|_{(\frac{64}{324},1)} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $2\cos x \sin y = 5$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $2\sin(10x + 10y) = 3xy$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $9\tan\left(\frac{x}{y}\right) = 9x$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Use implicit differentiation to find $\frac{dy}{dx}$ given $e^{9xy} = 3x$.

$\frac{dy}{dx} =$ [ ]
Tries 0/99

Quiz 5 — MA16010 — September 29, 2017

Alden Bradford

1. (1 point) Find $\dfrac{d^2y}{dx^2}$ if $y = 2\ln(3x)$.

2. (2 points) Find $f'(x)$ when $f(x) = (\cos(x))^3$.

3. (2 points) Find $g'(x)$ when $g(x) = x^4 e^{2x}$.

Related Rates

$3x^2 + y = -xy$ ← find $\frac{dx}{dt}$ when $x = 1$, $y = 2$, $\frac{dy}{dt} = -4$

At my old (crappy) studio apartment (don't rent from crestview) there was a leak in the ceiling dripping onto the counter, at 2mL/minute (2 cm³/minute) making a circular puddle that was $\frac{1}{4}$ cm tall. When I found it, the puddle was 60 cm across. How fast was the radius of the puddle increasing?

$$V = \pi r^2 h = \frac{\pi}{4} r^2$$

$$\frac{dV}{dt} = \frac{\pi}{2} r \frac{dr}{dt}$$

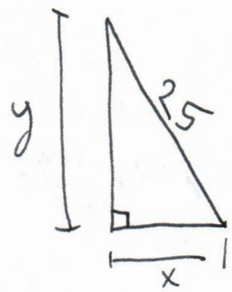
$$2 = \frac{\pi}{2} 30 \frac{dr}{dt}$$

$$\frac{4}{30\pi} = \frac{dr}{dt} = 0.042 \text{ cm/minute}$$

procedure

1. use geometry to relate the variables
2. differentiate with respect to time (t)
3. use given information to find the rate we need

A 25-foot ladder is resting against a wall. It starts to slide, with the top moving down at 6 inches per second. I am sitting 7 feet from the wall. How fast is the base of the ladder moving when it hits me?



$$x^2+y^2=25^2$$
$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$
$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$
$$x\frac{dx}{dt}=-y\frac{dy}{dt}$$
$$\frac{dx}{dt}=-\frac{y}{x}\frac{dy}{dt}$$
$$\frac{dx}{dt}=-\frac{24}{7}\left(-\frac{1}{2}\right)$$
$$=\frac{24}{14}$$
$$=1.71 \text{ feet/sec}$$

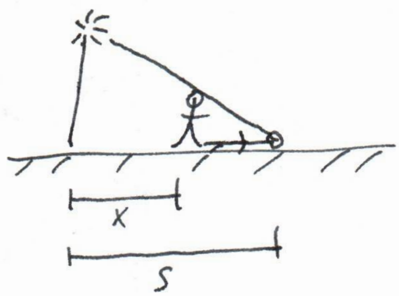
$$\frac{dy}{dt}=-\frac{1}{2}$$
$$x=7$$
$$7^2+y^2=25^2$$
$$y=24$$

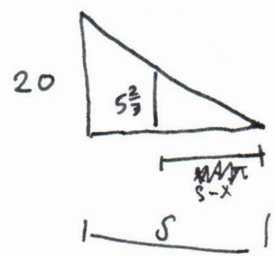
I left an ice cube sitting on the counter (oops!). Its side-length decreases at 2 $\frac{cm}{hour}$. How fast is the volume changing when the length is 1 cm?

$$V=l^3$$
$$\frac{dV}{dt}=3l^2\frac{dl}{dt}$$
$$\frac{dV}{dt}=3(1^2)(-2)$$
$$=-6 \text{ cm}^3/\text{hr}$$

My hummingbird feeder has an "ant-moat" above it, which is an inverted cone, 4 inches tall and 3 inches in diameter. Rain fills the moat at 2.5 in^3/hour. How fast is the height of the water increasing when the water is 3 inches deep?

I am 5'8" tall. One night (after a long evening of grading) I walked home from my office and to my surprise, the 20 foot tall street light turned on. As I walked away from it, I saw my shadow go in front of me. I walk 5 feet per second, but my shadow moves faster. How fast does the tip of my shadow move, exactly?





$$\frac{s-x}{5\frac{2}{3}} = \frac{s}{20}$$

$$20s - 20x = 5\tfrac{2}{3}s$$

$$14\tfrac{1}{3}s = 20x$$

$$\frac{ds}{dt} = \frac{20}{14\frac{1}{3}}\frac{dx}{dt}$$

$$= 6.98\ \frac{ft}{sec}$$

$$\cancel{\frac{x-s}{5\frac{2}{3}} = \frac{s}{20}}$$

$$\cancel{20x - 20s = 5\tfrac{2}{3}s}$$

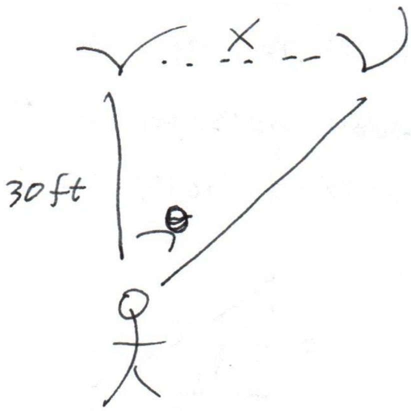
$$\cancel{20x = 5\tfrac{2}{3}s}$$

$$\cancel{20\frac{dx}{dt} = 5\tfrac{2}{3}\frac{ds}{dt}}$$

$$\cancel{20(5) = 5\tfrac{2}{3}\frac{ds}{dt}}$$

$$\cancel{\frac{ds}{dt} = \frac{100}{5\frac{2}{3}} = 17.65\ \frac{feet}{sec}}$$

I happend to see a heron the other day. I think it passed about 30 feet above my head. I estimated it was ~~travel~~ tracing an angle of ~~radian~~ 60°/sec when it was right above me. How fast was it flying?



$$\tan\theta = \frac{x}{30}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$$

$$\cancel{\underset{(1)}{\sec^2(0)}\,(1) = \frac{1}{30}\frac{dx}{dt}}$$

$$\cancel{\frac{dx}{dt} = 30\ \frac{ft}{sec}}$$

$$\frac{60^\circ}{sec} = \frac{\frac{60}{180}\pi \text{ rad}}{sec}$$

$$\frac{dx}{dt} = \sec^2(0)\left(\frac{\pi}{3}\right)(30)$$

$$= 10\pi \text{ ft/sec}$$

$$\approx 31.4 \text{ ft/sec}$$

Quiz 5 Key — MA16010 — September 29, 2017

Alden Bradford

Min	Mean	Max
1	3.75	5

1. 2 points ~~(1 point)~~ Find $\frac{d^2y}{dx^2}$ if $y = 2\ln(3x)$.

$$\boxed{\frac{d^2y}{dx^2} = -\frac{2}{x^2}}$$

2. (2 points) Find $f'(x)$ when $f(x) = (\cos(x))^3$.

$$\boxed{f'(x) = 3(\cos(x))^2(-\sin(x))}$$

~~3. (2 points) Find $g'(x)$ when $g(x) = x^4e^{2x}$.~~

$$\boxed{g'(x) = 4x^3e^{2x} + 2x^4e^{2x}}$$

+1 point FREE!

Due date: Mon Oct 2 10:00:00 pm 2017 (EDT)

Assume x and y are both differentiable functions of t and $6x^7y = 60$.

1. Find $\frac{dy}{dt}$ if $\frac{dx}{dt} = 6$ and $x = 1$

Tries 0/99

2. Find $\frac{dx}{dt}$ if $\frac{dy}{dt} = 5$ and $y = 2$

Tries 0/99

The radius of a circle is increasing at the rate of 3 cm/min.

1. Find the rate of change of the perimeter of the circle when $r = 8$.

cm/min

Tries 0/99

2. Find the rate of change of the area of the circle when $r = 8$.

cm
/min

Tries 0/99

All the edges of a cube are shrinking at the rate of 3 cm/sec.

1. How fast is the volume shrinking when each edge is 14 cm?

cm^3/sec

Tries 0/99

---

2. How fast is the surface area decreasing when each edge is 14 cm?

cm^2/sec

Tries 0/99

A spherical balloon is being deflated. The radius is decreasing at the rate of 1.5 cm/sec.

1. How fast is the volume decreasing when $r = 12$ cm? Note that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius of the sphere.

cm^3/sec

Tries 0/99

2. How fast is the surface area decreasing when $r = 12$ cm? Note that the surface area of a sphere is $A = 4\pi r^2$ where r is the radius of the sphere.

cm^2/sec

Tries 0/99

A cylindrical tank standing upright (with one circular base on the ground) has a radius of 21 cm for the base. How fast does the water level in the tank drop when the water is being drained at 24 cm^3/sec? Note that the volume of a cylinder is $V = \pi r^2 h$ where r is the radius of the base and h is the height of the cylinder.

cm/sec

Tries 0/99

Sand is poured onto a surface at 11 cm^3/sec, forming a conical pile whose base diameter is always equal to its altitude. How fast is the altitude of the pile increasing when the pile is 1 cm high? Note that the volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height of the cone.

cm/sec

Tries 0/99

Critical Points

Activity: find relative maxima & minima
find critical points

critical numbers are where $f'(x)=0$ or $f'(x)$ dne.
critical numbers are x-values.

examples
Find the critical numbers:

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 4$$

$$y = 4x^2 + \frac{1}{x}$$

$$y = \tan^2(x) \quad \text{on } (0, 2\pi)$$

$$y = 2\sin(3x) - 3\sqrt{3}\,x \quad \text{on } (0, \pi)$$

$$y = \frac{x^2+4}{2x}$$

Quiz 6 — MA16010 — October 4, 2017

Alden Bradford

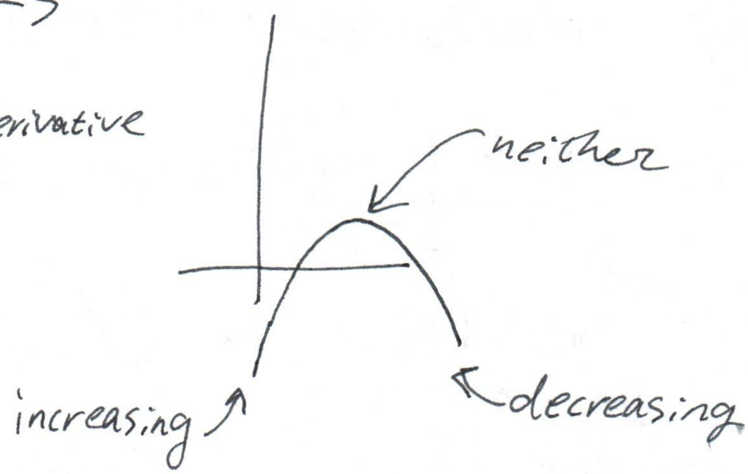
1. (2 points) Use implicit differentiation to find $\frac{dy}{dx}$ when $xy^2 = 2\ln(x)$.

2. (3 points) The sides of an ice cube are shrinking at a rate of 2 centimeters per hour. How fast is the volume of the ice cube decreasing when the side length of the cube is 3 centimeters?

increasing, decreasing, and 1st deriv. test

example

$y = -x^2 + 4x - 3$



idea: look at 1st derivative

$y' = -2x + 4$

$= -2(x-2)$

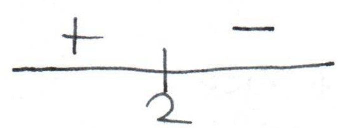
notice:

$y' = 0$ when $x = 2$

$y' > 0$ when $x < 2$

$y' < 0$ when $x > 2$

on a number line,



Facts

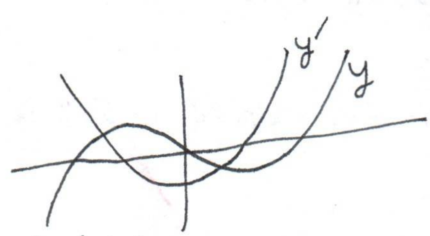
if $f'(x) > 0$ then $f(x)$ is increasing at x

if $f'(x) < 0$ then $f(x)$ is decreasing at x

if $f'(x) = 0$ then $f(x)$ is neither increasing nor decreasing

example

$y = x^3 - x + 2$

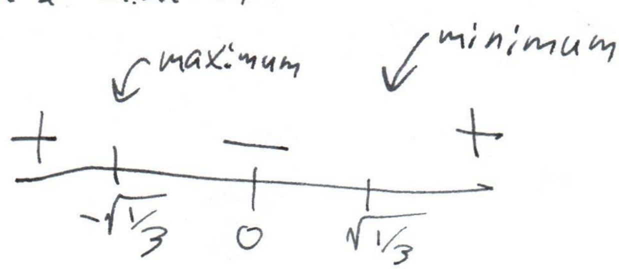


Goal: find maxima & minima

$y' = 3x^2 - 1$

$y' = 0 = 3x^2 - 1$

$x = \pm\sqrt{1/3}$



First Derivative Test

Step 1: find the critical points

Step 2: find where $f'(x)$ is positive and where $f'(x)$ is negative

Step 3: classify the critical points:

$f'(x)>0 \mid f'(x)<0$ at c ex.  $\Rightarrow$ relative max at $x=c$

$f'(x)<0 \mid f'(x)>0$ at c ex  $\Rightarrow$ relative min at $x=c$

$f'(x)>0 \mid f'(x)>0$ at c ex  $\Rightarrow$ neither

$f'(x)<0 \mid f'(x)<0$ ex  $\Rightarrow$ neither

example

apply the first derivative test to $y=-x^3$

example

apply the first derivative test to $y=3x^5-2x^4$

example

apply the first derivative test to $g(x)$ if $g'(x)=(x-3)^2(x-1)^3$

Due date: Fri Oct 6 10:00:00 pm 2017 (EDT)

2 Find the critical number(s) of $y = 4x^2 - 2x$ if there is/are any. Fill the extra blank(s), if any, with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • 1/4 • NONE

3 Find the critical numbers x_1 and x_2 of $y = 2 + 4x - 6x^3$. If there are less than two critical numbers, fill the remaining blanks with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • -sqrt(2)/3 • sqrt(2)/3

4 Find the critical number(s) of $f(x) = x^3 + 13x^2 + 9x$ if there is/are any. Fill the extra blank(s), if any, with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • (-2*sqrt(142)-26)/6 • (2*sqrt(142)-26)/6

5 Find the critical number(s) of $g(x) = 9x^4 - 4x^3$ if there is/are any. Fill the extra blank(s), if any, with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • 0 • 1/3

6 Find the critical number(s) of $y = 6x^2 - \frac{8}{x^2}$ if there is/are any. Fill the extra blank(s) with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • NONE • NONE

7 Find the critical numbers x_1 and x_2 of $y = \frac{3x^2+3}{9x}$. If there are less than two critical numbers, fill the remaining blanks with NONE.

$x_1=$ ____

$x_2=$ ____

Tries 0/99

Answer for Part: 11	• both • -1 • 1

8 Find the critical number(s) of $y = 7\cos(4x) + 14x$ on the interval of $(0, \pi)$ if there is/are any. Fill the extra blank(s), if any, with NONE.

$x_1=$ ____

$x_2=$ ____

$x_3=$ ____

$x_4=$ ____

Tries 0/99

Answer for Part: 11	• all • pi/24 • 5*pi/24 • 13*pi/24 • 17*pi/24

9 Find the critical numbers x_1 and x_2 of $y = 2 + 16x - 9x^3$. If there are fewer than two critical numbers, fill the remaining blanks with NONE.

x_1= [ ]

x_2= [ ]

Tries 0/99

Answer for Part: 0	• -4/3^(3/2), 4/3^(3/2)

10 Find the critical numbers x_1 and x_2 of $y = 2 + 5x - \frac{5x^3}{3}$. If there are fewer than two critical numbers, fill the remaining blanks with NONE.

x_1= [ ]

x_2= [ ]

Tries 0/99

Answer for Part: 0	• -1, 1

Quiz 6 Key — MA16010 — October 4, 2017

Alden Bradford

Min	Mean	Max
1	3.75	5

1. (2 points) Use implicit differentiation to find $\dfrac{dy}{dx}$ when $xy^2 = 2\ln(x)$.

$$\boxed{\frac{dy}{dx} = \frac{1}{2xy}\left(\frac{2}{x} - y^2\right)}$$

2. (3 points) The sides of an ice cube are shrinking at a rate of 2 centimeters per hour. How fast is the volume of the ice cube decreasing when the side length of the cube is 3 centimeters?

54 cubic centimeters per hour

1 If $h(t) = \sin(3t) + \cos(3t)$, find $h^{(3)}(t)$.

A. $-27\sin(3t) + 27\cos(3t)$

B. $-27\sin(3t) - 27\cos(3t)$

C. $27\sin(3t) + 27\cos(3t)$

D. $\sin(3t) + \cos(3t)$

E. $\sin(3t) - \cos(3t)$

F. $27\sin(3t) - 27\cos(3t)$

Tries 0/99

---

2 A toy rocket is launched from a platform on earth and flies straight up into the air.
Its height during the first 10 seconds after launching is given by: $s(t) = t^3 + 3t^2 + 4t + 100$, where s is measured in centimeters, and t is in seconds.
Find the velocity when the acceleration is 18 cm/s^2.

A. 2 cm/s

B. 44 cm/s

C. 13 cm/s

D. 28 cm/s

E. 16 cm/s

F. 32 cm/s

Tries 0/99

---

3 Find $\frac{dy}{dx}$ by implicit differentiation.

$$\ln(xy) + 2x = e^y$$

A. $\frac{dy}{dx} = \frac{-2-y}{x-e^y}$

B. $\frac{dy}{dx} = \frac{-2y}{1-ye^y}$

C. $\frac{dy}{dx} = \frac{1+2xy}{xye^y}$

D. $\frac{dy}{dx} = \frac{-2xy-y}{x-xye^y}$

E. $\frac{dy}{dx} = ye^y - \frac{y}{x} - 2y$

F. $\frac{dy}{dx} = \frac{-xy-y}{2x-xye^y}$

Tries 0/99

---

4 An airplane flies at an altitude of $y = 2$ miles straight towards a point directly over an observer. The speed of the plane is 500 miles per hour. Find the rate at which the observer's angle of elevation is changing when the angle is $\frac{\pi}{3}$.

A. $\frac{75}{4}$ radian per hour

B. $\frac{225}{8}$ radian per hour

C. $50\sqrt{3}$ radian per hour

D. $\frac{375}{2}$ radian per hour

E. $\frac{125\sqrt{3}}{2}$ radian per hour

F. $\frac{125}{2}$ radian per hour

Tries 0/99

5 Find the critical numbers of $y = x^2 e^x$.

A. -2 and 1

B. 0 and 2

C. -2 and 2

D. -2 and 0

E. 0 and 1

F. 1 and 2

Tries 0/99

---

6 Given $f(x) = \dfrac{2(3-x^2)}{\sqrt{3x^2+1}}$. Find $f'(1)$.

A. $-\dfrac{9}{4}$

B. $-\dfrac{1}{2}$

C. $-\dfrac{3}{4}$

D. $-\dfrac{7}{2}$

E. $-\dfrac{13}{6}$

F. $-\dfrac{3}{2}$

Tries 0/99

---

7 Find the largest open interval where $g(t)$ is increasing.

$$g(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2$$

A. $(-\infty, 0)$

B. $(3, \infty)$

C. $(0, \infty)$

D. $(0, 3)$

E. $(-\infty, 3)$

F. $(-\infty, 0) \cup (3, \infty)$

Tries 0/99

---

8 A spherical balloon is inflated with gas at a rate of 5 cubic centimeters per minute. How fast is the radius of the balloon changing at the instant when the radius is 4 centimeters?
The volume V of a sphere with a radius r is $V = \frac{4}{3}\pi r^3$.

A. $\frac{25}{4\pi}$ centimeters per minute

B. $\frac{5}{16\pi}$ centimeters per minute

C. $\frac{5}{4\pi}$ centimeters per minute

D. $\frac{5}{64\pi}$ centimeters per minute

E. $\frac{256\pi}{3}$ centimeters per minute

F. $\frac{5\pi}{64}$ centimeters per minute

Tries 0/99

---

9 Find $f'(2)$.

$$f(t) = \frac{2t-1}{(2t+1)^2}$$

A. $-\frac{2}{25}$

B. $\frac{22}{125}$

C. $\frac{4}{124}$

D. $-\frac{2}{125}$

E. $-\frac{1}{10}$

F. $\frac{2}{125}$

Tries 0/99

---

10 If $y = (\frac{2x-1}{2x+1})^3$, then $\frac{dy}{dx} =$

A. $\frac{48}{(2x+1)^4}$

B. $3(\frac{2x-1}{2x+1})^2$

C. $\frac{24x-12}{(2x+1)^3}$

D. $\frac{12(2x-1)^2}{(2x+1)^4}$

E. $\frac{6(2x-1)^2}{(2x+1)^3}$

F. $\frac{12(2x-1)^2}{(2x+1)^3}$

Tries 0/99

11 Given $f(x) = e^{5x}\ln(7x+e)$. Find $f'(0)$.

A. $1+\dfrac{1}{e}$

B. $\dfrac{1}{e}$

C. $\dfrac{35}{e}$

D. $5+\dfrac{7}{e}$

E. $\dfrac{5}{e}$

F. $1+\dfrac{7}{e}$

Tries 0/99

---

12 The price of a commodity is given by $p(t) = (t^2+2t)^2 + 100000$, where $p(t)$ is the price in dollars and t is years after 2000. At what rate is the price changing in the year of 2010?

A. \$5280/year

B. \$1680/year

C. \$2400/year

D. \$4800/year

E. \$2640/year

F. \$900/year

Tries 0/99

---

13 Find $g'(x)$ if $g(x) = \tan^2(3x^2 + 2)$.

A. $12x \tan(3x^2 + 2) \sec^2(3x^2 + 2)$

B. $12x \sec^2(3x^2 + 2)$

C. $2 \sec^2(6x)$

D. $2 \tan(6x)$

E. $6x \tan(3x^2 + 2) \sec^2(3x^2 + 2)$

F. $12x \tan(3x^2 + 2)$

Tries 0/99

---

14 Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + y^2 = 2xy + 5$.

A. 1

B. $\frac{x}{x-y}$

C. $\frac{x}{1-y}$

D. 0

E. $\dfrac{2x - 2y - 5}{2x - 2y}$

F. $\dfrac{2y - 2x + 5}{2y - 2x}$

Tries 0/99

15 All edges of a cube are expanding at a rate of 2 centimeters per second. How fast is the surface area changing when each edge is 3 centimeters?

A. 72 cm^2/sec

B. 36 cm^2/sec

C. 46 cm^2/sec

D. 12 cm^2/sec

E. 48 cm^2/sec

F. 54 cm^2/sec

Tries 0/99

---

16 Water flows into a right cylindrical shaped swimming pool with a circular base at a rate 4 m^3/min. The radius of the base is 3 m. How fast is the water level rising inside the swimming pool? The volume of a right cylinder with a circular base is $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

A. $\frac{4}{9\pi}$ m/min

B. $\frac{3}{8\pi}$ m/min

C. $\frac{2}{3\pi}$ m/min

D. $\frac{3}{16\pi}$ m/min

E. $\frac{2}{9\pi}$ m/min

F. $\frac{4}{3\pi}$ m/min

Tries 0/99

17 A 10-ft ladder, whose base is sitting on level ground, is leaning at an angle against a vertical wall when its base starts to slide away from the vertical wall. When the base of the ladder is 6 ft away from the bottom of the vertical wall, the base is sliding away at a rate of 4 ft/sec. At what rate is the vertical distance from the top of the ladder to the ground changing at this moment?

A. -3 ft/sec

B. 4 ft/sec

C. $\frac{1}{4}$ ft/sec

D. $-\frac{3}{4}$ ft/sec

E. $-\frac{1}{3}$ ft/sec

F. 8 ft/sec

Tries 0/99

---

18 Given $f(x) = \sin^3(2x)$, find $f'(\frac{\pi}{12})$.

A. $\frac{3\sqrt{3}}{4}$

B. $-\frac{3\sqrt{3}}{8}$

C. $\frac{3}{2}$

D. $\frac{9}{4}$

E. $-\frac{\sqrt{3}}{4}$

F. $\frac{1}{2}$

Tries 0/99

19 Given $f(x) = \ln \sqrt[3]{\dfrac{3+3x}{3-x}}$, find $f'(1)$.

A. $\frac{1}{3}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

E. $\frac{1}{4}$

F. $\frac{2}{3}$

Tries 0/99

---

20 Use implicit differentiation to find the equation of the tangent line to the graph at $(-2, 2)$.

$$x^2 + xy = 4 - y^2$$

A. $y = x + 4$

B. $y = -x$

C. $y = -x + 2$

D. $y = 2$

E. $y = -x + 4$

F. $y = x + 2$

Tries 0/99

21 Find $\frac{dy}{dx}$ by implicit differentiation.

$$e^{xy} = 8x - 8y$$

A. $\frac{dy}{dx} = \dfrac{8 - ye^{xy}}{8 + xe^{xy}}$

B. $\frac{dy}{dx} = \dfrac{8 - xe^{xy}}{8 + ye^{xy}}$

C. $\frac{dy}{dx} = \dfrac{8}{8 - xe^{xy}}$

D. $\frac{dy}{dx} = \dfrac{8}{8 + xe^{xy}}$

E. $\frac{dy}{dx} = \dfrac{8 + xe^{xy}}{8 - ye^{xy}}$

F. $\frac{dy}{dx} = \dfrac{8 + ye^{xy}}{8 - xe^{xy}}$

Tries 0/99

---

22 The position of an object moving on a straight line is given by $s(t) = 48 - 3t - 2t^2 - 6t^3$, where t is in minutes and $s(t)$ is in meters. What is the acceleration when $t = 3$ minutes?

A. -114 m/min^2

B. -108 m/min^2

C. -177 m/min^2

D. -76 m/min^2

E. -112 m/min^2

F. -110 m/min^2

Tries 0/99

---

23 The sides of an equilateral triangle are expanding at a rate of 2 cm per minute. Find the rate of change of the area when the length of each side is 3 cm. Use the fact that the area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}x^2$, where x is the length of a side.

A. $\sqrt{3}$ cm^2/min

B. $\frac{9\sqrt{3}}{2}$ cm^2/min

C. $\frac{3\sqrt{3}}{2}$ cm^2/min

D. $\frac{9\sqrt{3}}{4}$ cm^2/min

E. $3\sqrt{3}$ cm^2/min

F. $\frac{3\sqrt{3}}{4}$ cm^2/min

Tries 0/99

---

24 Given $f(x) = \dfrac{x^3}{3} + x + \sqrt{x^3}$. Find $f''(4)$.

A. $\frac{49}{8}$

B. $\frac{26}{3}$

C. $\frac{49}{6}$

D. $\frac{35}{4}$

E. $\frac{67}{8}$

F. $\frac{19}{2}$

Tries 0/99

---

25 Given $y = x \ln x$, find $y''(e)$.

A. $e + 1$

B. e

C. 2

D. 0

E. $\dfrac{1}{e}$

F. $\dfrac{1}{e} + 1$

Tries 0/99

---

26 Find the relative extrema of $g(x) = \dfrac{x}{x^2 + 9}$.

A. Relative maximum at $x = 3$; Relative minimum at $x = -\sqrt{3}$

B. Relative maximum at $x = -3$; Relative minimum at $x = 3$

C. Relative maximum at $x = -\sqrt{3}$; Relative minimum at $x = \sqrt{3}$

D. Relative maximum at $x = -3$; Relative minimum at $x = \sqrt{3}$

E. Relative maximum at $x = 3$; Relative minimum at $x = -3$

F. Relative maximum at $x = \sqrt{3}$; Relative minimum at $x = -\sqrt{3}$

Tries 0/99

---

27 Find the largest open interval(s) on which

$$f(x) = (3x - 4)(x + 2)$$

is increasing.

A. $(-\infty, 3)$

B. $(-\infty, -2)$ and $(\frac{4}{3}, \infty)$

C. $(-\infty, 3)$ and $(3, \infty)$

D. $(-\frac{1}{3}, \infty)$

E. $(-2, \frac{4}{3})$

Tries 0/99

---

28 If x and y are both functions of t and

$$x + y^2 = 4e^x,$$

find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 2$, $x = 0$, and $y = -2$.

A. -1

B. $-\frac{1}{2}$

C. 1

D. 0

E. $-\frac{3}{2}$

F. 3

Tries 0/99

---

29 Find $g'(1)$.

$$g(x) = \left(\frac{x^2}{x+2}\right)^3$$

A. $\frac{5}{3}$

B. $\frac{1}{3}$

C. $\frac{5}{9}$

D. $\frac{25}{3}$

E. $\frac{5}{27}$

F. $\frac{25}{27}$

Tries 0/99

---

30 The position of a particle on a straight line t seconds after it starts moving is $s(t) = 2t^3 - 3t^2 + 6t + 1$ feet. Find the acceleration of the particle when its velocity is 78 ft/sec.

A. 84 ft/sec^2

B. 105 ft/sec^2

C. 30 ft/sec^2

D. 258 ft/sec^2

E. 42 ft/sec^2

F. 46 ft/sec^2

Tries 0/99

---

31 Find the relative maximum of $f(x) = 2x^3 - 6x$.

A. $(1, 4)$

B. $(0, 0)$

C. $(-1, 0)$

D. $(1, 0)$

E. $(-1, 4)$

F. $(1, -4)$

Tries 0/99

---

32 Given that

$$y^2x - x^2 = y\ln(x) + 3,$$

use implicit differentiation to find $\frac{dy}{dx}$ at $(1, -2)$.

A. 5

B. $-\frac{2}{5}$

C. 1

D. -1

E. -2

F. 2

Tries 0/99

33

Find $f'(4)$ if $f(x) = (x^2 + 3)\sqrt{x^2 - 7}$.

A. $\frac{163}{6}$

B. $\frac{110}{3}$

C. $\frac{148}{3}$

D. $\frac{142}{3}$

E. $\frac{4}{3}$

F. $\frac{32}{3}$

Tries 0/99

34

Find the x value at which the function $f(x) = x^3 - 9x^2 - 120x + 3$ has a relative minimum.

A. $x = 4$

B. $x = -10$

C. $x = 10$

D. $x = -3$

E. $x = -4$

F. $x = 3$

Tries 0/99

35 Which of the following is a critical number of

$$y = \frac{1}{3}\sin(3x) - \frac{x}{2}?$$

A. 0

B. $\frac{\pi}{3}$

C. $\frac{\pi}{9}$

D. $\frac{\pi}{12}$

E. $\frac{\pi}{18}$

F. $\frac{\pi}{6}$

Tries 0/99

---

36 An observer stands 400 feet away from the point where a hot air balloon is launched. If the balloon ascends vertically at a (constant) rate of 30 feet per second, how fast is the balloon moving away from the observer 10 seconds after it is launched?

A. 40 ft/sec

B. 50 ft/sec

C. 18 ft/sec

D. 30 ft/sec

E. 24 ft/sec

F. 37.5 ft/sec

Tries 0/99

37 A spherical snowball grows in size as it rolls down a snow covered hill. If the volume of the snowball is increasing at a rate of 1 cubic inch per second, at what rate, in inches per second, is the radius of the snowball increasing when the radius is 3 inches? (Recall that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.)

A. 1

B. 9

C. $\frac{1}{36\pi}$

D. $\frac{1}{9\pi}$

E. $\frac{1}{4\pi}$

F. 0

Tries 0/99

---

38 Find the second derivative of $f(x) = \ln(4x) + e^{x^2}$.

A. $f''(x) = -\frac{1}{x^2} + 4x^2e^{x^2}$

B. $f''(x) = -\frac{1}{4x^2} + e^{x^2}(4x^2 + 2)$

C. $f''(x) = -\frac{1}{x^2} + e^{x^2}(4x^2 + 2)$

D. $f''(x) = -\frac{1}{x^2} + 2e^{x^2}$

E. $f''(x) = -\frac{1}{4x^2} + 4x^2e^{x^2}$

F. $f''(x) = -\frac{1}{4x^2} + 2e^{x^2}$

Tries 0/99

A tale of 2 parabolas

$f(x) = x^2$

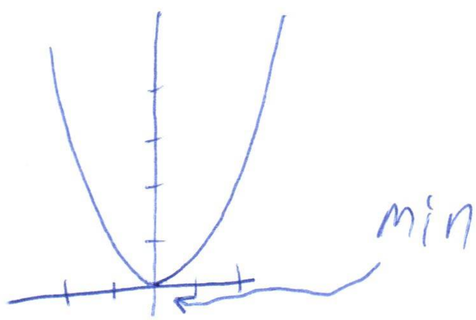
$f'(x) = 2x$

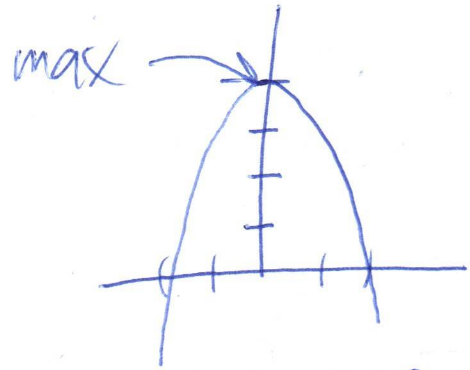
$f''(x) = 2$

$f(x) = 4 - x^2$

$f'(x) = -2x$

$f''(x) = -2$





concave up

$f''(x) > 0$

concave down

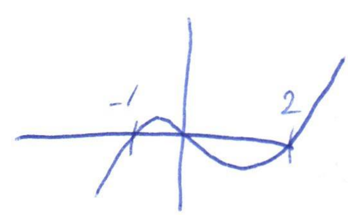
$f''(x) < 0$

Ask: what are similarities & differences between these functions?

Big idea: use these patterns to analyze other functions

example

$$f(x) = x^3 - x^2 - 2x$$
$$= x(x^2 - x - 2)$$
$$= x(x-2)(x+1)$$



- shape resembles 2 parabolas

$f'(x) = 3x^2 - 2x - 2$

$f''(x) = 6x - 2$

$f''(x) > 0$

$6x - 2 > 0$

$6x > 2$

$x > \frac{1}{3}$

concave up on $\left(\frac{1}{3}, \infty\right)$

concave down on $\left(-\infty, \frac{1}{3}\right)$

inflection point at $\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right) = \left(\frac{1}{3}, -\frac{20}{27}\right)$

Due date: Mon Oct 16 10:00:00 pm 2017 (EDT)

1 Consider the function $f(x) = 7x^2 + 7x + 7$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: (____ , ____) , (____ , ____)

Tries 0/99

Concave down: (____ , ____) , (____ , ____)

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: (____ , ____)

Tries 0/99

Inflection point: (____ , ____)

Tries 0/99

2 Consider the function $f(x) = 4x^3 + 6x^2 + 4$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: (____ , ____) , (____ , ____)

Tries 0/99

Concave down: (____ , ____) , (____ , ____)

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: (____ , ____)

Tries 0/99

Inflection point: (____ , ____)

Tries 0/99

Find the largest open intervals on which the function is both concave up and increasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up and increasing: (____ , ____), (____ , ____)

Tries 0/99

3 Consider the function $f(x) = 2x^4 + 4x^3 + 1$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: (____ , ____) , (____ , ____)

Tries 0/99

Concave down: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: ([ ] , [ ])

Tries 0/99

Inflection point: ([ ] , [ ])

Tries 0/99

Find the largest open intervals on which the function is both concave up and decreasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up and decreasing: ([ ] , [ ]), ([ ] , [ ])

Tries 0/99

---

4 Consider the function $f(x) = 8x^6 + 8x^4$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Concave down: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: ([ ] , [ ])

Tries 0/99

Inflection point: ([ ] , [ ])

Tries 0/99

Find the largest open intervals on which the function is both concave down and increasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave down and increasing: ([ ] , [ ]), ([ ] , [ ])

Tries 0/99

---

5 Consider the function $f(x) = \frac{x^5}{10} + \frac{x^4}{8} + 2$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Concave down: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: ([ ] , [ ])

Tries 0/99

Inflection point: ([ ] , [ ])

Tries 0/99

Find the largest open intervals on which the function is both concave down and increasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave down and increasing: ([ ] , [ ]), ([ ] , [ ])

Tries 0/99

6

Consider the function $f(x) = (x^2 - 12x + 37)e^x$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Concave down: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: ([ ] , [ ])

Tries 0/99

Inflection point: ([ ] , [ ])

Tries 0/99

7

Consider the function $f(x) = 5\ln(x^2 + 1)$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Concave down: ([ ] , [ ]) , ([ ] , [ ])

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: ([ ] , [ ])

Tries 0/99

Inflection point: ([ ] , [ ])

Tries 0/99

8 Consider the function $y = 5 + 6x - 8x^3$. Use the Second Derivative Test when applicable to find the relative extrema if they exist. Enter your answer in the format of (x, y). If not, enter NONE in the blanks.

Relative Maximum: (_____ , _____)

Tries 0/99

Relative Minimum: (_____ , _____)

Tries 0/99

9 Consider the function $y = \frac{1}{4}x^4 - 2x^3 + 3$. Use the Second Derivative Test when applicable to find the relative extrema if they exist. Enter your answer in the format of (x, y). If not, enter NONE in the blanks.

Relative Maximum: (_____ , _____)

Tries 0/99

Relative Minimum: (_____ , _____)

Tries 0/99

10 Consider the function $y = \frac{x^5}{5} - \frac{5x^4}{4} + 2$. Use the Second Derivative Test when applicable to find the relative extrema if they exist. Enter your answer in the format of (x, y). If not, enter NONE in the blanks.

Relative Maximum: (_____ , _____)

Tries 0/99

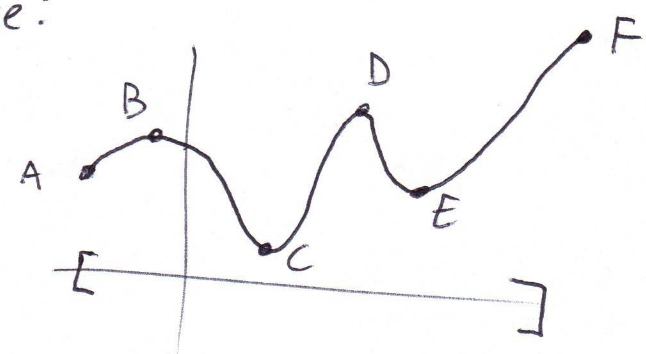
Relative Minimum: (_____ , _____)

Tries 0/99

Absolute Max and Min on an Interval

The Big Idea: compare fcn values at local extrema, & at endpoints.

picture:



<u>example</u> Find absolute extrema of $f(x) = 3x^2$ on $[-1, 2]$

<u>example</u> Find absolute extrema of $f(x) = \frac{x^3}{3} - 4x^2 + 12x$ on $[0, 3]$

<u>example</u> Find the maximum of $x^4 - 2x^2$ on $[-\frac{1}{2}, 2]$

<u>example</u> find the max & min values of $x^2 e^x$ on $[-5, 5]$

<u>example</u> find the minimum value of $\frac{x^2}{x+1}$ on $(-1, 4]$

<u>example</u> find the max value of $\ln(x) - x$ on $(0, 2)$

Quiz 7 — MA16010 — October 18, 2017

Alden Bradford

Let $f(x) = 3x^3 - 4x + 1$. Please answer the following problems in complete sentences.

1. Find $f'(x)$ and $f''(x)$.

2. Find any intervals where $f(x)$ is concave up and any intervals where $f(x)$ is concave down.

3. Give the coordinates of the inflection point.

4. Find any intervals where $f(x)$ is increasing and any intervals where $f(x)$ is decreasing.

5. On what interval is $f(x)$ concave up and decreasing?

Due date: Wed Oct 18 10:00:00 pm 2017 (EDT)

1

Find the absolute extrema of $y = 8x^2 - 24x + 6$ on the closed interval $[-3, 2]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

2

Find the absolute extrema of $y = \frac{1}{3}x^3 - 8x + 8$ on the closed interval $[-1, 6]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

3

Find the absolute extrema of $y = 2x^3 - 15x^2 - 300x + 9$ on the closed interval $[-6, 11]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

4

Find the absolute extrema of $f(x) = 9x^4 - 60x^3 + 8$ on the closed interval $[-1, 6]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

5

Find the absolute extrema of $g(x) = -6x^3 - 81x^2 - 360x + 1$ on the closed interval $[-5, 0]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

6

Find the absolute extrema of $f(x) = 3x^{\frac{4}{3}} - 3x^{\frac{1}{3}} + 8$ on the closed interval $[-1, 1]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

7

Find the absolute extrema of $f(x) = 5xe^{-x} + 5$ on the closed interval $[0, 3]$. Enter your answer in the format of (x, y)

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

8

Find the absolute extrema of $y = \frac{1}{3x^2+3}$ on the closed interval $[-1, 3]$. Enter your answer in the format of (x, y).

Absolute Minimum=(________ , ________)

Tries 0/99

Absolute Maximum=(________ , ________)

Tries 0/99

9

Find the absolute extrema of $f(x) = 5x^4e^x + 5$ on the closed interval $[-6, 4]$. Enter your answer in the format of (x, y).

Absolute Minimum=(,
)

Tries 0/99

Absolute Maximum=(,
)

Tries 0/99

---

10 Find the absolute maximum of $y = 3x^3 + 4x^2 + 5$ on the interval $\left(-\frac{16}{9}, -\frac{4}{9}\right)$. Enter your answer in the format of (x, y).

Absolute Maximum=(,
)

Tries 0/99

---

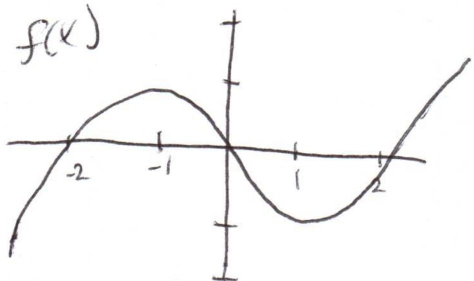
11 Find the absolute minimum of $y = \frac{5x^2}{x+1}$ on the interval $(-1, 5]$. Enter your answer in the format of (x, y).

Absolute Minimum=(,
)

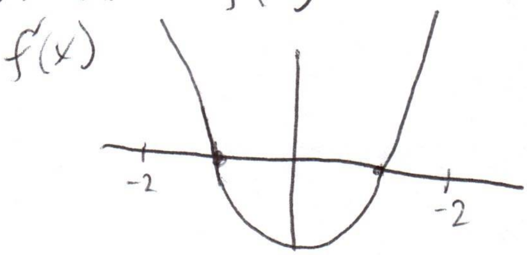
Tries 0/99

Graphical Interpretation of Derivatives

Here is a graph of $f(x)$



Now let's draw $f'(x)$



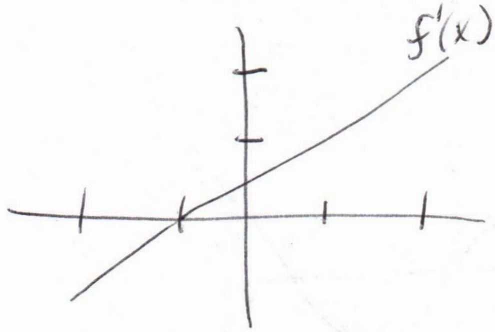
Notice

where $f(x)$ has a critical point, $f'(x)=0$
where $f(x)$ is increasing, $f'(x)$ is positive
(above the x-axis)

where $f(x)$ is decreasing, $f'(x)$ is negative
(below the x axis)

where $f(x)$ is concave up, $f'(x)$ is increasing
where $f(x)$ is concave down, $f'(x)$ is decreasing
where $f(x)$ has an inflection point, $f'(x)$ has a critical point
where $f(x)$ has a maximum, $f'(x)$ goes from positive to negative
where $f(x)$ has a minimum, $f'(x)$ goes from negative to positive

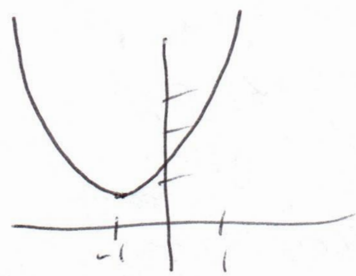
Now we go backwards

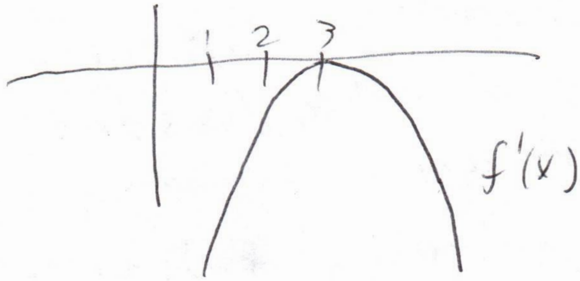


describe $f(x)$:

- critical point at $x=-1$
- increasing on $(-1,\infty)$
- decreasing on $(-\infty,-1)$
- concave up on $(-\infty,\infty)$
- never concave down
- no inflection point
- no maximum
- minimum at $x=-1$

draw $f(x)$:

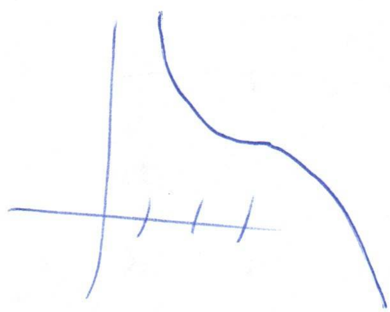




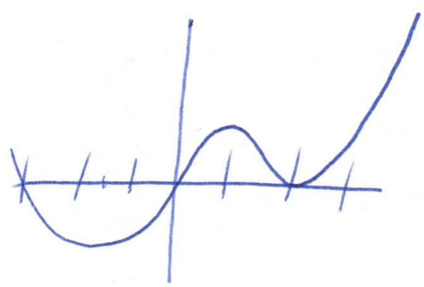
describe $f(x)$:

- critical point at $x=3$
- ~~dn~~ never increasing
- decreasing everywhere (except maybe at $x=3$)
- concave up on $(-\infty, 3)$
- concave down on $(3, \infty)$
- inflection point at $x=3$
- no maximum
- no minimum

draw $f(x)$:



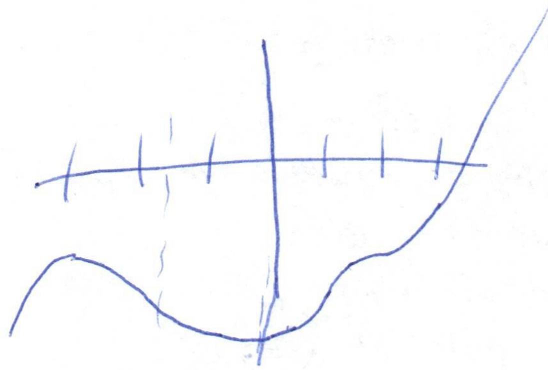
$f'(x)$ ~~#~~



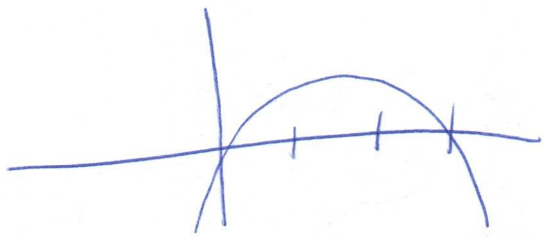
describe $f(x)$:

- critical points at $x = -3, 0, 2$
- increasing on $(-\infty, -3)$ and on $(0, \infty)$
- decreasing on $(-3, 0)$
- concave ~~dn~~ up on $(-\frac{3}{2}, 1)$ and on $(2, \infty)$
- concave down on $(-\infty, -\frac{3}{2})$ and on $(1, 2)$
- inflection points at $x = -\frac{3}{2}, 1$, and ~~#~~ 2
- maximum at $x = -3$
- minimum at $x = 0$

draw $f(x)$:



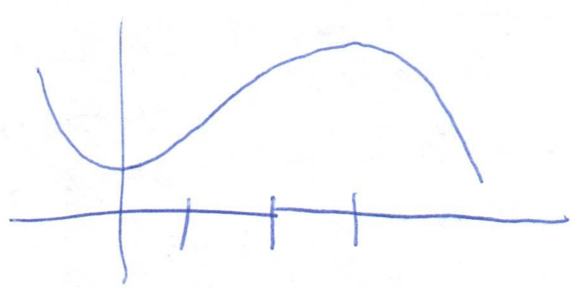
$f'(x)$:



describe $f(x)$:

- critical points at $x=0,3$
- increasing on $(0,3)$
- decreasing on $(-\infty,0)$ and $(3,\infty)$
- concave up on $(-\infty,\frac{3}{2})$
- concave down on $(\frac{3}{2},\infty)$
- inflection point at $x=3/2$
- maximum at $x=3$
- minimum at $x=0$

draw $f(x)$:



Quiz 7 Key — MA16010 — October 18, 2017

Alden Bradford

Min	Mean	Max
1	3.3	5

Let $f(x) = 3x^3 - 4x + 1$. Please answer the following problems in complete sentences.

1. Find $f'(x)$ and $f''(x)$.

   $f'(x) = 9x^2 - 4$ and $f''(x) = 18x$.

2. Find any intervals where $f(x)$ is concave up and any intervals where $f(x)$ is concave down.

   $f(x)$ is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$.

3. Give the coordinates of the inflection point.

   $f(x)$ has an inflection point at $(0, 1)$.

4. Find any intervals where $f(x)$ is increasing and any intervals where $f(x)$ is decreasing.

   $f(x)$ is increasing on $(-\infty, -\frac{2}{3})$ and on $(\frac{2}{3}, \infty)$. $f(x)$ is decreasing on $(-\frac{2}{3}, \frac{2}{3})$.

5. On what interval is $f(x)$ concave up and decreasing?

   $f(x)$ is concave up and decreasing on $(0, \frac{2}{3})$.

Limits at infinity

$$\lim_{x\to\infty} 8 - \frac{5}{x}$$

$$\lim_{x\to\infty} x^2 + 3$$

$$\lim_{x\to\infty} \frac{4x^3+3x-2}{10x^3+6}$$

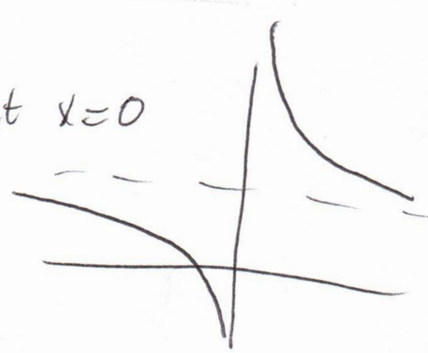
x	10	100	1000
f(x)	$\frac{4000+30-2}{10000+6}$	$\frac{4000000+300-2}{10000000}$	$\frac{4000000000+3000-2}{10000000000+6}$
f(x)	0.4026	0.40003	0.4000002996

$$\lim_{x\to\infty} \frac{4x^3+3x-2}{10x^3+6} = \lim_{x\to\infty} \frac{4x^3}{10x^3} = \lim_{x\to\infty} \frac{4}{10} = \frac{4}{10}$$

$$\lim_{x\to-\infty} \frac{5+x^2-x}{2x^3+5x} = \lim_{x\to-\infty} \frac{x^2}{2x^3} = \lim_{x\to-\infty} \frac{1}{2x} = 0$$

Asymptotes

$\frac{1+5x}{x}$ has a vertical asymptote at $x=0$



$$\lim_{x\to\infty} \frac{1+5x}{x} = \lim_{x\to\infty} \frac{5x}{x} = \lim_{x\to\infty} 5 = 5$$

has a horizontal asymptote at $y=5$

$$\lim_{x\to\infty} \frac{x^3+2x^2+2x+1}{2x^2+1} = \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \infty$$

has no horizontal asymptote!

$$\lim_{x\to\infty} \frac{x^3-x^2-x+2}{x^2-1} = \lim_{x\to\infty} \frac{x^3}{x^2} = \lim_{x\to\infty} x = \infty$$

has vertical asymptotes at $x=1$ and $x=-1$

has no horizontal asymptotes

Review: Long Division

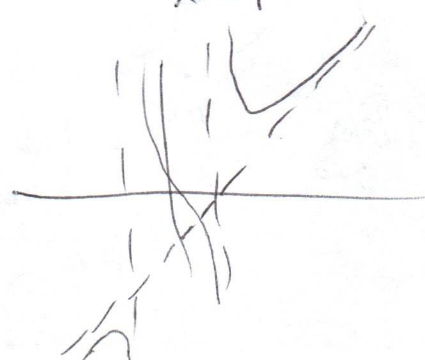
$$\begin{array}{r|l} & \;\;26 \\ \hline 21 & 547 \\ & \underline{-42}\;\; \\ & \;\;127 \\ & \underline{-126} \\ & \;\;\;\;\;1 \end{array}$$

$$\frac{541}{21} = 26 + \frac{1}{21}$$

Now: Polynomial Division (same thing)

$$\frac{x^3-x^2-x+2}{x^2-1} = x-1+\frac{1}{x^2-1}$$

$$\begin{array}{r|l} & \;\;x-1 \\ \hline x^2-1 & x^3-x^2-x+2 \\ & \underline{-(x^3\;\;\;\;\;\;-x)} \\ & \;\;0-x^2+0 \\ & \underline{-(-x^2\;\;\;\;+1)} \\ & \;\;\;\;\;\;\;\;\;\;1 \end{array}$$



Slant asymptote!

$$f(x) = \frac{3x^2 - 14x}{x-5} = 3x+1 + \frac{5}{x-5}$$

has a vertical asymptote at $x=5$

$$\begin{array}{r|l} & \;\;3x \quad + 1 \\ \hline x-5 & 3x^2 - 14x \\ & -(3x^2 - 15x) \\ \hline & \qquad\quad x \\ & \qquad -(x-5) \\ \hline & \qquad\qquad 5 \end{array}$$

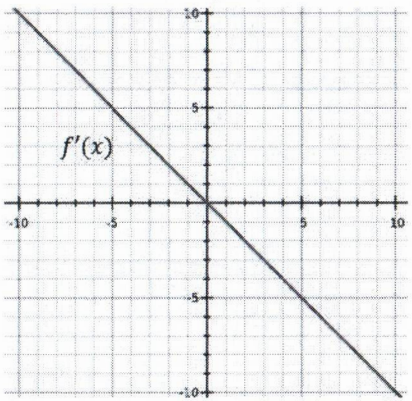
has a slant asymptote $y = 3x+1$

$$\frac{x^4 - x^3 + 9x^2 - 9x + 4}{x^2 + 9}$$

has neither!

Due date: Fri Oct 20 10:00:00 pm 2017 (EDT)

Given the graph of $f'(x)$ below, answer the following questions $f(x)$. If there are any extra blanks or if no answer exists at all, enter NONE. Use 'INF' for positive infinity and '-INF' for negative infinity. For numbers and intervals, always enter them from left to right as they appear on the real line.



Critical Number(s): x = , and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

Relative Maxima Occur at x = , and

Tries 0/99

Relative Minima occur at x = , and

Tries 0/99

Concave Up Interval(s): (,), (,), (,)

Tries 0/99

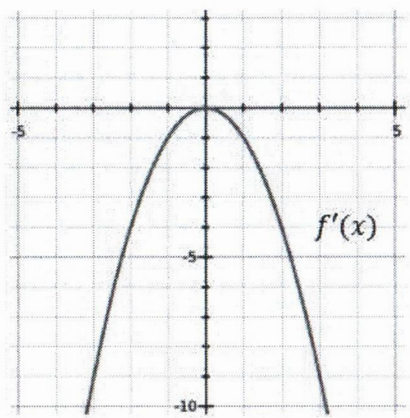
Concave Down Interval(s): (,), (,), (,)

Tries 0/99

Inflection Points Occur at x = , and

Tries 0/99

Given the graph of $f'(x)$ below, answer the following questions $f(x)$. If there are any extra blanks or if no answer exists at all, enter NONE. Use 'INF' for positive infinity and '-INF' for negative infinity. For numbers and intervals, always enter them from left to right as they appear on the real line.



Critical Number(s): x = , and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

Relative Maxima Occur at x = , and

Tries 0/99

Relative Minima Occur at x = , and

Tries 0/99

Concave Up Interval(s): (,), (,), (,)

Tries 0/99

Concave Down Interval(s): (,), (,), (,)

Tries 0/99

Inflection Points Occur at x = , and

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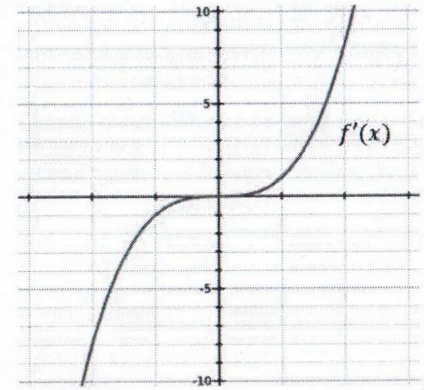


Critical Number(s): x = , and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

Relative Maxima Occur at x = , and

Tries 0/99

Relative Minima Occur at x = , and

Tries 0/99

Concave Up Interval(s): (,), (,), (,)

Tries 0/99

Concave Down Interval(s): (,), (

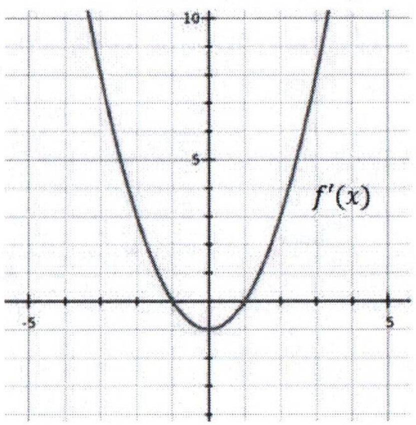
(,), (,)

Tries 0/99

Inflection Points Occur at $x =$, and

Tries 0/99

Given the graph of $f'(x)$ below, answer the following questions $f(x)$. If there are any extra blanks or if no answer exists at all, enter NONE. Use "INF" for positive infinity and "-INF" for negative infinity. For numbers and intervals, always enter them from left to right as they appear on the real line.



Critical Number(s): $x =$, and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

Relative Maxima Occur at $x =$, and

Tries 0/99

Relative Minima Occur at $x =$, and

Tries 0/99

Concave Up Interval(s): (,), (,), (,)

Tries 0/99

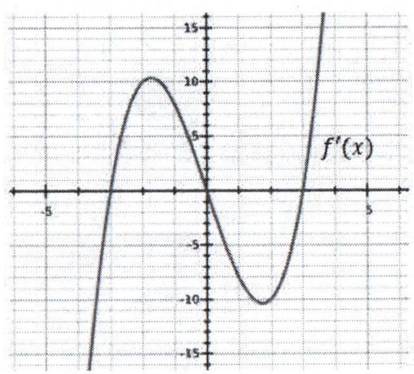
Concave Down Interval(s): (,), (,), (,)

Tries 0/99

Inflection Points Occur at $x =$, and

Tries 0/99

Given the graph of $f'(x)$ below, answer the following questions $f(x)$. If there are any extra blanks or if no answer exists at all, enter NONE. Use "INF" for positive infinity and "-INF" for negative infinity. For numbers and intervals, always enter them from left to right as they appear on the real line.



Critical Number(s): $x =$, and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

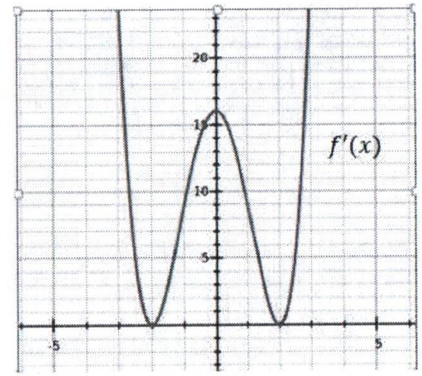
Relative Maxima Occur at $x =$, and

Tries 0/99

Relative Minima Occur at $x =$, and

Tries 0/99

Given the graph of $f'(x)$ below, answer the following questions $f(x)$. If there are any extra blanks or if no answer exists at all, enter NONE. Use "INF" for positive infinity and "-INF" for negative infinity. For numbers and intervals, always enter them from left to right as they appear on the real line.



Critical Number(s): $x =$, and

Tries 0/99

Increasing Interval(s): (,), (,), (,)

Tries 0/99

Decreasing Interval(s): (,), (,), (,)

Tries 0/99

Relative Maxima Occur at $x =$, and

Tries 0/99

Relative Minima Occur at $x =$, and

Tries 0/99

Concave Up Interval(s): (,), (,), (,)

Tries 0/99

Concave Down Interval(s): (,), (,), (,)

Due date: Mon Oct 23 10:00:00 pm 2017 (EDT)

1

Find the following limits.

Enter INF for positive infinity and -INF for negative infinity.

1. $\lim_{x\to\infty}\left(8+\frac{9}{x}\right) =$ ☐
Tries 0/99

2. $\lim_{x\to-\infty}\left(\frac{14}{x}-\frac{x}{19}\right) =$ ☐
Tries 0/99

2

Find the following limits.

Enter INF for positive infinity and -INF for negative infinity.

1. $\lim_{x\to\infty}\frac{9x+10}{19x^2+12} =$ ☐
Tries 0/99

2. $\lim_{x\to-\infty}\frac{9x+10}{19x^2+12} =$ ☐
Tries 0/99

3

Find the following limits.

Enter INF for positive infinity and -INF for negative infinity.

1. $\lim_{x\to\infty}\frac{4x^2+6}{19x^2+11} =$ ☐
Tries 0/99

2. $\lim_{x\to-\infty}\frac{4x^2+6}{19x^2+11} =$ ☐
Tries 0/99

4

Find the following limits.

Enter INF for positive infinity and -INF for negative infinity.

1. $\lim_{x\to\infty}\frac{4x^3+5}{17x^2+18} =$ ☐
Tries 0/99

2. $\lim_{x\to-\infty}\frac{4x^3+5}{17x^2+18} =$ ☐
Tries 0/99

5

Find the following limits.

Enter INF for positive infinity and -INF for negative infinity.

1. $\lim_{x\to\infty}\frac{6+60x-10x^4}{14+14x^2} =$ ☐
Tries 0/99

2. $\lim_{x\to-\infty}\frac{x-10x^2+60x^3+6}{14x^3-14} =$ ☐
Tries 0/99

6

Find the vertical, horizontal and slant asymptotes of $f(x) = \frac{-7x^2}{x^2-36}$ if they exist. If an asymptote does not exist, enter NONE. If there are extra blanks, enter NONE.

1. Vertical Asymptote(s): $x =$ ☐ and $x =$ ☐
Tries 0/99

2. Horizontal Asymptote(s): $y =$ ☐
Tries 0/99

3. Slant Asymptote(s): $y =$ ☐
Tries 0/99

7

Find the vertical, horizontal and slant asymptotes of $f(x) = \frac{-5x^2-8x-7}{x^2+4x-21}$ if they exist. If an asymptote does not exist, enter NONE. If there are extra blanks, enter NONE.

1. Vertical Asymptote(s): $x =$ ☐ and $x =$ ☐
Tries 0/99

2. Horizontal Asymptote(s): $y =$ ☐
Tries 0/99

3. Slant Asymptote(s): $y =$ [ ]
Tries 0/99

---

8 Find the vertical, horizontal and slant asymptotes of $f(x) = \frac{x^2+5x+24}{x-3}$ if they exist. If an asymptote does not exist, enter NONE. If there are extra blanks, enter NONE.

1. Vertical Asymptote(s): $x =$ [ ] and $x =$ [ ]
Tries 0/99

2. Horizontal Asymptote(s): $y =$ [ ]
Tries 0/99

3. Slant Asymptote(s): $y =$ [ ]
Tries 0/99

---

9 Find the vertical, horizontal and slant asymptotes of $f(x) = \frac{-5x^3+7x^2+19x+28}{x^2+4}$ if they exist. If an asymptote does not exist, enter NONE. If there are extra blanks, enter NONE.

1. Vertical Asymptote(s): $x =$ [ ] and $x =$ [ ]
Tries 0/99

2. Horizontal Asymptote(s): $y =$ [ ]
Tries 0/99

3. Slant Asymptote(s): $y =$ [ ]
Tries 0/99

Curve Sketching

useful properties:

x-intercepts
y-intercepts
increasing intervals/decreasing intervals
concave up/down intervals
asymptotes
inflection points

Functions to graph

$$y = \frac{3x+9}{x-2}$$

$$y = \frac{x^2-2x+5}{x^2-4}$$

$$y = \frac{4x}{x^2+6x+9}$$

$$y = \frac{x^2+8}{x-4}$$

Quiz 8 — MA16010 — October 23, 2017

Alden Bradford

For this quiz, $f(x) = x^2 e^{3x}$.

1. Find $f'(x)$.

2. Find the x-values of the critical points of $f(x)$.

3. Find the y-values of the critical points of $f(x)$.

4. Find $f(1)$ and $f(-1)$.

5. Find the absolute maximum and minimum values of $f(x)$ on the interval $[-1, 1]$.

Quiz 9 — MA16010 — October 25, 2017

Alden Bradford

1. Let $g(x) = \dfrac{x^3 + 11x^2 + 14x - 33}{x+9}$.

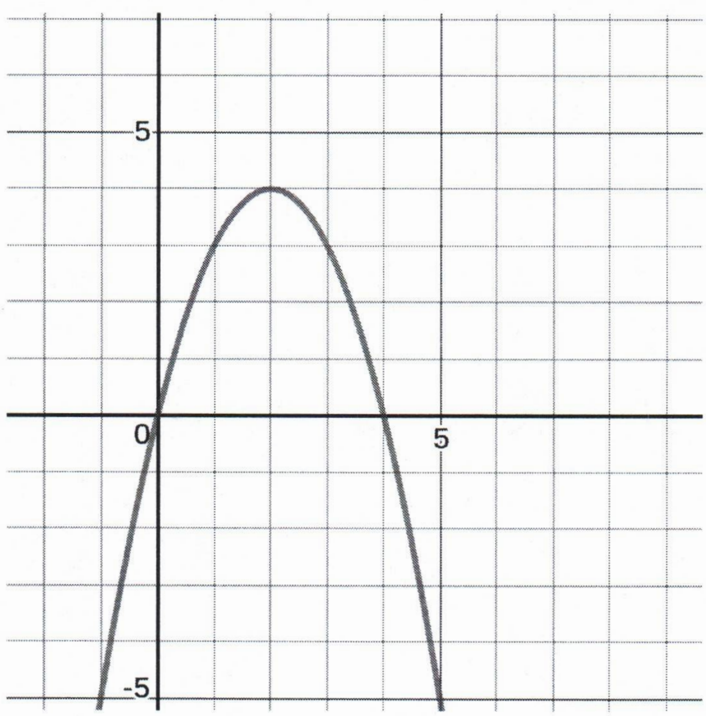
   (a) Perform long division on $g(x)$. Write your answer as
   $$g(x) = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}.$$

   (b) List all the vertical, horizontal, and/or slant asymptotes of $g(x)$.

2. A graph of $f'(x)$ is given below.

   (a) Find the x-coordinate of the local maximum of $f(x)$.

   (b) Find the interval on which $f(x)$ is concave up.



Quiz 8 — MA16010 — October 20, 2017

Alden Bradford

For this quiz, $f(x) = x^2e^{3x}$.

1. Find $f'(x)$.

2. Find the x-values of the critical points of $f(x)$.

3. Find the y-values of the critical points of $f(x)$.

4. Find $f(1)$ and $f(-1)$.

5. Find the maximum and minimum values of $f(x)$ on the interval $[-1, 1]$.

Quiz 8 Key — MA16010 — October 23, 2017

Alden Bradford

Min	Mean	Max
1	3.6	5

For this quiz, $f(x) = x^2e^{3x}$.

1. Find $f'(x)$.

$\boxed{f'(x) = 2xe^{3x} + 3x^2e^{3x} = x(2+3x)e^{3x}}$

2. Find the x-values of the critical points of $f(x)$.

$\boxed{x = 0 \text{ and } x = -2/3.}$

3. Find the y-values of the critical points of $f(x)$.

$\boxed{f(0) = 0,\ f(-2/3) = \frac{4}{9}e^{-2} = 0.06015...}$

4. Find $f(1)$ and $f(-1)$.

$\boxed{f(-1) = e^{-3} = 0.0498...,\ f(1) = e^3 = 20.0855...}$

5. Find the absolute maximum and minimum values of $f(x)$ on the interval $[-1, 1]$.

$\boxed{\text{Max: } e^3. \text{ Min: } 0.}$

Quiz 9 Key — MA16010 — October 25, 2017

Alden Bradford

Min	Mean	Max
0	3.6	5

1. Let $g(x) = \dfrac{x^3 + 11x^2 + 14x - 33}{x + 9}$.

   (a) Perform long division on $g(x)$. Write your answer as
   $$g(x) = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}.$$

   (b) List all the vertical, horizontal, and/or slant asymptotes of $g(x)$.

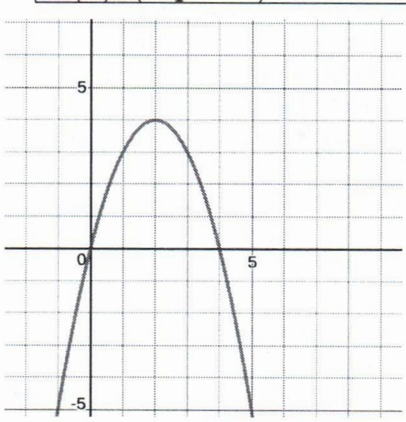
> (a) (1 point) $g(x) = x^2 + 2x - 4 + \dfrac{3}{x+9}$
>
> (b) (2 points) Vertical asymptote at $x = -9$

2. A graph of $f'(x)$ is given below.

   (a) Find the x-coordinate of the local maximum of $f(x)$.

   (b) Find the interval on which $f(x)$ is concave up.

> (a) (1 point) $x = 4$ (b) (1 point) $(-\infty, 2)$



Quiz 10 — MA16010 — October 30, 2017

Alden Bradford

A box with a square side (NOT necessarily a square base) and an open top has a volume of 36 cubic feet.

1. (1 point) Sketch the box and label the dimensions with appropriate variables.

2. (1 point) Write an equation for the volume of the box in terms of your variables.

3. (1 point) Write a formula for the amount of material used to make the box, in terms of your variables.

4. (2 points) Find the dimensions of the box which minimize the amount of material used.

Due date: Fri Oct 27 10:00:00 pm 2017 (EDT)

A piece of cardboard is 24 inches by 48 inches. A square is to be cut from each corner and the sides folded up to make an open-top box. What is the maximum possible volume of the box? Round your answer to the nearest four decimal places.

Volume= [ ] in^3

Tries 0/99

For rectangles that have a fixed perimeter of 520, what are the dimensions of the rectangle that has the largest possible area?

Length= [ ]

Width= [ ]

Tries 0/99

You have 90 ft of fence to make a rectangular vegetable garden alongside the wall of your house. The wall of the house bounds one side of the vegetable garden. What is the largest possible area of the vegetable garden?

Area= [ ] ft^2

Tries 0/99

You have $5L$ feet of fence to make a rectangular vegetable garden alongside the wall of your house, where L is a positive constant. The wall of the house bounds one side of the vegetable garden. What is the largest possible area for the vegetable garden?

Area= [ ] ft^2

Tries 0/99

A family wants to fence a rectangular play area alongside the wall of their house. The wall of their house bounds one side of the play area. If they want the play area to be exactly $1600ft^2$, what is the least amount of fencing needed to make this? Round your answer to the nearest two decimal places.

Amount of fencing= [ ] ft

Tries 0/99

You are designing a poster with an area of 625 cm^2 to contain a printing area in the middle and have the margins of 4cm at the top and bottom and 7cm on each side. Find the largest possible printing area. Round your answer to the nearest four decimal places.

Area= [ ] cm^2

Tries 0/99

Optimization

Two positive numbers multiply to make 64. Find their smallest possible sum:

A sheet of cardboard 10 cm by 15 cm has its corners cut and its edges turned up, to make a box. What is the largest box we can make this way?

~~I want to build a pen in the corner of my apartment to contain a cat. It needs to be a rectangle and it needs to contain 10 square feet. How much fencing do I need?~~

I have two couches, each 6 feet long. I will make a couch fort by putting them against the wall, forming a triangle. What's the biggest fort I can make this way?

~~Two hallways meet at a right angle, one 5 feet wide and the other 8 feet wide.~~

I want 10 square inches of baklava, but it's hard to cut.

I sell pies which cost \$4.50 to make. I know if I price them at \$p, then I will sell $20 - 5p$ pies. How much should I price them for, how many should I make?

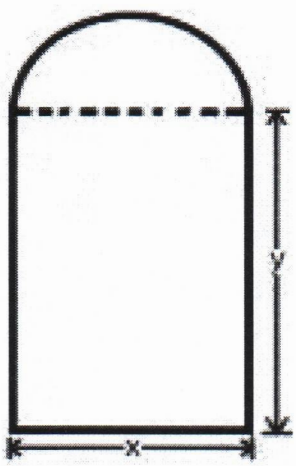
If I use $10x$ pounds of fertilizer, then I will get $10x$ pounds more zuchini, on top of the 8 pounds I already grow. However, some zuchini will rot, leaving only $e^{-x/30}$ times as much zucchini in the field.

Find the closest points to $(2,0)$

on the curve $y^2 - x^2 = 4$.

Due date: Mon Oct 30 10:00:00 pm 2017 (EDT)

A Norman window is constructed by adjoining a semicircle to the top of a rectangular window as shown in the figure below. If the perimeter of the Norman window is 25 ft, find the dimensions that will allow the window to admit the most light.



$x=$ [ ] ft

$y=$ [ ] ft

Tries 0/99

A box with a square base and no top is to be built with a volume of 10976 in^3. Find the dimensions of the box that requires the least amount of material. How much material is required at the minimum?

Width= [ ] in

Length= [ ] in

Height= [ ] in

Tries 0/99

Minimal Material= [ ] in^2

Tries 0/99

A rectangular box has a square base. If the sum of the height and the perimeter of the square base is 16 in, what is the maximum possible volume?

Volume= [ ] in^3

Tries 0/99

For a cylinder with a surface area of 60, what is the maximum volume that it can have? Round your answer to the nearest 4 decimal places.

Recall that the volume of a cylinder is $\pi r^2 h$ and the surface area is $2\pi rh + 2\pi r^2 whereristheradiusandhistheheight.$

Volume= [ ]

Tries 0/99

A company needs to make a cylindrical can that can hold precisely 2.1 liters of liquid. If the entire can is to be made out of the same material, find the dimensions of the can that will minimize the cost. Round your answer to the nearest two decimal places.

Recall that the volume of a cylinder is $\pi r^2 h$ and the surface area is $2\pi rh + 2\pi r^2 whereristheradiusandhistheheight.Alsonotethat1literisequalt$

Radius= [ ] cm

Tries 0/99

Height= [ ] cm

Tries 0/99

Due date: Wed Nov 1 10:00:00 pm 2017 (EDT)

A rectangular recreational field needs to be built outside of a gymnasium. Three walls of fencing are needed and the fourth wall is to be a wall of the gymnasium itself. The ideal area for such a field is exactly 40000ft^2. In order to minimize costs, it is necessary to construct the fencing using the least amount of material possible. Assuming that the material used in the fencing costs \$44/ft, what is the least amount of money needed to build this fence of ideal area? Round your answer to the nearest two decimal places.

Cost = \$ [ ]

Tries 0/99

Gloria would like to construct a box with volume of exactly 30ft^3 using only metal and wood. The metal costs $\$14/\text{ft}^2$ and the wood costs $\$5/\text{ft}^2$. If the wood is to go on the sides, the metal is to go on the top and bottom, and if the length of the base is to be 3 times the width of the base, find the dimensions of the box that will minimize the cost of construction. Round your answer to the nearest two decimal places.

Length= [ ] ft

Tries 0/99

Width= [ ] ft

Tries 0/99

Height= [ ] ft

Tries 0/99

A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2800 - 100p$ units. Each unit costs 4 dollars to make.

1. What price, p, should the company charge to maximize their revenue? \$ [ ]

Tries 0/99

---

2. What price, p, should the company charge to maximize their profits? \$ [ ]

Tries 0/99

Find the point on the curve $y = 7x + 5$ closest to the point $(0, 9)$.

(x, y)=([ ] , [ ])

Tries 0/99

Find the points on the curve $y = x^2 + 5$ closest to the point $(0, 9)$. Round to the nearest two decimal places. Write the point with the smaller x value first.

([ ] , [ ]) ,
([ ] , [ ])

Tries 0/99

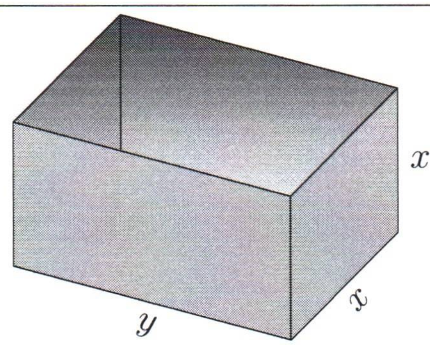
Quiz 10 Key — MA16010 — October 30, 2017

Alden Bradford

Min	Mean	Max
1	3.2	5

A box with a square side (NOT necessarily a square base) and an open top has a volume of 36 cubic feet.

1. (1 point) Sketch the box and label the dimensions with appropriate variables.



2. (1 point) Write an equation for the volume of the box in terms of your variables.

$36 = x^2 y$

3. (1 point) Write a formula for the amount of material used to make the box, in terms of your variables.

$M = 2x^2 + 3xy$

4. (2 points) Find the dimensions of the box which minimize the amount of material used.

$x = 3,\ y = 4$

Integration

if $f(x) = 12x^3$, what function has $f(x)$ as its derivative?

what function has $1 + \cos x$ as its derivative?

$\int 12x^3 dx = 3x^4 + C$

$\int (1+\cos x) dx = x + \sin x + C$

where C is any constant

examples: $\int (x^2 - 2e^x + 4) dx$

$\int x(x^3+3)\, dx$

$\int \frac{1 + x \sin x}{x} dx$

$\int \sqrt{x}\left(5 - \frac{3}{x^2}\right) dx$

$\int \sin x(\cot x + 8)\, dx$

Rules:- $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$ (unless $n = -1$)

$\int \cos x\, dx = \sin x + C$

$\int \frac{1}{x} dx = \ln x + C$

$\int \sin x\, dx = -\cos x + C$

$\int \sec^2 x\, dx = \tan x + C$

Due date: Fri Nov 3 10:00:00 pm 2017 (EDT)

Evaluate $\int(15x^2 + \frac{\sqrt[3]{x^2}}{8})\ dx$. Here C is the constant of integration.

[ ] + C

Tries 0/99

Evaluate $\int \sec x(7\sec x + 8\tan x)\ dx$. Here C is the constant of integration.

[ ] + C

Tries 0/99

Evaluate $\int \frac{x^2+4}{2}\ dx$. Here C is the constant of integration.

[ ] + C

Tries 0/99

Evaluate $\int \frac{x^6+x^3}{\sqrt{x}}\ dx$. Here C is the constant of integration.

[ ] + C

Tries 0/99

Evaluate $\int(2\tan x \cos x + 2)\ dx$. Here C is the constant of integration.

[ ] + C

Tries 0/99

Evaluate $\int \frac{3}{x}\ dx$. Here C is the constant of integration. Use abs(x) to denote $|x|$.

[ ] + C

Tries 0/99

Evaluate $\int \frac{1+8xe^x}{x}\ dx$. Here C is the constant of integration. Use abs(x) to denote $|x|$.

[ ] + C

Tries 0/99

Initial Value Problems

if $y' = 6x^2 + 2e^x - 1$

and $y(0) = 8$, find $y(x)$.

---

if $y'' = -2\cos x$

and $y'(\pi) = 3$ and $y(\frac{\pi}{2}) = e^2$

~~find y(x)~~ find $y(x)$.

---

if $y'' = 12x + e^x$

and $y(0) = 4$ and $y(2) = 6$

find $y(x)$

~~The speed of~~ acceleration of my car is given by $\frac{24}{x+5}$ feet per second per second.

If I start standing still, how fast am I going 40 seconds later?

Quiz 11 — MA16010 — November 3, 2017
Alden Bradford

1. (a) (1 point) Write a formula for the distance from any point (x, y) to the point $(2, 2)$.

   (b) (1 point) The point (x, y) is on the parabola $y = 16 - 4x^2$. Use this information to write your formula from part (a) in terms of only x.

   You DO NOT need to maximize or minimize the distance. You do not need to simplify. Just write the formulas.

2. (3 points) A newspaper costs \$0.50 to print per copy. If it is sold at a price of p dollars, then it will sell $(2000 - 10p)$ copies. Write formulas (which depend only on p) for the profit AND for the revenue made selling newspapers.

   You DO NOT need to maximize the profit nor revenue. Just give the formulas.

find $\sum_{i=1}^{5} 2i-1$, $\sum_{i=2}^{4} 3\sin\left(\frac{i\pi}{2}\right)$

write in sigma notation:

$$(e^2-3)+(e^3-2)+\dots+(e^{n+1}+n-4)$$

Quadrature

example: Find the signed area under the curve $y=x-1$ ~~between~~ $x=0$ and $x=3$

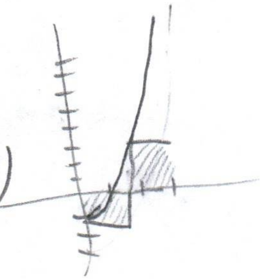
puzzle: Find the signed area under the curve $y=x^2-1$ between $x=0$ and $x=3$

Idea: break it up

Approximate the signed area with a left Riemann sum:

$-1\left(\frac{3}{2}\right)+1.25\left(\frac{3}{2}\right)$

$=3/8$



$n=2$

$n=3$

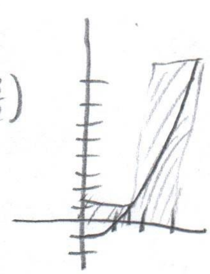
$-1(1)+0(1)+3(1)$

$=3$

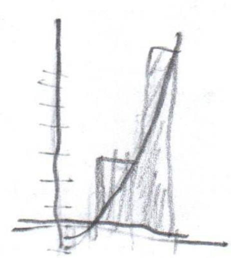
OR a right Riemann sum:

$1.25\left(\frac{3}{2}\right)+8\left(\frac{3}{2}\right)$

$=13.825$



$n=2$



$n=6$

$0(1)+3(1)+8(1)$

$=11$

use a left Riemann sum with 3 rectangles to approximate the signed area under $y=e^x$ between $x=1$ and $x=7$

Interval length?

$\Delta x = \frac{7-1}{3} = 2$

Points to check?

$x=1, 3, 5$

or $x = 1 + (\Delta x) \cdot i$, $i = 0, 1, 2$

left riemann sum $= e^1(2) + e^3(2) + e^5(2) = \sum_{i=0}^{2} e^{1+2i}(2)$

right riemann sum?

same interval length

now check $x = 3, 5, 7$ or $x = 1 + (\Delta x) \cdot i$, $i = 1, 2, 3$

right rieman sum $= e^3(2) + e^5(2) + e^7(2) = \sum_{i=1}^{3} e^{1+2i}(2)$

What if we want to split it into 100 rectangles?

left rieman sum: $\Delta x = \frac{7-1}{100} = 0.06$

points to check: $x = 1 + 0.06 \cdot i$, $i = 0, 1, \ldots, 99$

left riemann sum $= \sum_{i=0}^{99} e^{1+0.06 \cdot i} \, 0.06$

right riemann sum: points to check are $x = 1 + 0.06 \cdot i$, $i = 1, 2, \ldots, 100$

right riemann sum: $\sum_{i=1}^{100} e^{1+0.06 \cdot i} \, 0.06$

Quiz 12 — MA16010 — November 6, 2017

Alden Bradford

NOTE: if you forget the additive constant in either of these problems, then I will be sad.

5 points

1. (~~2 points~~) Evaluate $\int \sec x(4\sec x - 5\tan x)\, dx$.

2. ~~(2 points) Evaluate~~ $\cancel{\int \frac{x^5 + x^6}{\sqrt{x^3}}\, dx}$.

Quiz 11 Key — MA16010 — November 3, 2017

Alden Bradford

Min	Mean	Max
1	3.3	5

1. (a) (1 point) Write a formula for the distance from any point (x, y) to the point $(2, 2)$.

   (b) (1 point) The point (x, y) is on the parabola $y = 16 - 4x^2$. Use this information to write your formula from part (a) in terms of only x.

   You DO NOT need to maximize or minimize the distance. You do not need to simplify. Just write the formulas.

   (a) $\sqrt{(x-2)^2 + (y-2)^2}$

   (b) $\sqrt{(x-2)^2 + (16-4x^2-2)^2}$

2. (3 points) A newspaper costs \$0.50 to print per copy. If it is sold at a price of p dollars, then it will sell $(2000 - 10p)$ copies. Write formulas (which depend only on p) for the profit AND for the revenue made selling newspapers.

   You DO NOT need to maximize the profit nor revenue. Just give the formulas.

   profit$=(2000 - 10p)(p - 0.5)$, revenue$=(2000 - 10p)p$

Due date: Mon Nov 6 10:00:00 pm 2017 (EST)

Given $y' = \frac{21}{x}$ with $y(e) = 38$. Find $y(e^2)$.

$y(e^2) =$ [ ]

Tries 0/99

Solve the initial value problem $y'' = 5x + 8$ with $y'(1) = 2$ and $y(0) = 4$.

$y =$ [ ]

Tries 0/99

Solve the initial value problem $y' = 2\cos x + 2$ with $y\left(\frac{3\pi}{2}\right) = 4$.

$y =$ [ ]

Tries 0/99

Given $y'' = 9e^x + 4$ with $y'(0) = 6$ and $y(2) = 3$. Find $y(3)$.

$y(3) =$ [ ]

Tries 0/99

Given $y'' = 7e^x + 1$ with $y'(0) = 9$ and $y(2) = 6$. Find $y(3)$.

$y(3) =$ [ ]

Tries 0/99

The rate of growth $\frac{dP}{dt}$ of a population of bacteria is proportional to the square root of t with a constant coefficient of 2, where P is the population size and t is the time in days $(0 \leq t \leq 10)$. The initial size of the population is 200. Approximate the population after 7 days. Round the answer to the nearest integer.

[ ]

Tries 0/99

A hot air balloon is rising vertically with a velocity of 2.0 feet per second. A very small ball is released from the hot air balloon at the instant when it is 1280 feet above the ground. Use $a(t) = -32$ ft/sec^2 as the acceleration due to gravity.

1. How many seconds after its release will the ball strike the ground? Round the answer to the nearest two decimal places.

2. At what velocity will it hit the ground?

---

1. [ ] sec

Tries 0/99

2. [ ] ft/sec

Tries 0/99

Quiz 12 Key — MA16010 — November 6, 2017

Alden Bradford

Min	Mean	Max
1	3.3	5

NOTE: if you forget the additive constant in this problem, then I will be sad.

(5 points) Evaluate $\int \sec x(4\sec x - 5\tan x)\,dx$.

$\boxed{4\tan x - 5\sec x + C}$

Due date: Wed Nov 8 10:00:00 pm 2017 (EST)

Evaluate $\sum_{i=1}^{4}(i^2-2)$.

Tries 0/99

Evaluate $\sum_{i=3}^{6} i(\sqrt{i}+5)$. Round your answer to the second decimal place.

Tries 0/99

Use the sigma notation to write the sum.

$$(1-1)^2+(2-1)^2+...+(n-1)^2$$

$\sum_{i=1}^{n}$ ( )

Tries 0/99

Use the sigma notation to write the sum.

$$(3\sqrt{0}+1)+(3\sqrt{1}+2)+...+(3\sqrt{n}+n+1)$$

$\sum_{i=0}^{n}$ ( )

Tries 0/99

Use the Left and Right Riemann Sums with 3 rectangles to estimate the (signed) area under the curve of $y=5x^2$ on the interval of $[0,6]$.

Left Riemann Sum=

Tries 0/99

Right Riemann Sum=

Tries 0/99

Use the Left and Right Riemann Sums with 3 rectangles to estimate the area under the curve of $y=\ln x$ on the interval of $[2,8]$. Round your answers to the second decimal place.

Left Riemann Sum=

Tries 0/99

Right Riemann Sum=

Tries 0/99

Use the Left and Right Riemann Sums with 4 rectangles to estimate the (signed) area under the curve of $y=\sqrt[3]{x-1}$ on the interval of $[3,6]$. Round your answers to the second decimal place.

Left Riemann Sum=

Tries 0/99

Right Riemann Sum=

Tries 0/99

Use the Left and Right Riemann Sums with 100 rectangles to estimate the (signed) area under the curve of $y=-2x+3$ on the interval $[0,50]$. Write your answer using the sigma notation.

Left Riemann Sum $=\sum_{i=0}^{99}$( )

Tries 0/99

Right Riemann Sum $=\sum_{i=1}^{100}$( )

Tries 0/99

Use the Left and Right Riemann Sums with 80 rectangles to estimate the (signed) area under the curve of $y=e^{2x}-13$ on the interval of $[10,20]$.
Write your answer using the sigma notation.

Left Riemann Sum= $\sum_{i=0}^{79}$( )

Tries 0/99

Right Riemann Sum= $\sum_{i=1}^{80}($ [ ] $)$

Tries 0/99

Use the Left and Right Riemann Sums with 40 rectangles to estimate the (signed) area under the curve of $y = -2\sin(x + \pi)$ on the interval of $[0, 2\pi]$.

A $\frac{\pi}{10}\sum_{i=0}^{39} -2\sin(\frac{\pi}{10}i + \pi)$

B $\frac{\pi}{20}\sum_{i=1}^{40} -2\cos(\frac{\pi}{20}i + \pi)$

C $\frac{\pi}{10}\sum_{i=1}^{40} -2\cos(\frac{\pi}{10}i + \pi)$

D $\frac{\pi}{20}\sum_{i=0}^{39} -2\cos(\frac{\pi}{20}i + \pi)$

E $\frac{\pi}{20}\sum_{i=1}^{40} -2\sin(\frac{\pi}{20}i + \pi)$

F $\frac{\pi}{20}\sum_{i=0}^{39} -2\sin(\frac{\pi}{20}i + \pi)$

G $\frac{\pi}{10}\sum_{i=1}^{40} -2\sin(\frac{\pi}{10}i + \pi)$

H $\frac{\pi}{10}\sum_{i=0}^{39} -2\cos(\frac{\pi}{10}i + \pi)$

[ ] Left Riemann Sum
[ ] Right Riemann Sum

Tries 0/99

Quiz 13 — MA16010 — November 8, 2017

Alden Bradford

1. (2 points) Find the derivative of $-\frac{1}{2}\cos(x^2)$.

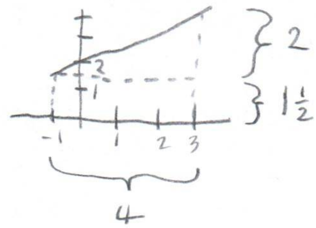
2. (2 points) Use your answer from part 1 to find $\int 8x\sin(x^2)\,dx$.

3. (1 point) $y'(x) = 8x\sin(x^2)$ and $y(0) = 4$. Use your answer from part 2 to find $y(x)$.

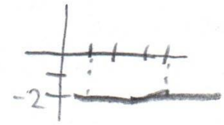
Definite Integrals

"It's the area under the curve!"

Example: $\int_{-1}^{3} \frac{1}{2}x+2 \, dx$



Example: $\int_{1}^{4} -2 \, dx$



Algebra with definite integrals!

ex $\int_{-1}^{2} x^2 \, dx = 3$ and $\int_{2}^{3} x^2 dx = 6 + \frac{1}{3}$.

Find $\int_{-1}^{3} x^2 dx$.

ex $\int_{1}^{5} f(x) \, dx = 6$ and $\int_{2}^{5} f(x) \, dx = 4$.

Find $\int_{1}^{2} f(x) \, dx$.

Questions for You (work together!)

1. $f(x) = \begin{cases} -2x-2 & \text{if } x<0 \\ x+1 & \text{if } x>0 \end{cases}$

find $\int_{-2}^{2} f(x) \, dx$

2. $\int_{0}^{2} g(x) dx = 6$, $\int_{1}^{3} g(x) dx = 5$, and $\int_{0}^{3} g(x) dx = 9$.

Find $\int_{1}^{2} g(x) \, dx$.

More Algebra

$$\int_0^2 2+x\,dx = \int_0^2 2\,dx + \int_0^2 x\,dx$$

in general: $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

$$\int_{-1}^2 2x\,dx = 2\int_{-1}^2 x\,dx$$

in general: $\int_a^b c\,f(x)dx = c\int_a^b f(x)dx$

examples

$\int_{-5}^{20} f(x)dx = 3$ and $\int_{-5}^{20} g(x)dx = -1$

Find $\int_{-5}^{20} (2f(x) + 8g(x))\,dx$

$\int_{-2}^4 f(x)\,dx = 10$, $\int_3^4 f(x) = 6$, and

Find $\int_{-2}^{3} (2f(x)+1)\,dx$

Due date: Wed Nov 15 10:00:00 pm 2017 (EST)

Evaluate $\int_0^4 2x\,dx$ by using geometric formulas.

Tries 0/99

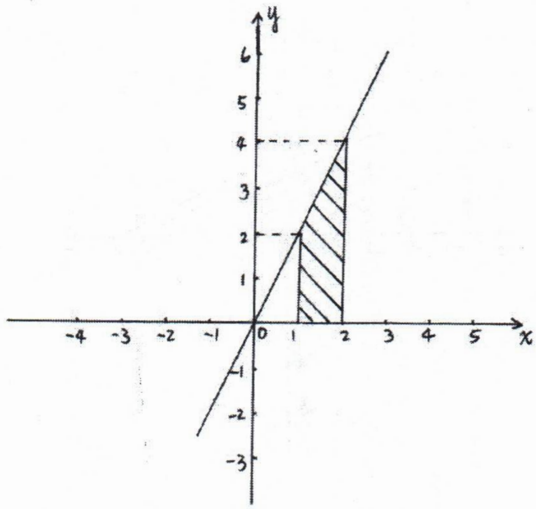
Evaluate $\int_3^7 -4\,dx$ by using geometric formulas.

Tries 0/99

Evaluate $\int_{-9}^{-3} (-3x+4)\,dx$ by using geometric formulas.

Tries 0/99

Choose the definite integral that represents the shaded area below.



A. $\int_1^2 2x\ dx$

B. $\int_2^4 2x\ dx$

C. $\int_2^4 4x\ dx$

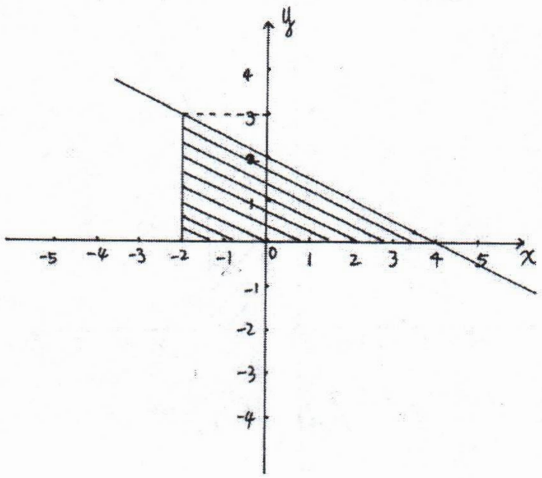
D. $\int_2^4 \frac{1}{2}x\ dx$

E. $\int_1^2 4x\ dx$

F. $\int_1^2 \frac{1}{2}x\ dx$

Tries 0/99

Choose the definite integral that represents the shaded area below.



A. $\int_{-2}^{4}(-\frac{1}{2}x+2)\ dx$

B. $\int_{0}^{3} -2x\ dx$

C. $\int_{-2}^{4}(-2x+2)\ dx$

D. $\int_{-2}^{4} -2x\ dx$

E. $\int_{0}^{3}(-\frac{1}{2}x+2)\ dx$

F. $\int_{0}^{3} -\frac{1}{2}x\ dx$

G. $\int_{-2}^{4} -\frac{1}{2}x\ dx$

H. $\int_{0}^{3}(-2x+2)\ dx$

Tries 0/99

Given $\int_3^6 x^2\ dx = 63$, $\int_3^6 x\ dx = \frac{27}{2}$, $\int_3^6 1\, dx = 3$.

Compute $\int_3^6(-6x^2+5x-6)\ dx$.

Tries 0/99

Given $\int_2^3 f(x)\ dx = 4$, $\int_3^4 f(x)\ dx = 12$, $\int_2^4 g(x)\ dx = 2$.

Compute $\int_2^4[5f(x)-2g(x)]\ dx$.

Tries 0/99

Given $\int_{-7}^{-1} f(x)\ dx = 5$, $\int_{-8}^{-4} f(x)\ dx = 8$, $\int_{-8}^{-1} f(x)\ dx = 3$.

Compute $\int_{-4}^{-7} f(x)\ dx$.

Tries 0/99

Quiz 13 Key — MA16010 — November 8, 2017

Alden Bradford

Min	Mean	Max
1	3.4	5

1. (2 points) Find the derivative of $-\frac{1}{2}\cos(x^2)$.

$\boxed{x \sin x^2}$

2. (2 points) Use your answer from part 1 to find $\int 8x\sin(x^2)\,dx$.

$\boxed{-4\cos(x^2) + C}$

3. (1 point) $y'(x) = 8x\sin(x^2)$ and $y(0) = 4$. Use your answer from part 2 to find $y(x)$.

$\boxed{-4\cos(x^2) + 8}$

1 Find the open interval where the function $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x - 7$ is concave down.

A. $(5, \infty)$

B. $(1, \infty)$

C. $(3, \infty)$

D. $(1, 5)$

E. $(-\infty, 3)$

F. $(-\infty, 1)$

Tries 0/99

---

2 Find the x-coordinate of the inflection point of $y = e^{2x} - 8x^2$.

A. $x = \ln 4$

B. $x = 2\ln 4$

C. $x = e^2$

D. $x = e$

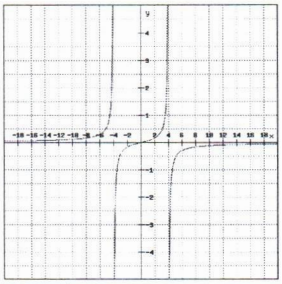
E. $x = \frac{1}{2}\ln 4$

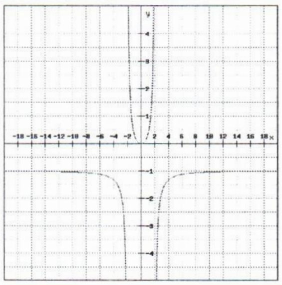
F. $x = 0$

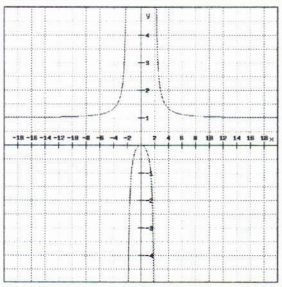
Tries 0/99

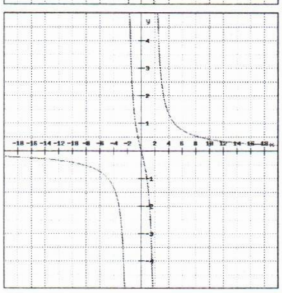
3

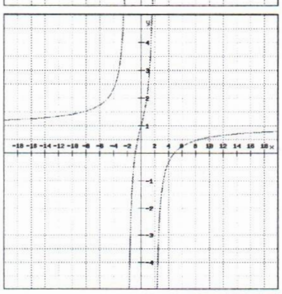
Given the function $f(x) = \dfrac{4x}{x^2-4}$ with its first and second derivatives $f'(x) = \dfrac{-4(x^2+4)}{(x^2-4)^2}$ and $f''(x) = \dfrac{8x(x^2+12)}{(x^2-4)^3}$.
Find the graph of $f(x)$.

A. 

B. 

C. 

D. 

E. 

Tries 0/99

4 Let $f(x)$ be a polynomial whose derivative is always increasing. Choose the correct statement(s).

[I.] $f(x)$ has an inflection point.

[II.]$f(x)$ has a relative maximum.

[III.] $f(x)$ is always concave up.

A. Only I is correct.

B. Only II is correct.

C. Only III is correct.

D. I and II are correct.

E. II and III are correct.

F. I and III are correct.

Tries 0/99

Which of the following limits equals to $-\infty$?

5

A. $\lim_{x\to\infty} \frac{x^3-1}{x^2+1}$

B. $\lim_{x\to-\infty} \frac{2x^2}{x^2+2}$

C. $\lim_{x\to\infty} \left(\frac{2}{x} - \frac{x}{6}\right)$

D. $\lim_{x\to\infty} \frac{-x^3+2x^2-3x}{3x^4-5x^3+1}$

E. $\lim_{x\to-\infty} \frac{1-x^2}{x-1}$

F. $\lim_{x\to\infty} \frac{x-1}{x^3-1}$

Tries 0/99

---

6 Consider the function $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 1}$. Which of the statements are true?

[I.] f has a vertical asymptote at $x = 1$.

[II.] f has a horizontal asymptote at $y = 0$.

[III.] f has a vertical asymptote at $x = -1$.

[IV.] f has a horizontal asymptote at $y = 1$.

A. II and IV

B. I and II

C. I and IV

D. II and III

E. I and III

F. III and IV

Tries 0/99

---

7 An open-top box with a square base is made using 48 ft^2 of material. Find the maximum possible volume of this box.

A. 96 ft^3

B. 16 ft^3

C. 32 ft^3

D. 64 ft^3

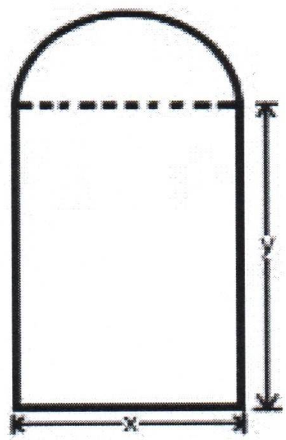
E. 48 ft^3

F. 80 ft^3

Tries 0/99

---

8 Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. See figure below. Find x which maximizes the area of this window if the total perimeter is 10 feet.



A. $\dfrac{20}{\pi}$ ft

B. $\dfrac{10}{(\pi+2)^2}$ ft

C. $\dfrac{20}{\pi+4}$ ft

D. $\dfrac{10}{\pi}$ ft

E. $\dfrac{20}{(\pi+4)^2}$ ft

F. $\dfrac{20}{\pi+2}$ ft

Tries 0/99

9 Find the x-coordinate of the point on the graph of $y = \sqrt{x} + 2$ that is the closest to the point $(3, 2)$.

A. $\frac{9}{2}$

B. 0

C. $\frac{5}{2}$

D. $\frac{3}{2}$

E. $\frac{1}{2}$

F. $\frac{7}{2}$

Tries 0/99

---

10 SheSellsSeaShells is an ocean boutique offering shells and handmade shell crafts on Sanibel Island in Florida. Find the price SheSellsSeaShells should charge to maximize revenue if $p(x) = 160 - 2x$, where $p(x)$ is the price in dollars at which x shells will be sold per day.

A. \$120

B. \$20

C. \$80

D. \$60

E. \$40

F. \$100

Tries 0/99

---

11 Find the open interval where $f(x) = \frac{1}{2}x^4 + 2x^3$ is concave downward.

A. (–3, ∞)

B. (–∞, –3)

C. (–2, 0)

D. (–3, 0)

E. (–2, ∞)

F. (–3, –2)

Tries 0/99

---

12 Let $f(x) = -x^3 + 12x$. The y values of the absolute minimum and the absolute maximum of $f(x)$ over the closed interval $[-3, 5]$ are respectively:

A. -65 and -16

B. -65 and -9

C. -65 and 16

D. -16 and 16

E. -16 and -9

F. -9 and 16

Tries 0/99

13 $\lim_{x\to\infty} f(x) = \infty$ is true for which of the following functions?

A. $f(x) = \dfrac{2x^2}{x^2 + x}$

B. $f(x) = \dfrac{2x^3 + x^2 - 2}{-3x^3 + 7}$

C. $f(x) = \dfrac{x - x^2}{-x + 5}$

D. $f(x) = \dfrac{x + 9}{x^2 + x + 6}$

E. $f(x) = \dfrac{2}{x} + 3$

F. $f(x) = \dfrac{x^3 + x^2 - 2}{-x + 5}$

Tries 0/99

---

14 Choose the correct statement regarding the asymptotes of $f(x)$.

$f(x) = \dfrac{x^2 - 2x + 6}{x + 1}$

A. Horizontal Asymptote: $y = 0$; Vertical Asymptote: $x = -1$; Slant Asymptote: None

B. Horizontal Asymptote: None; Vertical Asymptote: $x = -1$; Slant Asymptote: None

C. Horizontal Asymptote: $y = -1$; Vertical Asymptote: $x = 1$; Slant Asymptote: None

D. Horizontal Asymptote: $y = 0$; Vertical Asymptote: $x = 1$; Slant Asymptote: $y = x - 3$

E. Horizontal Asymptote: $y = -1$; Vertical Asymptote: $x = 1$; Slant Asymptote: $y = x - 3$

F. Horizontal Asymptote: None; Vertical Asymptote: $x = -1$; Slant Asymptote: $y = x - 3$

Tries 0/99

---

15 Find the point on the graph of $y = 5x + 2$ that is the closest to the point (0,4).

A. $(\frac{5}{13}, \frac{51}{26})$

B. $(\frac{10}{13}, \frac{102}{13})$

C. $(\frac{5}{26}, \frac{51}{13})$

D. $(\frac{10}{13}, \frac{51}{13})$

E. $(\frac{5}{13}, \frac{102}{13})$

F. $(\frac{5}{13}, \frac{51}{13})$

Tries 0/99

---

16 $f(x)$ is a polynomial and

$$f'(2) = 0, \qquad f'(5) = 0$$

$$f''(3.5) = 0,\ f''(x) < 0 \text{ on } (-\infty, 3.5) \text{ and } f''(x) > 0 \text{ on } (3.5, \infty)$$

Which of the following statements are true?

I. $(2, f(2))$ is an inflection point of $f(x)$.

II. $(3.5, f(3.5))$ is an inflection point of $f(x)$.

III. $f(x)$ has a relative maximum at $x = 2$.

IV. $f(x)$ has a relative minimum at $x = 5$.

A. Only I and IV are true.

B. Only II and III are true.

C. Only I and III are true.

D. Only II and IV are true.

E. Only I, II and IV are true.

F. Only II, III and IV are true.

Tries 0/99

17

$$\int \frac{\sin x - 2\cos x}{4} dx =$$

A. $\dfrac{2\sin x - \cos x}{4} + C$

B. $\dfrac{-\sin x + 2\cos x}{4} + C$

C. $\dfrac{2\sin x + \cos x}{4} + C$

D. $\dfrac{\sin x + 2\cos x}{4} + C$

E. $\dfrac{-2\sin x + 2\cos x}{4} + C$

F. $\dfrac{-2\sin x - \cos x}{4} + C$

Tries 0/99

---

18 An evergreen nursery usually sells a certain shrub after 5 years of growth and shaping. The growth rate during those 5 years is approximated by

$$\frac{dh}{dt} = 1.4t + 8,$$

where t is the time in years and h is the height in centimeters. The seedlings are 14 centimeters tall when planted. How tall are the shrubs when they are sold?

A. 36 cm

B. 57.5 cm

C. 29 cm

D. 42 cm

E. 92.5 cm

F. 71.5 cm

Tries 0/99

---

19 A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2800 - 200p$ units. Each unit costs \$10 to make. What is the maximum profit that the company can make?

A. 980 dollars

B. 1000 dollars

C. 600 dollars

D. 880 dollars

E. 1200 dollars

F. 800 dollars

Tries 0/99

---

20 Find the absolute extrema of $f(x) = 2x^3 + 3x^2 - 36x$ on the closed interval $[0, 4]$.

A. absolute minimum: $(0, 0)$; absolute maximum: $(4, 32)$

B. absolute minimum: $(-3, 0)$; absolute maximum: $(2, 0)$

C. absolute minimum: $(2, -44)$; absolute maximum: $(0, 0)$

D. absolute minimum: $(-3, 0)$; absolute maximum: $(0, 0)$

E. absolute minimum: $(2, -44)$; absolute maximum: $(-3, 81)$

F. absolute minimum: $(2, -44)$; absolute maximum: $(4, 32)$

Tries 0/99

---

21 A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 160 m of wire at your disposal, what is the **largest area** you can enclose?

A. $6400\,\mathrm{m}^2$

B. $4800\,\mathrm{m}^2$

C. $1600\,\mathrm{m}^2$

D. $3600\,\mathrm{m}^2$

E. $4000\,\mathrm{m}^2$

F. $3200\,\mathrm{m}^2$

Tries 0/99

---

22 A rectangular box with square base and top is to be constructed using sturdy metal. The volume is to be 16 m^3. The material used for the sides costs \$4 per square meter, and the material used for the top and bottom costs \$1 per square meter. What is the least amount of money that can be spent to construct the box?

A. \$55

B. \$136

C. \$30

D. \$120

E. \$160

F. \$96

Tries 0/99

---

23 Choose the correct statement(s) about the function $f(x) = 2x^3 - 9x^2$.

[I.]$f(x)$ has a relative maximum at $x = 0$.

[II.] $f(x)$ has a relative minimum at $x = 3$.

[III.] $f(x)$ is concave downward on $(-\infty, \frac{3}{2})$.

A. I only

B. II only

C. I & III only

D. II & III only

E. All of the statements are true.

F. I & II only

Tries 0/99

24 Find the point of inflection of $h(x) = xe^{-2x}$.

A. $\left(-\frac{1}{2}, -\frac{e}{2}\right)$

B. $(0, 0)$

C. $\left(-1, -e^2\right)$

D. $\left(\frac{1}{2}, \frac{e}{2}\right)$

E. $\left(\frac{1}{2}, \frac{1}{2e}\right)$

F. $\left(1, \frac{1}{e^2}\right)$

Tries 0/99

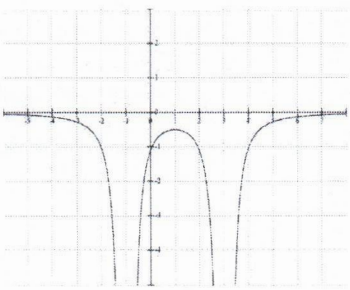
25 A function $f(x)$ satisfies the following conditions:

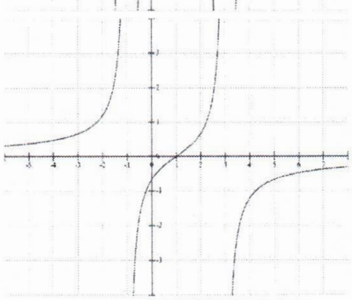
$f'(x) > 0$ on $(-\infty, -1)$

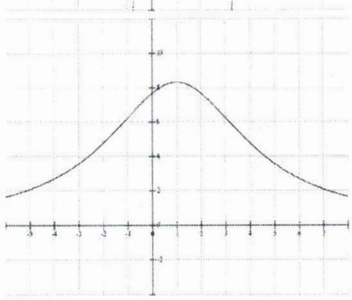
$f''(x) < 0$ on $(-1, 0)$

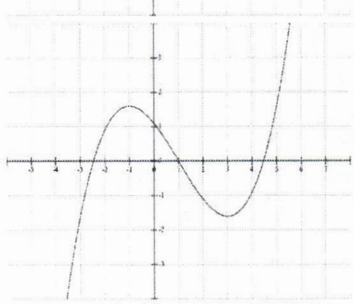
$f'(x) = 0$ at $x = 1$

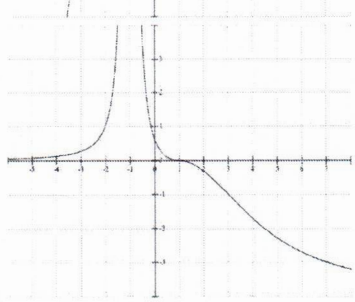
Which of the following graphs is a possible graph of $f(x)$?

A. 

B. 

C. 

D. 

E. 

26 Which of the following functions satisfies $\lim_{x\to\infty} f(x) = -\infty$?

A. $f(x) = \dfrac{2x-5}{x^2+25}$

B. $f(x) = \dfrac{x^2-3x}{x-5x^2}$

C. $f(x) = \dfrac{x^2-10}{2x^3+x}$

D. $f(x) = \dfrac{x^3-27x}{7-4x^2}$

E. $f(x) = \dfrac{x^4-16}{6x+2}$

F. $f(x) = \dfrac{6}{x} + 3$

Tries 0/99

---

27 Which of the following describes all the asymptotes of the function $f(x) = \dfrac{-2x^2-5x+7}{x+3}$?

A. $x = 3, y = 0$

B. $x = -2, y = 0$

C. $x = 3, y = -2$

D. $x = -3, y = -2x+1$

E. $x = -3, y = -2$

F. $x = -2, y = 2x+1$

Tries 0/99

28 A box with a square base and open top is to be made from 300 square inches of material. What is the volume of the largest box that can be made.

A. 600 cubic inches

B. 560 cubic inches

C. 400 cubic inches

D. 500 cubic inches

E. 472 cubic inches

F. 532 cubic inches

Tries 0/99

---

29 A poster is to have an area of 200 square inches with 1 inch margins on the left and right sides, and 2 inch margins on the top and bottom. Varying the dimensions of the poster changes the area of the region inside the margins. What is the maximum area inside the margins?

A. 168 square inches

B. 148 square inches

C. 138 square inches

D. 128 square inches

E. 88 square inches

F. 108 square inches

Tries 0/99

---

30 Find the x-coordinate of the point on the line of $y = 2x + 1$ that is closest to the point (5,1).

A. 4

B. 3

C. 5

D. 1

E. 0

F. 2

Tries 0/99

---

31 $\int \frac{3x^2-4}{2\sqrt{x}}dx =$

A. $\frac{3}{7}\sqrt{x^7} - \frac{4}{3}\sqrt{x^3} + C$

B. $\frac{9}{4}\sqrt{x} + \frac{1}{\sqrt{x^3}} + C$

C. $\frac{3}{5}\sqrt{x^3} - \frac{4}{3}\sqrt{x} + C$

D. $\frac{3}{5}\sqrt{x^5} - 4\sqrt{x} + C$

E. $\frac{3}{4}\sqrt{x^3} - \frac{3}{\sqrt{x}} + C$

F. $\frac{9}{4}\sqrt{x^5} + \sqrt{x} + C$

Tries 0/99

---

32 Find the particular solution that satisfies the following differential equation and the initial conditions.

$$f''(x) = 3\cos(x), \quad f'(0) = 4, \quad f(0) = 7$$

A. $f(x) = 3\cos(x) + x + 7$

B. $f(x) = 3\cos(x) + 4x + 10$

C. $f(x) = -3\cos(x) + x + 7$

D. $f(x) = -3\cos(x) + 4x + 10$

E. $f(x) = -3\cos(x) + 4x + 7$

F. $f(x) = 3\cos(x) + 4x + 7$

Tries 0/99

---

33 Find the inflection point of $y = x^3 + 3x^2$.

A. $(-1, 0)$

B. $(0, 0)$

C. $(0, 2)$

D. $(-1, 2)$

E. $(-2, 4)$

F. $(-2, 0)$

Tries 0/99

---

34 A particle is moving on a straight line with an initial velocity of 10 ft/sec and an acceleration of

$$a(t) = \sqrt{t} + 2,$$

where t is time in seconds and $a(t)$ is in ft/sec^2. What is its velocity after 9 seconds?

A. 24 ft/sec

B. 135 ft/sec

C. 72 ft/sec

D. 46 ft/sec

E. 90 ft/sec

F. 140 ft/sec

Tries 0/99

---

35 Which of the following limits equals $-\infty$?

A. $\lim\limits_{x\to-\infty} \dfrac{x^3+5x^2-7x}{-2x^2-5x+6}$

B. $\lim\limits_{x\to-\infty} \dfrac{-x^3+8}{x^2+x-2}$

C. $\lim\limits_{x\to-\infty} \dfrac{-2x^2+7x}{x^3+5x^2+1}$

D. $\lim\limits_{x\to-\infty} \dfrac{x^4+8x}{x^3+1}$

E. $\lim\limits_{x\to-\infty} \dfrac{x^2-4}{x^2+1}$

F. $\lim\limits_{x\to-\infty} \dfrac{x^2+4x-5}{x^4-1}$

Tries 0/99

---

36 Choose the correct statement regarding the y values of the absolute maximum and the absolute minimum of $f(x) = x^3 - 3x + 10$ on the interval of $[0, 3]$.

A. The y values of the absolute maximum and the absolute minimum are 28 and 10 respectively.

B. The y values of the absolute maximum and the absolute minimum are 28 and 8 respectively.

C. The y values of the absolute maximum and the absolute minimum are 12 and 12 respectively.

D. The y values of the absolute maximum and the absolute minimum are 12 and 8 respectively.

E. The y values of the absolute maximum and the absolute minimum are 28 and 12 respectively.

F. The y values of the absolute maximum and the absolute minimum are 12 and 10 respectively.

Tries 0/99

---

37 Which of the following statements is true regarding the function $f(x) = \dfrac{2x^2 - 3x + 4}{x - 1}$?

A. $f(x)$ has a slant asymptote which is $y = x - 1$.

B. $f(x)$ has a slant asymptote which is $y = 2x - 1$.

C. $f(x)$ has a slant asymptote which is $y = x + 1$.

D. $f(x)$ has a horizontal asymptote which is $y = 2$.

E. $f(x)$ has a horizontal asymptote which is $y = 3$.

F. $f(x)$ has a horizontal asymptote which is $y = \frac{1}{2}$.

Tries 0/99

---

38 Find the x values at which the inflection points of $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{15}{2}x^2 + 7$ occur.

A. $x = 0$ and $x = 3$

B. $x = -3$ and $x = \frac{5}{3}$

C. $x = -3$ and $x = 0$

D. $x = -5$ and $x = -3$

E. $x = -5$ and $x = 3$

F. $x = 0$ and $x = \frac{5}{3}$

Tries 0/99

---

39 Find the largest open interval(s) where $f(x) = 4x^5 - 5x^4$ is concave upward.

A. $(-\infty, 0)$ and $(1, \infty)$

B. $(\frac{3}{4}, \infty)$

C. $(-\infty, \frac{3}{4})$ and $(1, \infty)$

D. $(-\infty, 0)$ and $(\frac{3}{4}, \infty)$

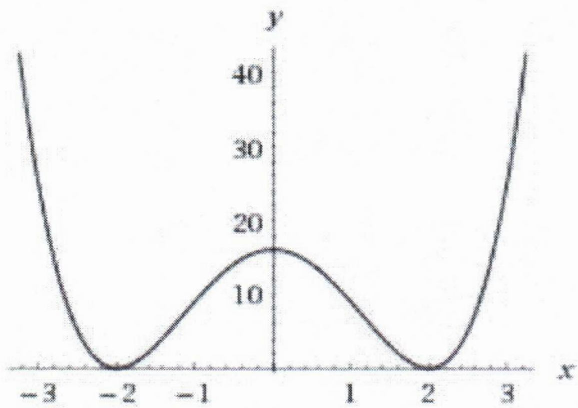
E. $(0, \infty)$

F. $(-\infty, \frac{3}{4})$

Tries 0/99

---

40 The following graph is of $f'(x)$. Choose the correct statement(s) about $f(x)$.



I. On $(-2, 2)$, $f(x)$ is increasing.

II. On $(-\infty, -2)$, $f(x)$ is concave up.

III. $f(x)$ has a relative maximum at $x = 0$.

A. I, II only

B. I only

C. I, III only

D. II, III only

E. II only

F. III only

Tries 0/99

41 Evaluate the indefinite integral $\int \sec x(\tan x - \sec x)\, dx$.

A. $\sec x + \tan x + C$

B. $\sec x - \tan x + C$

C. $-\sec x + \tan x + C$

D. $\sec x + \cot x + C$

E. $\csc x + \tan x + C$

F. $-\sec x - \tan x + C$

Tries 0/99

---

42 Solve the following initial value problem

$$y' = \frac{1}{x^2} + x, \qquad y(2) = 1$$

A. $y = -\frac{2}{x^3} + 4$

B. $y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$

C. $y = -\frac{1}{x} + \frac{x^2}{2} + \frac{1}{2}$

D. $y = -\frac{2}{x^3} + \frac{x^2}{2} + \frac{7}{2}$

E. $y = -\frac{2}{x^3} + \frac{x^2}{2} - \frac{3}{4}$

F. $y = -\frac{1}{x} + \frac{x^2}{2} + \frac{5}{2}$

Tries 0/99

43 Solve the initial value problem $y'' = 2 + 4e^x$ with $y'(0) = 1$ and $y(0) = 4$.

A. $y = 8e^{2x}$

B. $y = x^2 + 4e^x - 3x$

C. $y = 8e^{2x} + 2x$

D. $y = x^2 + 4e^x - 4x + 3$

E. $y = x^2 + 4e^x - 4$

F. $y = x^2 + 4e^x - 3x - 4$

Tries 0/99

---

44 A family wants to fence a rectangular play area alongside the wall of their house. The wall of their house bounds one side of the play area. If they want the play area to be exactly 2500 ft^2, what is the least amount of fencing needed? Round your answer to the nearest tenth place.

A. 70.7 ft

B. 141.4 ft

C. 93.3 ft

D. 212.1 ft

E. 106.1 ft

F. 186.6 ft

Tries 0/99

---

45 A box with a square base and an open top must have a volume of 4000 cm^3. If the cost of the material used is \$1 per cm^2, the smallest possible cost of the box is

A. \$500

B. \$1200

C. \$1500

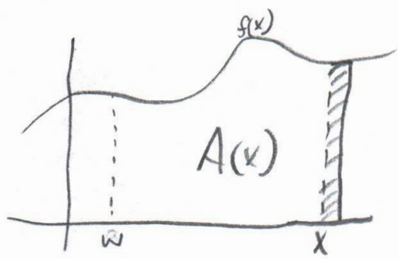
D. \$1000

E. \$600

F. \$2000

Tries 0/99

The Fundamental Theorem of Calculus

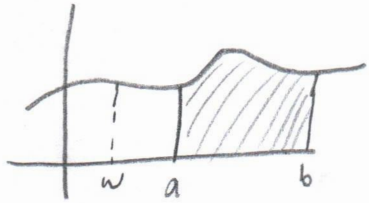


The big idea:

$A'(x) = f(x)$.

or, another way:

$A(x) = \int f(x)\,dx$



$\int_a^b f(x)\,dx = A(b) - A(a)$

> The fundamental theorem, version 1:
> if $F(x)$ is any antiderivative of $f(x)$,
> then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Another way to write it: $\int_a^b f(x)\,dx = F(x)\Big|_a^b$

---

ex (use the FTC)

$$\int_3^8 2\,\frac{1}{x}\,dx = 2\ln(x)\Big|_3^8 = 2\ln(8) - 2\ln(3)$$

$$= 2(\ln(8) - \ln(3))$$

$$= 2\ln(8/3).$$

$$\int_{-1}^{2} \frac{x^3 - 5}{7}\,dx$$

$$\int_0^{\frac{\pi}{2}} \sin(x)\,dx$$

ex Find the area bounded by the lines
$y=0$, $x=2$, $x=4$, and The curve $y=2x^2$

Find the area bounded by the lines
$y=0$, $x=-1$, $x=3$, and the curve $y=e^x+1$.

Fundamental Theorem 2

version 1:

$\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is an antiderivative of $f(x)$

version 2:

$\int_a^b f'(x)\,dx = f(b) - f(a)$

The idea: add up little changes, get the total change.

Examples

The amount of bacteria on the grape I dropped behind the cabinet grows at a rate of $0.3e^{t} + 5$ bacteria/day. If I drop it on a monday, how many bacteria will grow in the first week? in the second week?

A particle's velocity is given by $3t^2 - 10t$. How much does its displacement change between $t = 3$ minutes and $t = 10$ minutes?

The amount of heat entering my tea kettle is $1500(60 - t)$ watts. How much heat flows into my water in its third minute of use?

At my old job, the pay rate increased depending on how long you had been there. They paid $\$10 + \$1.5t$/hr, where t is in years. If I worked for 10 hrs/week, how much did I earn at that job?

Due date: Fri Nov 17 10:00:00 pm 2017 (EST)

1 Evaluate $\int_0^8 (3e^x + 7)\ dx$.

Tries 0/99

2 Evaluate $\int_8^{27} (3x^2 + \frac{\sqrt[3]{x^2}}{4}) dx$.

Tries 0/99

3 Evaluate $\int_6^9 \frac{x^2+4}{3}\ dx$.

Tries 0/99

4 Evaluate $\int_1^4 \frac{x^3+x^4}{\sqrt{x}}\ dx$.

Tries 0/99

5 Evaluate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (5 \tan x \cos x + 4)\ dx$.

Tries 0/99

6 Evaluate $\int_0^{\frac{\pi}{3}} \sec x (6 \sec x + 5 \tan x)\ dx$.

Tries 0/99

7 Find the area of the region bounded by the graphs of the following equations.

$$y = 7x + 4, \quad y = 0, \quad x = 2 \quad \text{and} \quad x = 8.$$

Tries 0/99

8 Find the area of the region bounded by the graphs of the following equations.

$$y = 4e^x, \quad y = 0, \quad x = 4 \quad \text{and} \quad x = 6.$$

Tries 0/99

9 Find the area enclosed by the graphs of the following equations.

$$y = 5(\frac{\sqrt{x}}{5} - \frac{x}{4})^2,\ y = 0,\ x = 1 \text{ and } x = 4$$

Tries 0/99

Due date: Mon Nov 27 10:00:00 pm 2017 (EST)

The growth rate of the population of a county is

$$P'(t) = \sqrt{t}(2270t + 4800),$$

where t is time in years. How much does the population increase from $t = 1$ year to $t = 4$ years?

Tries 0/99

The velocity function, in meters per second, of a particle moving along a straight line is

$$v(t) = 6t - 1,$$

where t is time in minutes.

1. Find the displacement of the particle from $t = 2$ minutes to $t = 5$ minutes?

meters

Tries 0/99

2. Find the time t when the displacement is zero after the particle starts moving?

minutes

Tries 0/99

The acceleration of a car t seconds after the driver steps on the brake, before the car comes to a full stop, is $a(t) = -(t-5)^2$ mph per second. What's the decrease in velocity in mph 4 seconds after the brake is applied?

mph

Tries 0/99

A faucet is turned on at 9:00am and water starts to flow into a tank at the rate of

$$r(t) = 10\sqrt{t},$$

where t is time in hours after 9:00am and the rate $r(t)$ is in cubic feet per hour.

1. How much water, in cubic feet, flows into the tank from 10:00am to 1:00pm?

cubic feet

Tries 0/99

2. How many hours after 9:00am will there be 205 cubic feet of water in the tank? Round your answer to the nearest tenth.

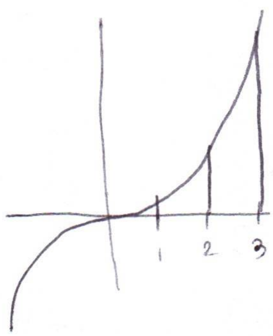
hours

Tries 0/99

All About Numerical Integration

Example

approximate $\int_0^3 x^3\,dx$ by left riemann sums, right riemann sums, and the trapezoid rule using 3 subintervals. Compare with the correct value, found with FTC.



LRS:

$(\Delta x)(f(0)+f(1)+f(2))$

$1(0+1+8)$

9

RRS: $(\Delta x)(f(1)+f(2)+f(3))$

$1(1+8+27)$

36

Trap rule: $\frac{\Delta x}{2}\left(f(0)+2f(1)+2f(2)+f(3)\right)$

$\frac{1}{2}(0+2+16+27)$

22.5

pretty close!

actual value: $\left.\frac{x^4}{4}\right|_0^3 = \frac{81}{4} = 20.25$

Notice: trap rule is the average of right & left riemann sums.

example

$$\int_0^1 e^{-x^2}\,dx \quad \leftarrow \text{The "Gaussian Integral"}$$

→ I can't solve this using the FTC (nobody can!)

→ approximate with trap rule, 4 subintervals

$$f(x) = e^{-x^2}$$

$$f(0) = 1.000$$

$$f(\tfrac{1}{4}) = 0.939$$

$$f(\tfrac{1}{2}) = 0.779$$

$$f(\tfrac{3}{4}) = 0.570$$

$$f(1) = 0.368$$

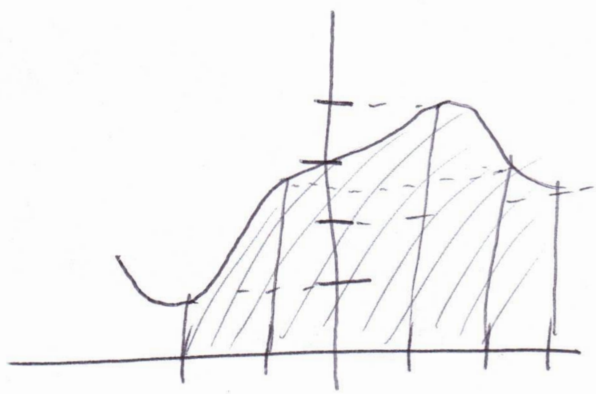
$$\int_0^1 e^{-x^2}\,dx \approx \frac{\Delta x}{2}\left(f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1)\right)$$

$$= 0.743$$

Notice: wolfram alpha gives 0.746824.

Pretty darn close!

example



use ~~tr~~ trap rule with 5 subintervals to approximate the area above.

$$A \approx \frac{\Delta x}{2}\Big(f(-2)+2f(-1)+2f(0)+2f(1)+2f(2)+f(3)\Big)$$

$$= \frac{1}{2}(1+5+6+8+5+2)$$

$$= \frac{27}{2} = 13.5$$

Due date: Wed Nov 29 10:00:00 pm 2017 (EST)

Use the Trapezoidal Rule to approximate $\int_3^5 (8x^2+1)\,dx$ using $n = 4$. Round your answer to the nearest tenth. Evaluate the exact value of $\int_3^5 (8x^2+1)\,dx$ and compare the results.

Trapezoidal Approximation ≈ [ ]

Tries 0/99

Exact Value= [ ]

Tries 0/99

Use the Trapezoidal Rule to approximate $\int_{-1}^{0} e^{x^2}\,dx$ using $n = 4$. Round your answer to the nearest hundredth.

[ ]

Tries 0/99

Use the Trapezoidal Rule to approximate $\int_3^7 \ln(x^2+5)\,dx$ using $n = 3$. Round your answer to the nearest hundredth.

[ ]

Tries 0/99

Use the Trapezoidal Rule to approximate $\int_2^3 \sqrt{x^2+7}\,dx$ using $n = 3$. Round your answer to the nearest hundredth.

[ ]

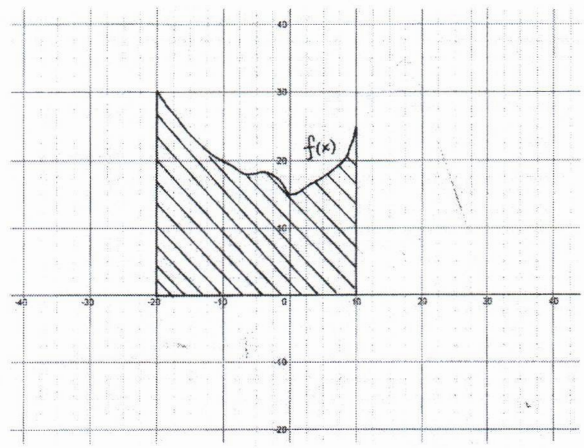
Tries 0/99

Use the Trapezoidal Rule to approximate $\int_{-0.9}^{1.3} \frac{\tan x}{x+2}\,dx$ using $n = 3$. Round your answer to the fourth decimal place.

[ ]

Tries 0/99

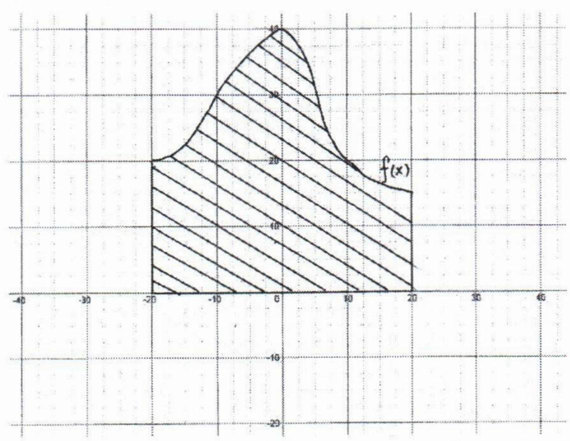
Approximate the area of the shaded region by using the Trapezoidal Rule with $n = 3$.



[ ]

Tries 0/99

Approximate the area of the shaded region by using the Trapezoidal Rule with $n = 4$.



[ ]

Tries 0/99

Exponential Growth

A very important function:

$$y = Ce^{kt}$$

notice:
$$y' = Ce^{kt}k$$
$$= ky$$

The Big Idea: If anything follows the rule $\frac{dy}{dt} = ky$, then we have a solution!

Example $\frac{dy}{dt} = 3y$ and $y(0) = 70$. find $y(t)$.

ex ~~The number of people infected with SARS during an outbreak is proportional to the number of people~~

The rate at which people were infected with SARS during the 2003 China outbreak was proportional to the number of people already infected, with proportionality constant 0.53, where t is in days. If 3 people in a town were infected on the first day, how many were infected after a week? how long did it take the ~~p~~ number of infected people to double?

My current savings account gives 0.1% interest. How long would it take the money in that account to double?

Suppose sam deposits $5000 in an account which will double her money in just 16 years. How much will she have in 10 years?

Quiz 14 — MA16010 — November 29, 2017

Alden Bradford

1. (2 points) Given that $\int_0^2 f(x)\,dx = 5$ and $\int_0^1 f(x)\,dx = -2$, find $\int_1^2 f(x)\,dx$.

2. (3 points) Find $\int_0^{3\pi/2} 4\sin x\,dx$.

Quiz 15 — MA16010 — December 1, 2017

Alden Bradford

1. (2 points) Find $\int_{-1}^{2}(x^2+1)\,dx$ using the fundamental theorem of calculus.

2. (3 points) Use the trapezoid rule with 3 subintervals to approximate $\int_{-1}^{2}(x^2+1)\,dx$.

Quiz 14 Key — MA16010 — November 29, 2017

Alden Bradford

Min	Mean	Max
1	4.2	5

1. (2 points) Given that $\int_0^2 f(x)\,dx = 5$ and $\int_0^1 f(x)\,dx = -2$, find $\int_1^2 f(x)\,dx$.

$\boxed{7}$

2. (3 points) Find $\int_0^{3\pi/2} 4\sin x\,dx$.

$\boxed{4}$

Due date: Fri Dec 1 10:00:00 pm 2017 (EST)

Given $\frac{dy}{dt} = 9y$ and $y(0) = 300$. Find $y(t)$.

$y =$

Tries 0/99

Given $\frac{dy}{dt} = 2y$ and $y(2) = 400$. Find $y(6)$.

$y =$

Tries 0/99

The population of a culture of bacteria, $P(t)$, where t is time in days, is growing at a rate that is proportional to the population itself and the growth rate is 0.3. The initial population is 30.

(1) What is the population after 20 days? (Do not round your answer.)

Tries 0/99

(2) How long does it take for the population to double? (Round your answer to one decimal place.) days

Tries 0/99

The rate of change of the population of a small town is $\frac{dP}{dt} = kP$, where P is the population, t is time in years and k is the growth rate.
If $P = 40000$ when $t = 2$ and $P = 50000$ when $t = 4$, what is the population when $t = 10$? Round your answer to the nearest integer.

Tries 0/99

Mike deposited \$10000 in a saving account in which interest is compounded continuously. The annual rate of interest is 7%.

---

(1) How much does he have in this account after 13 years? (Round your answer to two decimal places.) dollars

Tries 0/99

(2) How long does it take for his money to double? (Round your answer to two decimal places.) years

Tries 0/99

Bob deposited \$1600 in a saving account in which interest is compounded continuously. After 23 years, he has \$4000 in this account.

(1) What is the annual rate of interest? (Round your answer to one decimal place.) %

Tries 0/99

(2) How long does it take for his money to double? (Round your answer to two decimal places.) years

Tries 0/99

Suppose you deposited \$19000 in a saving account in which interest is compounded continuously. It takes 14 years to double your money in this account.

(1) What is the annual rate of interest? (Round your answer to one decimal place.) %

Tries 0/99

(2) How much will you have in this account after 29 years? (Round your answer to two decimal places.) dollars

Tries 0/99

Exponential decay

same differential equation:

$$\frac{dy}{dt} = ky$$

but k is negative!

ex suppose $y(2)=5$ and $y(4)=3$, and $\frac{dy}{dt}=ky$. Find $y(10)$.

ex carbon-11 has a half-life of 20 minutes. If my sample has 30 grams of carbon-11 now, how much will there be in an hour and a half?

ex iodine 131 has a half-life of 8 days. If my sample contains 8 grams now, how much did it have 30 days ago?

ex I measured that my sample of cobalt-60 had 50 mg this time last year, and it has 43.8 mg now. What is the halflife of cobalt-60?

ex Aluminum-26 has a halflife of 717000 years. Aluminum-26 only forms in space; it decays into Mg^{26}. Some scientists found a meteorite where only 0.05% of the aluminum-26 had not decayed (the rest was Mg^{26}). How long ago did the meteorite fall?

$$t = 717000\frac{\ln(0.0005)}{\ln(\frac{1}{2})} = 7.86 \text{ million years}$$

Due date: Mon Dec 4 10:00:00 pm 2017 (EST)

Find the following limits analytically if they exist.

Enter INF for positive infinity and -INF for negative infinity.

If a limit does not exist, enter DNE.

$$f(x) = \frac{x^2-11x}{x^2+14x}$$

(a) $\lim_{x\to 0} f(x) =$ [ ]

Tries 0/99

(b) $\lim_{x\to -14} f(x) =$ [ ]

Tries 0/99

(c) $\lim_{x\to 11} f(x) =$ [ ]

Tries 0/99

Find the derivative of $9\sin x\,(2\cos x + 8\sin x)$ at $x = \frac{\pi}{6}$.

[ ]

Tries 0/99

Find the derivative of $y = \frac{8\sqrt[3]{x}}{x^2+3}$.

$y' =$ [ ]

Tries 0/99

Find the derivative of $y = 6\tan^2(5x)$ at $x = \frac{\pi}{15}$.

$y'\left(\frac{\pi}{15}\right) =$ [ ]

Tries 0/99

Find the derivative of $f(x) = \sqrt{2x}\ln(7x)$.

$f'(x) =$ [ ]

Tries 0/99

A particle is traveling on a straight line with a position function of

$$s(t) = \frac{20}{3}t^3 + 600t^2,$$

where t is time in seconds and $s(t)$ is position in feet. Answer the following questions:

(a) What is the particle's acceleration?

$a(t) =$ [ ] ft/sec^2

Tries 0/99

(b) What is the acceleration when the velocity of the particle is 54000 ft/sec?

[ ] ft/sec^2

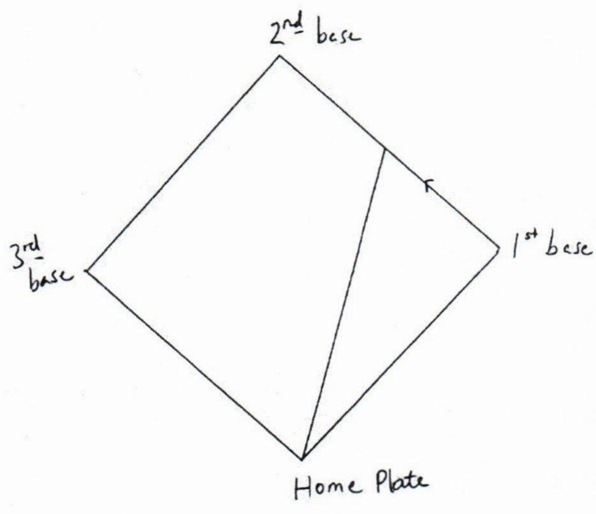
Tries 0/99

Use implicit differentiation to find the slope of the tangent line to the graph of $10\sqrt{x} + 7\sqrt{y} = 10$ at $(\frac{9}{100}, 1)$.

$\left.\frac{dy}{dx}\right|_{(\frac{9}{100},1)} =$ [ ]

Tries 0/99

A baseball diamond is a square 90 ft on a side. A player runs from first base to second base at 14 ft/sec. At what rate is the player's distance from home base increasing when he is half way from first to second base?



[ ] ft/sec

Tries 0/99

Find the critical numbers x_1 and x_2 of $y = \frac{5x^2+6}{4x}$. If there are less than two critical numbers, fill the remaining blanks with NONE.

$x_1=$

$x_2=$

Tries 0/99

The derivative of a polynomial is $f'(x) = (x-8)^2(x+6)$. Find the largest open intervals on which the function $f(x)$ is increasing or decreasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Increasing: (,) , (,)

Tries 0/99

Decreasing: (,) , (,)

Tries 0/99

Apply the First Derivative Test to find the relative extrema if they exist. If not, enter NONE in the blanks.

Relative Maximum occurs at $x=$

Tries 0/99

Relative Minimum occurs at $x=$

Tries 0/99

Consider the function $f(x) = 2x^4 + 5x^3 + 4$.

Find the largest open intervals on which the function is concave up or concave down. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up: (,) , (,)

Tries 0/99

Concave down: (,) , (,)

Tries 0/99

Find all the inflection points. Enter your answer in the format of (x, y) and enter the inflection point with the smaller x-value first. If there are no inflection points, enter NONE.

Inflection point: (,)

Tries 0/99

Inflection point: (,)

Tries 0/99

Find the largest open intervals on which the function is both concave up and decreasing. If there is more than one interval, enter your intervals from left to right as they appear on the real line. Enter INF for ∞ and -INF for $-\infty$. If there are extra blanks, enter NONE.

Concave up and decreasing: (,), (,)

Tries 0/99

Find the absolute extrema of $f(x) = 15x^4 - 60x^3 + 8$ on the closed interval $[-1, 4]$. Enter your answer in the format of (x, y)

Absolute Minimum=(,)

Tries 0/99

Absolute Maximum=(,)

Tries 0/99

Quiz 15 Key — MA16010 — December 1, 2017

Alden Bradford

Min	Mean	Max
1	4.1	5

1. (2 points) Find $\int_{-1}^{2}(x^2+1)\,dx$ using the fundamental theorem of calculus.

   $\boxed{6}$

2. (3 points) Use the trapezoid rule with 3 subintervals to approximate $\int_{-1}^{2}(x^2+1)\,dx$.

   $\boxed{6.5}$

Due date: Wed Dec 6 10:00:00 pm 2017 (EST)

Given $\frac{dy}{dt} = -2y$ and $y(2) = 200$. Find $y(6)$.

$y =$

Tries 0/99

The population, P, of a species of fish is decreasing at a rate that is proportional to the population itself. If $P = 400000$ when $t = 2$ and $P = 350000$ when $t = 5$, what is the population when $t = 10$?
Round your answer to the nearest integer.

Tries 0/99

The radioactive isotope ^{226}Ra has a half-life of approximately 1599 years. There are 35g of ^{226}Ra now.

(1) How much of it remains after 2000 years? (Round your answer to three decimal places.) g

Tries 0/99

(2) How much of it remains after 20000 years? (Round your answer to three decimal places.) g

Tries 0/99

The radioactive isotope ^{226}Ra has a half-life of approximately 1599 years. After 1800 years, there are 4g of ^{226}Ra.

(1) What was the initial quantity? (Round your answer to three decimal places.) g

Tries 0/99

(2) How much of it remains after 18000 years? (Round your answer to three decimal places.) g

Tries 0/99

The radioactive isotope ^{14}C has a half-life of approximately 5715 years. Now there are 25g of ^{14}C.

---

(1) How much of it remains after 1400 years? (Round your answer to three decimal places.) g

Tries 0/99

(2) How much of it remains after 14000 years? (Round your answer to three decimal places.) g

Tries 0/99

The radioactive isotope ^{239}Pu has a half-life of approximately 24100 years. After 1300 years, there are 1.5g of ^{239}Pu.

(1) What was the initial quantity? (Round your answer to three decimal places.) g

Tries 0/99

(2) How much of it remains after 13000years? (Round your answer to three decimal places.) g

Tries 0/99

Radioactive radium has a half-life of approximately 1599 years. What percent of a given amount remains after 800 years? (Round your answer to two decimal places.)

%

Tries 0/99

The radioactive isotope ^{14}C has a half-life of approximately 5715 years. A piece of ancient charcoal contains only 79% as much of the radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal? (Round your answer to the nearest integer.)

years

Tries 0/99

---

1 If $\frac{dy}{dt} = -3y$ and $y(1) = 5$, find $y(10)$.

A. $y(10) = e^{-27}$

B. $y(10) = e^{-29}$

C. $y(10) = 5e^{-29}$

D. $y(10) = e^{-30}$

E. $y(10) = 5e^{-30}$

F. $y(10) = 5e^{-27}$

Tries 0/99

---

2 Given that the radioactive isotope Plutonium-240 has a half-life of 6563 years, what is its decay rate, k?

A. $-\frac{1}{2}$

B. $\frac{\ln 2}{6563}$

C. $-\frac{2}{6563}$

D. -6563

E. $-\frac{1}{6563}$

F. $\frac{\ln \frac{1}{2}}{6563}$

Tries 0/99

---

3 Evaluate $\int_0^2 (3x^2 - 2x + 7e^x)\, dx$.

A. $7e^2 + 13$

B. $7e^2 - 1$

C. $7e^2 - 8$

D. $7e^2 + 11$

E. $7e^2 + 4$

F. $7e^2 - 3$

Tries 0/99

---

4 Find the Left Riemann Sum that approximates the area under the curve of $y = e^{4x}$ on the interval $[0, 2]$ with 8 rectangles.

A. $\sum_{i=0}^{9} \frac{1}{4} e^{i}$

B. $\sum_{i=1}^{9} \frac{1}{4} e^{i}$

C. $\sum_{i=1}^{7} \frac{1}{4} e^{2i}$

D. $\sum_{i=0}^{8} \frac{1}{4} e^{\frac{i}{2}}$

E. $\sum_{i=1}^{8} \frac{1}{4} e^{i}$

F. $\sum_{i=0}^{7} \frac{1}{4} e^{i}$

Tries 0/99

---

5 If $\int_a^b 2f(x)\, dx = 1$ and $\int_a^c -\frac{1}{2}f(x)\, dx = 1$, where a, b and c are some constants, find $\int_b^c f(x)\, dx$.

A. $-\frac{3}{2}$

B. 1

C. 0

D. $\frac{5}{2}$

E. $\frac{3}{2}$

F. $-\frac{5}{2}$

Tries 0/99

6

Compute the signed area of the region bounded by $y = 8 - 2x^2$ and $y = 0$ for $0 < x < 3$.

A. -18

B. 12

C. $\frac{40}{3}$

D. 6

E. -12

F. $\frac{46}{3}$

Tries 0/99

7

Evaluate the definite integral $\int_0^1 \frac{x^2 - x^3}{2\sqrt{x}}\, dx$.

A. $\frac{4}{35}$

B. $\frac{4}{15}$

C. $\frac{12}{35}$

D. $\frac{2}{35}$

E. $\frac{2}{15}$

F. $\frac{4}{5}$

Tries 0/99

8

The rate of change of a certain population of bacteria is modeled by $P'(t) = 2\sqrt{t}\,(10t + 3)$, where t is in hours. What is the increase in the bacteria population between $t = 4$ and $t = 9$ hours?

A. 386

B. 558

C. 2052

D. 1764

E. 172

F. 852

Tries 0/99

9

Evaluate $\int_0^{\frac{\pi}{3}} (\sec^2 t + \sin t)\, dt$.

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{2} + \frac{1}{2}$

C. $\sqrt{3} - \frac{1}{2}$

D. $\sqrt{3} + \frac{1}{2}$

E. $\frac{\sqrt{3}}{3} - \frac{1}{2}$

F. $\frac{\sqrt{3}}{4}$

Tries 0/99

10

Use the Trapezoidal Rule to approximate the integral $\int_1^4 e^{(x^2-1)}\, dx$ with 3 trapezoids.

A. $T_3 = e^3 + e^8 + e^{15}$

B. $T_3 = \frac{1}{2} + 2e^3 + 2e^8 + 2e^{15}$

C. $T_3 = 1 + e^3 + e^8 + e^{15}$

D. $T_3 = \frac{1}{2} + e^3 + e^8 + \frac{1}{2}e^{15}$

E. $T_3 = \frac{1}{4} + \frac{1}{2}e^3 + \frac{1}{2}e^8 + \frac{1}{4}e^{15}$

F. $T_3 = e^3 + e^8 + \frac{1}{2}e^{15}$

Tries 0/99

11

Given $\frac{dy}{dt} = \frac{1}{2}y$ and $y(0) = 2$, find $y(4)$.

A. $y(4) = e^2$

B. $y(4) = 2e^{\frac{1}{2}}$

C. $y(4) = e^4$

D. $y(4) = 2e^2$

E. $y(4) = 2e^4$

F. $y(4) = 4e^{\frac{1}{2}}$

Tries 0/99

12

If we deposit \$1000 into a savings account which compounds interests continually, at an annual rate of 5%, how many years will it take for the money to double?

A. $\frac{\ln 2}{5}$ years

B. $2\ln 2$ years

C. $5\ln 2$ years

D. $20\ln 2$ years

E. $\ln 2$ years

F. $\frac{\ln 2}{2}$ years

Tries 0/99

13

Use the Left Riemann Sum to approximate $\int_1^3 x^3\,dx$ with four rectangles.

A. 54

B. 20

C. 27

D. 14

E. 28

F. 10

Tries 0/99

14

Compute

$$\int_{-\pi}^{1} (3x + \pi)\,dx.$$

A. $\dfrac{3}{2}(1 - \pi^2)$

B. $\dfrac{3}{2}(1 + \pi^2)$

C. $\frac{3}{2} + 2\pi - \frac{\pi^2}{2}$

D. $\frac{3}{2} + \pi - \frac{\pi^2}{2}$

E. $\frac{3}{2} + \pi + \frac{\pi^2}{2}$

F. $\frac{3}{2} + 2\pi + \frac{\pi^2}{2}$

Tries 0/99

15

Find the area of the region bounded by $x = 0$, $x = 10$, $y = 0$, and $y = x^3 + 5$.

A. 600

B. 1050

C. 1850

D. 2550

E. 300

F. 2200

Tries 0/99

16

Compute

$$\int_0^{\ln(10)} e^x\, dx$$

A. $e^{10} - 1$

B. $9 - e$

C. 11

D. 9

E. $e^{\ln(10)}$

F. $11 - e$

Tries 0/99

17

You have just placed \$300 into a bank account that earns interest at an annual rate of 7% compounded continuously. How much money will be in your bank account 3.5 years from now? Round your answer to the nearest cent.

A. \$363.34

B. \$437.36

C. \$415.79

D. \$383.29

E. \$321.74

F. \$453.42

Tries 0/99

18

You have just placed \$800 into a bank account that accumulates interest with continuous compounding. If 20 years from now you will have \$1200, how much money will you have in your bank account 30 years from now? Round your answer to the nearest cent.

A. \$1469.69

B. \$1400.00

C. \$600.00

D. \$1546.63

E. \$1342.23

F. \$1537.32

Tries 0/99

19

Suppose that the half life of some radioactive isotope is 50,000 years. If you start out with 2,500 grams of this radioactive isotope, how much will be left after 65,000 years? Round your answer to the nearest whole number.

A. 1015 grams

B. 983 grams

C. 1005 grams

D. 994 grams

E. 1028 grams

F. 920 grams

Tries 0/99

20 Suppose that the half-life of some radioactive material is 2 years. If you start with 1,024,000 pounds of this radioactive material, how long will it be until there are only 1,000 pounds left? Round your answer until the nearest year.

A. 20 years

B. 40years

C. 15 years

D. 100 years

E. 60 years

F. 10 years

Tries 0/99

---

21 Evaluate the following integral

$$\int_1^4 \frac{x+x^2}{\sqrt{x}}\,dx.$$

A. $\frac{256}{15}$

B. $\frac{268}{15}$

C. 8

D. $\frac{3}{4}$

E. 26

F. 38

Tries 0/99

---

22 Let $f(x)$ and $g(x)$ be functions with antiderivatives $F(x)$ and $G(x)$ respectively.

Given that $F(3) = 3$, $G(3) = 5$, $F(6) = 1$, and $G(6) = 6$, evaluate the following integral

$$\int_3^6 (3f(x) - 4g(x))\,dx$$

A. -10

B. 20

C. -32

D. -50

E. -9

F. 11

Tries 0/99

---

23 Given that $\int_1^8 f(x)\,dx = 3$, $\int_0^4 f(x)\,dx = -7$, and $\int_0^8 f(x)\,dx = 10$, find

$$\int_1^4 f(x)\,dx.$$

A. -14

B. 0

C. 10

D. -20

E. -8

F. 6

Tries 0/99

24 Find the expression that represents the Left Riemann Sum of the (signed) area under the curve of $y = x^2 \sin x$ on the interval of $[5, 15]$ with 80 rectangles.

A. $\sum_{i=0}^{79} \frac{1}{8}\left(5+\frac{i}{8}\right)^2 \sin\left(5+\frac{i}{8}\right)$

B. $\sum_{i=0}^{79} \frac{1}{8}\left(5^2+\frac{i}{8}\right) \sin\left(5+\frac{i}{8}\right)$

C. $\sum_{i=0}^{79} \left(5+\frac{i}{8}\right)^2 \sin\left(5+\frac{i}{8}\right)$

D. $\sum_{i=0}^{79} (5+i)^2 \sin(5+i)$

E. $\sum_{i=0}^{79} \frac{1}{8}\left(\frac{i}{8}\right)^2 \sin\left(\frac{i}{8}\right)$

F. $\sum_{i=0}^{79} \left(\frac{i}{8}\right)^2 \sin\left(\frac{i}{8}\right)$

Tries 0/99

25 Find the signed area of the region bounded by the graphs of the following equations

$$x = 3,\ x = 5,\ y = 0,\ y = 3x^2 + 4x.$$

A. 130

B. $\frac{604}{3}$

C. 166

D. 56

E. 86

F. 358

Tries 0/99

26 The velocity function, in meters per second, of a particle moving is given by

$$v(t) = 5t - 2,$$

where t is time in seconds. Find the displacement of the particle from $t = 0$ seconds to $t = 6$ seconds.

A. 78 meters

B. 88 meters

C. 28 meters

D. $\frac{4}{5}$ meters

E. $\frac{10}{9}$ meters

F. 102 meters

Tries 0/99

27 A faucet is turned on and water begins to flow into a tank at a rate of

$$r(t) = 10\sqrt{t}$$

cubic feet per hour, where t is in hours. How many hours later will there be $\frac{160}{3}$ cubic feet of water in the tank?

A. 4

B. 8

C. 3

D. 2

E. 6

F. $16\sqrt{2}$

Tries 0/99

28 Which of the following gives the correct expression for the approximation of

$$\int_1^3 \ln(x^2+3)\,dx$$

using the Trapezoidal Rule with $n = 4$ trapezoids?

A. $\frac{1}{4}(\ln 4 + 2\ln\frac{21}{4} + 2\ln 7 + 2\ln\frac{37}{4} + \ln 12)$

B. $\frac{1}{2}(\ln 4 + 2\ln\frac{21}{4} + 2\ln 7 + 2\ln\frac{37}{4} + \ln 12)$

C. $\frac{1}{2}(\ln 4 + \ln\frac{9}{2} + \ln 5 + \ln\frac{11}{2} + \ln 6)$

D. $\frac{1}{4}(\ln 4 + \ln\frac{9}{2} + \ln 5 + \ln\frac{11}{2} + \ln 6)$

E. $\frac{1}{4}(\ln 4 + 2\ln\frac{9}{2} + 2\ln 5 + 2\ln\frac{11}{2} + \ln 6)$

F. $\frac{1}{4}(\ln 4 + \ln\frac{21}{4} + \ln 7 + \ln\frac{37}{4} + \ln 12)$

Tries 0/99

---

29 Given that $\frac{dy}{dt} = 60y$ and $y(0) = 120$, find $y(t)$.

A. $y(t) = \ln(60t + 120)$

B. $y(t) = 120e^{60t}$

C. $y(t) = 120e^{t} + 60t$

D. $y(t) = e^{60t} + 120 - e$

E. $y(t) = 60t + 120$

F. $y(t) = e^{60t} + 120$

Tries 0/99

---

30 Evaluate

$$\sum_{i=2}^{5} \frac{i(i+1)}{2}.$$

A. 9

B. 34

C. 28

D. 58

E. 47

F. 72

Tries 0/99

---

31 The half-life of carbon-14 is about 5715 years. Explorers found a mummy containing only 70% of the amount of this isotope that is normally found in living human beings. How old is this mummy? Round your answer to the nearest integer.

A. 853 years old

B. 2941 years old

C. 3202 years old

D. 2536 years old

E. 1245 years old

F. 2823 years old

Tries 0/99

32

Joe has invested \$50,000 in a fund which pays him 8% a year, continuously compounded. He estimates that he can retire with \$200,000. How long will that take?

A. 13.65 years

B. 17.33 years

C. 19.42 years

D. 25.96 years

E. 15.21 years

F. 30.26 years

Tries 0/99

33

The population of a city has been growing at a rate that is proportional to the population itself. According to census data, the population was 150 thousand people in 2000, and 170 thousand in 2010. Assuming that trend continues, how many people will be in this city by the year 2020 when the next census will take place? Round your answer to the nearest integer.

A. 182,365 people

B. 192,667 people

C. 178,389 people

D. 201,624 people

E. 195,223 people

F. 261,325 people

Tries 0/99

34

Use the left Riemann sum with $n = 5$ to approximate the area under the graph of the curve $f(x) = x \ln x$ over the interval $[1, 11]$. Give your answer with two decimal digits of accuracy.

A. 26.38

B. 89.48

C. 94.21

D. 71.12

E. 44.74

F. 142.23

Tries 0/99

35

Use the right Riemann sum with $n = 200$ to approximate the area under $f(x) = x^2 e^{2x}$ over the interval $[0, 100]$.

A. $R_{200} = \frac{1}{2} \sum_{i=0}^{199} x_i^2 e^{2x_i}$

B. $R_{200} = \frac{1}{2} \sum_{i=1}^{200} x_i^2 e^{2x_i}$

C. $R_{200} = \frac{1}{2} \sum_{i=1}^{200} x_i^3 e^{2x_i}$

D. $R_{200} = \sum_{i=0}^{199} x_i^2 e^{2x_i}$

E. $R_{200} = \sum_{i=1}^{200} x_i^2 e^{2x_i}$

F. $R_{200} = \sum_{i=1}^{200} x_i^3 e^{2x_i}$

Tries 0/99

36

Compute the signed area of the region bounded by the curves $y = 2 + \sin x$, $y = 0$, $x = 0$ and $x = 2\pi$.

A. $4\pi - 2$

B. 4π

C. $8\pi - 2$

D. 8π

E. 0

F. $\frac{3\pi}{2}$

Tries 0/99

37

Compute the definite integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (2\sec^2\theta + 3\theta)\, d\theta$.

A. $\frac{52\sqrt{3}}{27} + \frac{\pi^2}{2}$

B. $\frac{4\sqrt{3}}{3} + \frac{\pi^2}{8}$

C. $\frac{3\sqrt{3}}{3} + \frac{\pi^2}{4}$

D. $\frac{4\sqrt{3}}{3} + \frac{\pi^2}{4}$

E. $\frac{52\sqrt{3}}{27} + \frac{\pi^2}{8}$

F. $\frac{3\sqrt{3}}{3} + \frac{\pi^2}{2}$

Tries 0/99

38

Evaluate $\int_1^8 (2x - \frac{\sqrt[3]{x}}{3})\, dx$.

A. 15

B. $\frac{237}{4}$

C. $\frac{138}{3}$

D. $\frac{256}{81}$

E. $\frac{25}{16}$

F. $\frac{153}{5}$

Tries 0/99

39

Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x(2\csc x + 3\cot x)\, dx$.

A. $2\sqrt{3} - 1$

B. $3\sqrt{2} - 1$

C. $3\sqrt{5} - 3$

D. $\frac{3\sqrt{2}}{5} - 4$

E. $2\sqrt{2} - 3$

F. $2\sqrt{5} - 1$

Tries 0/99

40

Find the signed area enclosed by the region bounded by the curves of

$$y = \frac{x + \sqrt[4]{x}}{\sqrt{x}}, \; y = 0, \; x = 0 \text{ and } x = 16.$$

A. $\frac{80}{3}$

B. 120

C. $\frac{21}{20}$

D. 240

E. $\frac{160}{3}$

F. $\frac{156}{5}$

Tries 0/99

41

The growth rate of the population of a country is given by $P'(t) = \sqrt[3]{t}\,(2651t + 2210)$, where t is in years and $t = 0$ corresponds to 2010. How much did the population grow from 2010 to 2013? Round your answer to the nearest integer.

A. 35,774 people

B. 19,030 people

C. 17,925 people

D. 42,555 people

E. 21,919 people

F. 23,223 people

Tries 0/99

42

Use the trapezoidal rule with $n = 5$ to approximate the integral $\int_1^6 e^x \ln x \, dx$. Round your answer to the nearest integer.

A. 564

B. 321

C. 80

D. 1406

E. 703

F. 111

Tries 0/99

43

Given $\frac{dy}{dt} = -4y$ and $y(3) = 42$, find $y(5)$.

A. $42e^{-20}$

B. $42e^{14}$

C. $42e^{12}$

D. $42e^{-36}$

E. $42e^{-8}$

F. $42e^{32}$

Tries 0/99

44

Jasmine and Bella start at the same spot and run back and forth on a straight road. Jasmine's velocity is $v(t) = t + 2$ feet per second, and Bella's velocity is $v(t) = 3\sqrt{t}$ feet per second. Which of the following statements is true about their displacements after $t = 4$ seconds?

A. Jasmine's displacement is negative while Bella's displacement is positive.

B. Jasmine's displacement is positive while Bella's displacement is negative.

C. Jasmine's displacement is greater than Bella's displacement.

D. Both of their displacements are negative.

E. Jasmine's displacement equals to Bella's displacement.

F. Jasmine's displacement is less than Bella's displacement.

Tries 0/99

45

Evaluate the sum $\sum_{i=0}^{3}(-i^2 + 2i + 1)$.

A. -30

B. -27

C. 27

D. -2

E. 2

F. 30

Tries 0/99

46

Use the left Riemann sum with 120 rectangles to estimate the signed area under $y = 2\cos(3x)$ on the interval $[0, 2\pi]$. Give the answer in sigma notation.

A. $\sum_{i=0}^{120} \frac{\pi}{30} \cos(\frac{1}{60}\pi i)$

B. $\sum_{i=0}^{119} \frac{\pi}{60} \cos(\frac{1}{60}\pi i)$

C. $\sum_{i=0}^{120} \frac{\pi}{60} \cos(\frac{1}{20}\pi i)$

D. $\sum_{i=0}^{119} \frac{\pi}{30} \cos(\frac{1}{60}\pi i)$

E. $\sum_{i=0}^{119} \frac{\pi}{30} \cos(\frac{1}{20}\pi i)$

F. $\sum_{i=1}^{120} \frac{\pi}{30} \cos(\frac{1}{20}\pi i)$

Tries 0/99

47

Given $\int_{-5}^{-2} 2f(x)\,dx = 6$ and $\int_{0}^{-2} f(x)\,dx = 1$, compute $\int_{-5}^{0} 3f(x)\,dx$.

A. 21

B. 18

C. 5

D. 7

E. 6

F. 15

Tries 0/99

48

Given $\int_1^3 f(x)\,dx = 7$, $\int_3^1 g(x)\,dx = 2$ and $\int_1^3 h(x)\,dx = -3$, compute $\int_1^3 (f(x) + g(x) + 2h(x))\,dx$.

A. 2

B. -5

C. 6

D. -3

E. -1

F. 3

Tries 0/99

49

Consider $f(x) = (x+1)^2$. Ethan approximates the area under $f(x)$ on the interval $[0, 2]$ using the right Riemann sum with 2 rectangles. Vincent approximates the area under $f(x)$ on the interval $[0, 2]$ using the Trapezoidal Rule with 2 trapezoids. Let E be Ethan's estimate and V be Vincent's estimate. What is $E - V$?

A. 9

B. 8

C. 14

D. 13

E. 4

F. 5

Tries 0/99

1

Given $f(x) = \dfrac{x-1}{\sqrt{x}-1}$. Find $\lim\limits_{x\to 1} f(x)$ numerically.

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
$f(x)$									

A. 2

B. -1

C. $-\infty$

D. ∞

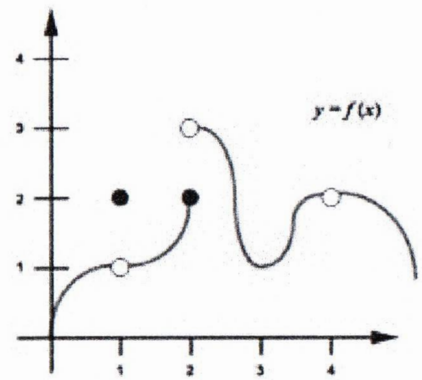
E. 0

F. 1

Tries 0/99

2

Choose the correct statement(s) regarding $f(x)$.



I. $f(x)$ is discontinuous at $x = 1, x = 2$ and $x = 4$.
II. $\lim\limits_{x\to 2} f(x) = 2$.
III. $\lim\limits_{x\to 4} f(x)$ does not exist
IV. $\lim\limits_{x\to 1} f(x) = 2$.

A. I only

B. II and IV only

C. II and III only

D. I and IV only

E. I and II only

F. IV only

Tries 0/99

3

Which of the following function has a vertical asymptote at $x = -3$?

A. $y = \frac{x-3}{x^2-9}$

B. $y = \frac{x^2-9}{x-3}$

C. $y = \frac{x+3}{3-x}$

D. $y = \frac{x+3}{x-3}$

E. $y = \frac{x^2+3x}{x+3}$

F. $y = x + 3$

Tries 0/99

4

Which of following does NOT equal to positive infinity $(+\infty)$?

A. $\lim\limits_{x \to 2} \frac{x+2}{x-2}$

B. $\lim\limits_{x \to 0^+} \frac{1}{x}$

C. $\lim\limits_{x \to 1} \frac{1}{(x-1)^2}$

D. $\lim\limits_{x \to 3^+} \frac{x}{\sqrt{x^2-9}}$

E. $\lim\limits_{x \to 1^+} \frac{1}{x-1}$

F. $\lim\limits_{x \to 0} \frac{1}{x^2}$

Tries 0/99

5

The population P, in thousands, of a small city is given by $P(t) = 10 + \dfrac{50t}{2t^2+9}$ where t is the time in years. What is the rate of change of the population at $t = 2$ yr? Round your answer to the third decimal place.

A. 0.173 thousand per year

B. 0.346 thousand per year

C. 5.882 thousand per year

D. 2.941 thousand per year

E. 3.214 thousand per year

F. -1.557 thousand per year

Tries 0/99

6

Find the limit:

$$\lim_{x \to 2} \frac{x^2}{x^2+1}$$

A. $\frac{4}{5}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. 4

E. $\frac{16}{5}$

F. 1

Tries 0/99

7

A student used the limit process to find the derivative of $f(x) = \frac{x^2}{2}$ and his work is shown below. Which of the following statements is true?

$f'(x) = \lim\limits_{h\to 0} \frac{\frac{(x+h)^2}{2} - \frac{x^2}{2}}{h}$..................(1)
$= \lim\limits_{h\to 0} \frac{x^2+2xh+h^2-x^2}{2h}$......................(2)
$= \lim\limits_{h\to 0} \frac{2xh+h^2}{2h}$...........................(3)
$= \lim\limits_{h\to 0} (x + h^2)$..........................(4)
$= x$....................................(5)

A. He made a mistake in Line (1).

B. He made a mistake in Line (2).

C. He made a mistake in Line (3).

D. He made a mistake in Line (4).

E. He made a mistake in Line (5).

F. He did not make any mistake.

Tries 0/99

8

Given $f(x) = \dfrac{1}{x+1}$, and $g(x) = \dfrac{x-1}{x^2-1}$, which of the following statements is false?

A. $\lim\limits_{x\to -1} f(x)$ does not exist.

B. $\lim\limits_{x\to -1} g(x)$ does not exist.

C. $g(x)$ has a vertical asymptote at $x = -1$

D. $f(x)$ has a vertical asymptote at $x = -1$

E. $\lim\limits_{x\to 1} g(x)$ does not exist.

F. $g(x)$ has a hole at $x = 1$

Tries 0/99

9

For:

$$f(x) = \begin{cases} x+2 & : x < -1 \\ -x-2 & : x \geq -1 \end{cases}$$

Choose the number of correct statements below.

I. f is not continuous at $x = -1$.
II. $\lim\limits_{x\to -1^+} f(x) = 1$.
III. $\lim\limits_{x\to -1} f(x) = 1$.
IV. $\lim\limits_{x\to -1} f(x) \neq \lim\limits_{x\to -1^+} f(x)$.

A. Only one of the above statements is true.

B. All of the above statements are true.

C. Only three of the above statements are true.

D. Only two of the above statements are true.

E. None of the above statements is true.

Tries 0/99

10

Find the derivative of $y = (\sin x + \tan x)e^x$.

A. $y' = (\sin x + \cos x + 2\tan x+)e^x$

B. $y' = (\cos x + \tan x \sec x)e^x$

C. $y' = (\sin x + \cos x + \tan x + \sec x \tan x)e^x$

D. $y' = (\sin x + \cos x + \tan x + \sec x)e^x$

E. $y' = (\sin x + \cos x + \tan x + \sec^2 x)e^x$

F. $y' = (\cos x + \sec^2 x)e^x$

Tries 0/99

11

Find the equation of the tangent line to the graph of $g(x) = \frac{x^2+32\sqrt{x}}{8}$ at $x = 4$.

A. $y = 2x - 18$

B. $y = 5x + 18$

C. $y = 2x + 10$

D. $y = 5x - 10$

E. $y = 2x + 2$

F. $y = 5x - 30$

Tries 0/99

12

Given the piecewise function:

$$f(x) = \begin{cases} x+4 & \text{if } x \le -2 \\ -x-2 & \text{if } -2 < x \le 2 \\ x-2 & \text{if } x > 2 \end{cases}$$

Which of the following statements is false?

A. $\lim_{x\to 0} f(x) = -2$

B. $\lim_{x\to 0^+} f(x) = -2$

C. $\lim_{x\to 2^+} f(x) = 0$

D. $\lim_{x\to -2^+} f(x) = 0$

E. $\lim_{x\to 2} f(x) = -2$

F. $\lim_{x\to -2} f(x) = 2$

Tries 0/99

13

Which of following does NOT equal to positive infinity $(+\infty)$?

A. $\lim_{x\to 4} \frac{x^2}{\sqrt{16-x^2}}$

B. $\lim_{x\to 0^+} \frac{x+3}{x^2}$

C. $\lim_{x\to 2} \frac{1}{(x-2)^2}$

D. $\lim_{x\to 3^+} \frac{3}{x-3}$

E. $\lim_{x\to 2^+} \frac{x+8}{2-x}$

F. $\lim_{x\to 0} \frac{5x+4}{x^2}$

Tries 0/99

14

Find the limit: $\lim_{x\to 1} \frac{-4x+4}{x^2-4x+3}$

A. $\frac{4}{3}$

B. -2

C. DNE

D. 1

E. 2

F. 0

Tries 0/99

15

Consider the function $f(x) = \dfrac{1}{2x-1}$. When using the definition of derivative (the limit process) to compute $f'(x)$, we would need to find the following limit:

A. $\lim\limits_{h\to 0} \dfrac{h}{(2x+2h-1)(2x-1)}$

B. $\lim\limits_{h\to 0} \dfrac{-h}{(2x+2h-1)(2x-1)}$

C. $\lim\limits_{h\to 0} \dfrac{-h}{(2x+h-1)(2x+2h-1)}$

D. $\lim\limits_{h\to 0} \dfrac{-1}{(2x+h-1)(2x-1)}$

E. $\lim\limits_{h\to 0} \dfrac{-2}{(2x+2h-1)(2x-1)}$

F. $\lim\limits_{h\to 0} \dfrac{h-1}{(2x+h-1)(2x-1)}$

Tries 0/99

16

Given $f(x) = \dfrac{x^2-9}{x}$, and $f'(x) = \dfrac{x^2+9}{x^2}$. Find the equation of the tangent line to the graph of $f(x)$ at $x = -1$.

A. $y = 8x - 18$

B. $y = 10x - 18$

C. $y = 10x - 2$

D. $y = 8x - 2$

E. $y = 10x + 18$

F. y = 8x + 18

Tries 0/99

17

Use the graph of f(x) to choose the correct statements.

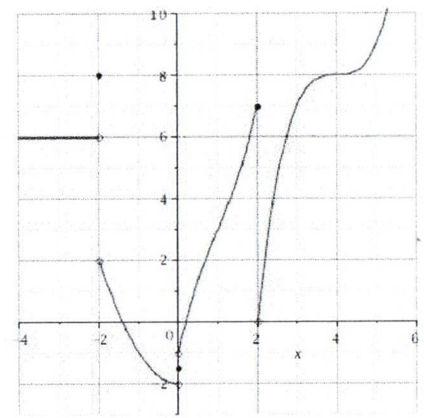


Figure 1: Graph of f(x)

I. f(x) is discontinuous at x=-2, x=0 and x=4.
II. $\lim\limits_{x\to 0^+} f(x) = -1$.
III. $\lim\limits_{x\to 2} f(x) = 0$.
IV. $\lim\limits_{x\to 4} f(x) = f(4)$.

A. I and IV only.

B. III and IV only.

C. II and III only.

D. I and III only.

E. II and IV only.

F. I and II only.

Tries 0/99

18

Which of the following is FALSE for the function

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$$

A. $\lim_{x\to 1} f(x) = \lim_{x\to 1^+} f(x)$

B. $\lim_{x\to -1} f(x) = 1$

C. f(x) is continuous at every point except x=0

D. $\lim_{x\to -1} f(x) = 1.$

E. $\lim_{x\to 0} f(x) = 1.$

F. $\lim_{x\to 1} f(x) = 1.$

Tries 0/99

19

Which of the following is TRUE regarding $f(x) = \dfrac{x+4}{x^2+x-12}$

A. $\lim_{x\to 3} f(x)$ does not exist and $f(x)$ has a vertical asymptote at $x = -4$

B. $\lim_{x\to -4} f(x)$ does not exist and $f(x) hasaholeat x = -4$

C. $\lim_{x\to 3} f(x)$ does not exist and $f(x)$ has a vertical asymptote at $x = 3$

D. $\lim_{x\to -4} f(x)$ does not exist and $f(x)$ has a vertical asymptote at $x = -4$

E. $\lim_{x\to 3} f(x)$ does not exist and $f(x)$ has a hole at $x = -3$

F. $\lim_{x\to -4} f(x)$ does not exist and $f(x)$ has a vertical asymptote at $x = 3$

Tries 0/99

20

Which of the following limits does NOT equal to $(+\infty)$?

A. $\lim_{x\to 0} \dfrac{-2}{x}$

B. $\lim_{x\to 1^+} \dfrac{2x+1}{(x-1)^2}$

C. $\lim_{x\to 1} \dfrac{3x}{x^2-1}$

D. $\lim_{x\to 2^+} \dfrac{2}{x-2}$

E. $\lim_{x\to 2} \dfrac{x^2}{\sqrt{4-x^2}}$

F. $\lim_{x\to 0} \dfrac{x+5}{x^2}$

Tries 0/99

21

Find the equation of the tangent line to $f(x) = x - 5\sin x$ at $x = \dfrac{\pi}{2}$.

A. $y = x + \dfrac{\pi}{2}$

B. $y = x - \pi + 5$

C. $y = x - 5$

D. $y = x$

E. $y = \dfrac{\pi}{2}$

F. $y = x + 5$

Tries 0/99

---

22 Let $f(x) = \cos x - 2x$, $g(x) = x - \frac{1}{3x^3}$ and $h(x) = e^x$. Which of the following is NOT true for all values of $x < 0$?

A. $h'(x) < 1$

B. $f'(x) < h'(x)$

C. $g'(x) < h'(x)$

D. $g'(x) > 0$

E. $f'(x) < g'(x)$

F. $h'(x) > 0$

Tries 0/99

---

23 The population P, in thousands, of a certain species of birds is given by:

$$P(t) = 2t^3 + 3t^2 - 4t + 10$$

where t is the number of years. What is the rate of change of population at $t = 2$?

A. 20 thousand per year

B. 10 thousand per year

C. 32 thousand per year

D. 24 thousand per year

E. 46 thousand per year

F. 30 thousand per year

Tries 0/99

---

24 Given $f(x) = \frac{x^2}{\sin x}$. Find $f'(x)$.

A. $f'(x) = \frac{x^2 \cos x + 2x \sin x}{\sin^2 x}$

B. $f'(x) = \frac{x^2 \cos x - 2x \sin x}{\sin^2 x}$

C. $f'(x) = \frac{2x \sin x - x^2 \cos x}{\sin^2 x}$

D. $f'(x) = \frac{2x \sin x - x^2 \cos x}{x^4}$

E. $f'(x) = \frac{2x \sin x + x^2 \cos x}{\sin^2 x}$

F. $f'(x) = \frac{2x}{\cos x}$

Tries 0/99

---

25 Find the slope of the tangent line to the graph of $y = x \cot x$ at $x = \frac{\pi}{4}$.

A. 1

B. -1

C. $1 - \frac{\pi}{2}$

D. 3

E. $\frac{1}{2}$

F. $1 + \frac{\pi}{8}$

Tries 0/99

---

26 A bowling ball is launched off the top of a 240-foot tall building. The height of the bowling ball above the ground t seconds after being launched is $s(t) = -16t^2 + 32t + 240$ feet above the ground. What is the velocity of the ball as it hits the ground?

A. 64 ft/s

B. 0 ft/s

C. −128 ft/s

D. 32 ft/s

E. 5 ft/s

F. −76 ft/s

Tries 0/99

27 Given $f(x) = e^x(3x^2 - x + 1)$, find $f'(x)$.

A. $e^x(3x - 1)$

B. $e^x(6x - 1)$

C. $e^x(3x^2 + 5x)$

D. $e^x(3x^2 - x + 1)$

E. $e^x(x - 1)$

F. $e^x(3x^2 - 7x + 2)$

Tries 0/99

28 Given $h(t) = \dfrac{3t - 1}{\sqrt{t} - 2}$, find $h'(16)$.

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{32}$

D. $\frac{1}{8}$

E. $\frac{1}{16}$

F. $\frac{1}{4}$

Tries 0/99

29 A ball is thrown straight up from the top of a 64-foot building with an initial velocity of 32 feet per second. Use the position function below for free-falling objects and find its velocity after 2 seconds.

$$s(t) = -16t^2 + 32t + 64$$

A. 32 ft/sec

B. 64 ft/sec

C. −32 ft/sec

D. 48 ft/sec

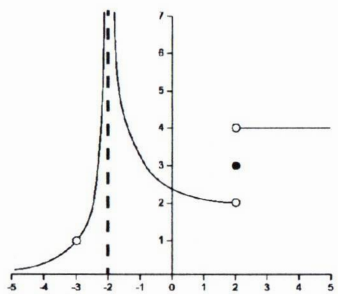
E. −16 ft/sec

F. −64 ft/sec

Tries 0/99

30

If $f(x)$ has the graph sketched below, then $\lim_{x \to 2} f(x) =$



A. 4

B. ∞

C. 3

D. 1

E. The limit does not exist.

F. 2

Tries 0/99

31

Find

$$\lim_{x \to 1} \frac{x^2 + 2x - 3}{x^2 + x - 2}.$$

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $+\infty$

D. $\frac{2}{3}$

E. DNE

F. $\frac{4}{3}$

Tries 0/99

32

Given

$$f(x) = \begin{cases} 2\cos(x) & x \le 0 \\ x + 2 & 0 < x < 2 \\ 3 & x \ge 2 \end{cases}$$

Find the discontinuities of $f(x)$.

A. f(x) has jumps at $x = 0$ and at $x = 2$.

B. f(x) has a jump at $x = 2$ and a hole at $x = 0$.

C. f(x) has a hole at $x = 0$.

D. f(x) has a jump at $x = 0$.

E. f(x) has a hole at $x = 2$.

F. f(x) has a jump at $x = 2$.

Tries 0/99

33

John is deriving the derivative of $f(x)$ using the limit definition and he writes, correctly,

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}.$$

Which of the following functions could be $f(x)$?

A. $x^3 - 3x^2$

B. $3x^2 - x^3$

C. x^3

D. $3x^3$

E. $3x^2 + x^3$

F. $3x^2$

Tries 0/99

34

The derivative of $f(x) = 3e^x + \cos(x) - 2x^3$ is:

A. $f'(x) = 3xe^x - \cos(x) - 3x^2$

B. $f'(x) = 3e^x + \sin(x) - 6x^2$

C. $f'(x) = 3xe^{x-1} - \sin(x) - 6x^2$

D. $f'(x) = 3e^x + \sin(x) - 6x$

E. $f'(x) = 3e^x - \cos(x) - 3x^2$

F. $f'(x) = 3e^x - \sin(x) - 6x^2$

Tries 0/99

35

Given $f(x) = 6\sin x$. Find the equation of the tangent line to the graph of $f(x)$ at $x = \frac{\pi}{3}$.

A. $y = 3\sqrt{3}x + 3$

B. $y = 3\sqrt{3}x - \pi$

C. $y = 3x - \sqrt{3}\pi$

D. $y = 3x - \pi$

E. $y = 3\sqrt{3}x - \sqrt{3}\pi + 3$

F. $y = 3x - \pi + 3\sqrt{3}$

Tries 0/99

36

The position of a particle moving on a straight line is given by $s(t) = 3t^2 - 12t + 9$, where t is the time in minutes and s is the position in meters. At what time is the velocity zero?

A. $t = 5$ minutes

B. $t = 3$ minutes

C. $t = 1$ minutes

D. $t = 6$ minutes

E. $t = 4$ minutes

F. $t = 2$ minutes

Tries 0/99

37

The population of a herd of cattle over time (in years) is given by $p(t) = 70\left(4 + 0.1t + 0.01t^2\right)$. What is the growth rate (in cattle per year) when $t = 5$ years?

A. 124

B. 294

C. 78

D. 62

E. 332.5

F. 14

Tries 0/99

38

Find the derivative of $f(x) = (x^2 - 3x)(5x + 2)$.

A. $10x - 15$

B. $15x^2 - 11x$

C. $5x^3 - 13x^2 - 6x$

D. $10x^2 - 15x + 6$

E. $5x^2 - 15x$

F. $15x^2 - 26x - 6$

Tries 0/99

39

Given $g(x) = \dfrac{2\sqrt{x}}{x^2 - 8}$. Find $g'(4)$.

A. $-\frac{1}{4}$

B. $-\frac{3}{32}$

C. $\frac{9}{16}$

D. $\frac{7}{8}$

E. $-\frac{9}{8}$

F. $-\frac{7}{16}$

Tries 0/99

40

Given $y = \tan x(\sec x + 1)$. Find y'.

A. $y' = (\sec x)^3 \tan x$

B. $y' = 2(\sec x)^2 \tan x + \sec x \tan x$

C. $y' = (\sec x)^2 \tan x + 2 \sec x \tan x$

D. $y' = (\sec x)^3 + (\sec x)^2 + (\tan x)^2 \sec x$

E. $y' = (\sec x)^3 + \sec x \tan x + (\tan x)^2 \sec x$

F. $y' = (\tan x)^3 + \tan x \sec x + (\sec x)^2 \tan x$

Tries 0/99

1

If $h(t) = \sin(3t) + \cos(3t)$, find $h^{(3)}(t)$.

A. $-27\sin(3t) + 27\cos(3t)$

B. $-27\sin(3t) - 27\cos(3t)$

C. $27\sin(3t) + 27\cos(3t)$

D. $\sin(3t) + \cos(3t)$

E. $\sin(3t) - \cos(3t)$

F. $27\sin(3t) - 27\cos(3t)$

Tries 0/99

2

A toy rocket is launched from a platform on earth and flies straight up into the air.
Its height during the first 10 seconds after launching is given by: $s(t) = t^3 + 3t^2 + 4t + 100$, where s is measured in centimeters, and t is in seconds.
Find the velocity when the acceleration is 18 cm/s^2.

A. 2 cm/s

B. 44 cm/s

C. 13 cm/s

D. 28 cm/s

E. 16 cm/s

F. 32 cm/s

Tries 0/99

3

Find $\frac{dy}{dx}$ by implicit differentiation.

$$\ln(xy) + 2x = e^y$$

A. $\frac{dy}{dx} = \frac{-2-y}{x-e^y}$

B. $\frac{dy}{dx} = \frac{-2y}{1-ye^y}$

C. $\frac{dy}{dx} = \frac{1+2xy}{xye^y}$

D. $\frac{dy}{dx} = \frac{-2xy-y}{x-xye^y}$

E. $\frac{dy}{dx} = ye^y - \frac{y}{x} - 2y$

F. $\frac{dy}{dx} = \frac{-xy-y}{2x-xye^y}$

Tries 0/99

4

An airplane flies at an altitude of $y = 2$ miles straight towards a point directly over an observer. The speed of the plane is 500 miles per hour. Find the rate at which the observer's angle of elevation is changing when the angle is $\frac{\pi}{3}$.

A. $\frac{75}{4}$ radian per hour

B. $\frac{225}{8}$ radian per hour

C. $50\sqrt{3}$ radian per hour

D. $\frac{375}{2}$ radian per hour

E. $\frac{125\sqrt{3}}{2}$ radian per hour

F. $\frac{125}{2}$ radian per hour

Tries 0/99

5

Find the critical numbers of $y = x^2 e^x$.

A. -2 and 1

B. 0 and 2

C. -2 and 2

D. -2 and 0

E. 0 and 1

F. 1 and 2

Tries 0/99

6

Given $f(x) = \frac{2(3-x^2)}{\sqrt{3x^2+1}}$. Find $f'(1)$.

A. $-\frac{9}{4}$

B. $-\frac{1}{2}$

C. $-\frac{3}{4}$

D. $-\frac{7}{2}$

E. $-\frac{13}{6}$

F. $-\frac{3}{2}$

Tries 0/99

7

Find the largest open interval where $g(t)$ is increasing.

$$g(t) = -\frac{1}{3}t^3 + \frac{3}{2}t^2$$

A. $(-\infty, 0)$

B. $(3, \infty)$

C. $(0, \infty)$

D. $(0, 3)$

E. $(-\infty, 3)$

F. $(-\infty, 0) \cup (3, \infty)$

Tries 0/99

8

A spherical balloon is inflated with gas at a rate of 5 cubic centimeters per minute. How fast is the radius of the balloon changing at the instant when the radius is 4 centimeters?

The volume V of a sphere with a radius r is $V = \frac{4}{3}\pi r^3$.

A. $\frac{25}{4\pi}$ centimeters per minute

B. $\frac{5}{16\pi}$ centimeters per minute

C. $\frac{5}{4\pi}$ centimeters per minute

D. $\frac{5}{64\pi}$ centimeters per minute

E. $\frac{256\pi}{3}$ centimeters per minute

F. $\frac{5\pi}{64}$ centimeters per minute

Tries 0/99

9

Find $f'(2)$.

$$f(t) = \frac{2t - 1}{(2t+1)^2}$$

A. $-\frac{2}{25}$

B. $\frac{22}{125}$

C. $\frac{4}{124}$

D. $-\frac{2}{125}$

E. $-\frac{1}{10}$

F. $\frac{2}{125}$

Tries 0/99

10

If $y = (\frac{2x-1}{2x+1})^3$, then $\frac{dy}{dx} =$

A. $\frac{48}{(2x+1)^4}$

B. $3(\frac{2x-1}{2x+1})^2$

C. $\frac{24x-12}{(2x+1)^3}$

D. $\frac{12(2x-1)^2}{(2x+1)^4}$

E. $\frac{6(2x-1)^2}{(2x+1)^3}$

F. $\frac{12(2x-1)^2}{(2x+1)^3}$

Tries 0/99

11

Given $f(x) = e^{5x}\ln(7x + e)$. Find $f'(0)$.

A. $1 + \frac{1}{e}$

B. $\frac{1}{e}$

C. $\frac{35}{e}$

D. $5 + \frac{7}{e}$

E. $\frac{5}{e}$

F. $1 + \frac{7}{e}$

Tries 0/99

12

The price of a commodity is given by $p(t) = (t^2 + 2t)^2 + 100000$, where $p(t)$ is the price in dollars and t is years after 2000. At what rate is the price changing in the year of 2010?

A. \$5280/year

B. \$1680/year

C. \$2400/year

D. \$4800/year

E. \$2640/year

F. \$900/year

Tries 0/99

13

Find $g'(x)$ if $g(x) = \tan^2(3x^2 + 2)$.

A. $12x \tan(3x^2 + 2) \sec^2(3x^2 + 2)$

B. $12x \sec^2(3x^2 + 2)$

C. $2 \sec^2(6x)$

D. $2 \tan(6x)$

E. $6x \tan(3x^2 + 2) \sec^2(3x^2 + 2)$

F. $12x \tan(3x^2 + 2)$

Tries 0/99

14

Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + y^2 = 2xy + 5$.

A. 1

B. $\dfrac{x}{x-y}$

C. $\dfrac{x}{1-y}$

D. 0

E. $\dfrac{2x - 2y - 5}{2x - 2y}$

F. $\dfrac{2y - 2x + 5}{2y - 2x}$

Tries 0/99

15

All edges of a cube are expanding at a rate of 2 centimeters per second. How fast is the surface area changing when each edge is 3 centimeters?

A. 72 cm^2/sec

B. 36 cm^2/sec

C. 46 cm^2/sec

D. 12 cm^2/sec

E. 48 cm^2/sec

F. 54 cm^2/sec

Tries 0/99

16

Water flows into a right cylindrical shaped swimming pool with a circular base at a rate 4 m^3/min. The radius of the base is 3 m. How fast is the water level rising inside the swimming pool? The volume of a right cylinder with a circular base is $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

A. $\frac{4}{9\pi}$ m/min

B. $\frac{3}{8\pi}$ m/min

C. $\frac{2}{3\pi}$ m/min

D. $\frac{3}{16\pi}$ m/min

E. $\frac{2}{9\pi}$ m/min

F. $\frac{4}{3\pi}$ m/min

Tries 0/99

17

A 10-ft ladder, whose base is sitting on level ground, is leaning at an angle against a vertical wall when its base starts to slide away from the vertical wall. When the base of the ladder is 6 ft away from the bottom of the vertical wall, the base is sliding away at a rate of 4 ft/sec. At what rate is the vertical distance from the top of the ladder to the ground changing at this moment?

A. -3 ft/sec

B. 4 ft/sec

C. $\frac{1}{4}$ ft/sec

D. $-\frac{3}{4}$ ft/sec

E. $-\frac{1}{3}$ ft/sec

F. 8 ft/sec

Tries 0/99

18

Given $f(x) = \sin^3(2x)$, find $f'(\frac{\pi}{12})$.

A. $\frac{3\sqrt{3}}{4}$

B. $-\frac{3\sqrt{3}}{8}$

C. $\frac{3}{2}$

D. $\frac{9}{4}$

E. $-\frac{\sqrt{3}}{4}$

F. $\frac{1}{2}$

Tries 0/99

19

Given $f(x) = \ln \sqrt[3]{\dfrac{3+3x}{3-x}}$, find $f'(1)$.

A. $\frac{1}{3}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{2}$

E. $\frac{1}{4}$

F. $\frac{2}{3}$

Tries 0/99

20

Use implicit differentiation to find the equation of the tangent line to the graph at $(-2, 2)$.

$$x^2 + xy = 4 - y^2$$

A. $y = x + 4$

B. $y = -x$

C. $y = -x + 2$

D. $y = 2$

E. $y = -x + 4$

F. $y = x + 2$

Tries 0/99

21

Find $\frac{dy}{dx}$ by implicit differentiation.

$$e^{xy} = 8x - 8y$$

A. $\frac{dy}{dx} = \dfrac{8 - ye^{xy}}{8 + xe^{xy}}$

B. $\frac{dy}{dx} = \dfrac{8 - xe^{xy}}{8 + ye^{xy}}$

C. $\frac{dy}{dx} = \dfrac{8}{8 - xe^{xy}}$

D. $\frac{dy}{dx} = \dfrac{8}{8 + xe^{xy}}$

E. $\frac{dy}{dx} = \dfrac{8 + xe^{xy}}{8 - ye^{xy}}$

F. $\frac{dy}{dx} = \dfrac{8 + ye^{xy}}{8 - xe^{xy}}$

Tries 0/99

22

The position of an object moving on a straight line is given by $s(t) = 48 - 3t - 2t^2 - 6t^3$, where t is in minutes and $s(t)$ is in meters. What is the acceleration when $t = 3$ minutes?

A. -114 m/min^2

B. -108 m/min^2

C. -177 m/min^2

D. -76 m/min^2

E. -112 m/min^2

F. -110 m/min^2

Tries 0/99

23

The sides of an equilateral triangle are expanding at a rate of 2 cm per minute. Find the rate of change of the area when the length of each side is 3 cm. Use the fact that the area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}x^2$, where x is the length of a side.

A. $\sqrt{3}$ cm^2/min

B. $\frac{9\sqrt{3}}{2}$ cm^2/min

C. $\frac{3\sqrt{3}}{2}$ cm^2/min

D. $\frac{9\sqrt{3}}{4}$ cm^2/min

E. $3\sqrt{3}$ cm^2/min

F. $\frac{3\sqrt{3}}{4}$ cm^2/min

Tries 0/99

24

Given $f(x) = \dfrac{x^3}{3} + x + \sqrt{x^3}$. Find $f''(4)$.

A. $\frac{49}{8}$

B. $\frac{26}{3}$

C. $\frac{49}{6}$

D. $\frac{35}{4}$

E. $\frac{67}{8}$

F. $\frac{19}{2}$

Tries 0/99

25 Given $y = x \ln x$, find $y''(e)$.

A. $e + 1$

B. e

C. 2

D. 0

E. $\frac{1}{e}$

F. $\frac{1}{e} + 1$

Tries 0/99

---

26 Find the relative extrema of $g(x) = \dfrac{x}{x^2+9}$.

A. Relative maximum at $x = 3$; Relative minimum at $x = -\sqrt{3}$

B. Relative maximum at $x = -3$; Relative minimum at $x = 3$

C. Relative maximum at $x = -\sqrt{3}$; Relative minimum at $x = \sqrt{3}$

D. Relative maximum at $x = -3$; Relative minimum at $x = \sqrt{3}$

E. Relative maximum at $x = 3$; Relative minimum at $x = -3$

F. Relative maximum at $x = \sqrt{3}$; Relative minimum at $x = -\sqrt{3}$

Tries 0/99

---

27 Find the largest open interval(s) on which

$$f(x) = (3x-4)(x+2)$$

is increasing.

A. $(-\infty, 3)$

B. $(-\infty, -2)$ and $(\frac{4}{3}, \infty)$

C. $(-\infty, 3)$ and $(3, \infty)$

D. $(-\frac{1}{3}, \infty)$

E. $(-2, \frac{4}{3})$

Tries 0/99

---

28 If x and y are both functions of t and

$$x + y^2 = 4e^x,$$

find $\frac{dy}{dt}$ when $\frac{dx}{dt} = 2$, $x = 0$, and $y = -2$.

A. -1

B. $-\frac{1}{2}$

C. 1

D. 0

E. $-\frac{3}{2}$

F. 3

Tries 0/99

---

29 Find $g'(1)$.

$$g(x) = \left(\frac{x^2}{x+2}\right)^3$$

A. $\frac{5}{3}$

B. $\frac{1}{3}$

C. $\frac{5}{9}$

D. $\frac{25}{3}$

E. $\frac{5}{27}$

F. $\frac{25}{27}$

Tries 0/99

---

30 The position of a particle on a straight line t seconds after it starts moving is $s(t) = 2t^3 - 3t^2 + 6t + 1$ feet. Find the acceleration of the particle when its velocity is 78 ft/sec.

A. 84 ft/sec^2

B. 105 ft/sec^2

C. 30 ft/sec^2

D. 258 ft/sec^2

E. 42 ft/sec^2

F. 46 ft/sec^2

Tries 0/99

---

31 Find the relative maximum of $f(x) = 2x^3 - 6x$.

A. $(1,4)$

B. $(0,0)$

C. $(-1,0)$

D. $(1,0)$

E. $(-1,4)$

F. $(1,-4)$

Tries 0/99

---

32 Given that

$$y^2x - x^2 = y\ln(x) + 3,$$

use implicit differentiation to find $\frac{dy}{dx}$ at $(1,-2)$.

A. 5

B. $-\frac{2}{5}$

C. 1

D. -1

E. -2

F. 2

Tries 0/99

---

33 Find $f'(4)$ if $f(x) = (x^2 + 3)\sqrt{x^2 - 7}$.

A. $\frac{163}{6}$

B. $\frac{110}{3}$

C. $\frac{148}{3}$

D. $\frac{142}{3}$

E. $\frac{4}{3}$

F. $\frac{32}{3}$

Tries 0/99

---

34 Find the x value at which the function $f(x) = x^3 - 9x^2 - 120x + 3$ has a relative minimum.

A. $x = 4$

B. $x = -10$

C. $x = 10$

D. $x = -3$

E. $x = -4$

F. $x = 3$

Tries 0/99

35 Which of the following is a critical number of

$$y = \frac{1}{3}\sin(3x) - \frac{x}{2}?$$

A. 0

B. $\frac{\pi}{3}$

C. $\frac{\pi}{9}$

D. $\frac{\pi}{12}$

E. $\frac{\pi}{18}$

F. $\frac{\pi}{6}$

Tries 0/99

---

36 An observer stands 400 feet away from the point where a hot air balloon is launched. If the balloon ascends vertically at a (constant) rate of 30 feet per second, how fast is the balloon moving away from the observer 10 seconds after it is launched?

A. 40 ft/sec

B. 50 ft/sec

C. 18 ft/sec

D. 30 ft/sec

E. 24 ft/sec

F. 37.5 ft/sec

Tries 0/99

---

37 A spherical snowball grows in size as it rolls down a snow covered hill. If the volume of the snowball is increasing at a rate of 1 cubic inch per second, at what rate, in inches per second, is the radius of the snowball increasing when the radius is 3 inches? (Recall that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.)

A. 1

B. 9

C. $\frac{1}{36\pi}$

D. $\frac{1}{9\pi}$

E. $\frac{1}{4\pi}$

F. 0

Tries 0/99

---

38 Find the second derivative of $f(x) = \ln(4x) + e^{x^2}$.

A. $f''(x) = -\frac{1}{x^2} + 4x^2e^{x^2}$

B. $f''(x) = -\frac{1}{4x^2} + e^{x^2}(4x^2 + 2)$

C. $f''(x) = -\frac{1}{x^2} + e^{x^2}(4x^2 + 2)$

D. $f''(x) = -\frac{1}{x^2} + 2e^{x^2}$

E. $f''(x) = -\frac{1}{4x^2} + 4x^2e^{x^2}$

F. $f''(x) = -\frac{1}{4x^2} + 2e^{x^2}$

Tries 0/99

---

1 Find the open interval where the function $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x - 7$ is concave down.

A. $(5, \infty)$

B. $(1, \infty)$

C. $(3, \infty)$

D. $(1, 5)$

E. $(-\infty, 3)$

F. $(-\infty, 1)$

Tries 0/99

---

2 Find the x-coordinate of the inflection point of $y = e^{2x} - 8x^2$.

A. $x = \ln 4$

B. $x = 2\ln 4$

C. $x = e^2$

D. $x = e$

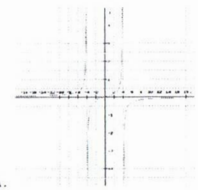
E. $x = \frac{1}{2}\ln 4$

F. $x = 0$

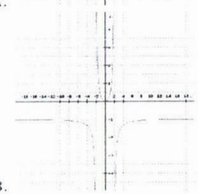
Tries 0/99

---

3 Given the function $f(x) = \frac{4x}{x^2 - 4}$ with its first and second derivatives $f'(x) = \frac{-4(x^2+4)}{(x^2-4)^2}$ and $f''(x) = \frac{8x(x^2+12)}{(x^2-4)^3}$.
Find the graph of $f(x)$.



A.



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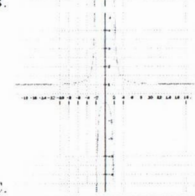
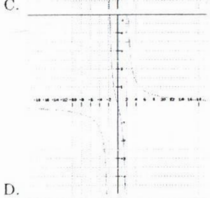


C.



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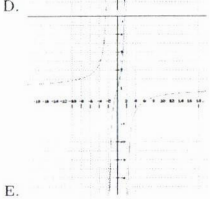


E.

Tries 0/99

4

Let $f(x)$ be a polynomial whose derivative is always increasing. Choose the correct statement(s).

[I.] $f(x)$ has an inflection point.

[II.]$f(x)$ has a relative maximum.

[III.] $f(x)$ is always concave up.

A. Only I is correct.

B. Only II is correct.

C. Only III is correct.

D. I and II are correct.

E. II and III are correct.

F. I and III are correct.

Tries 0/99

5

Which of the following limits equals to $-\infty$?

A. $\lim\limits_{x\to\infty} \dfrac{x^3-1}{x^2+1}$

B. $\lim\limits_{x\to-\infty} \dfrac{2x^2}{x^2+2}$

C. $\lim\limits_{x\to\infty} \left(\dfrac{2}{x} - \dfrac{x}{6}\right)$

D. $\lim\limits_{x\to\infty} \dfrac{-x^3+2x^2-3x}{3x^4-5x^3+1}$

E. $\lim\limits_{x\to-\infty} \dfrac{1-x^2}{x-1}$

F. $\lim\limits_{x\to\infty} \dfrac{x-1}{x^3-1}$

Tries 0/99

6

Consider the function $f(x) = \dfrac{x^2+3x+2}{x^2-1}$. Which of the statements are true?

[I.] f has a vertical asymptote at $x = 1$.

[II.] f has a horizontal asymptote at $y = 0$.

[III.] f has a vertical asymptote at $x = -1$.

[IV.] f has a horizontal asymptote at $y = 1$.

A. II and IV

B. I and II

C. I and IV

D. II and III

E. I and III

F. III and IV

Tries 0/99

7

An open-top box with a square base is made using 48 ft^2 of material. Find the maximum possible volume of this box.

A. 96 ft^3

B. 16 ft^3

C. 32 ft^3

D. 64 ft^3

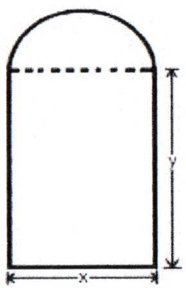
E. 48 ft^3

F. 80 ft^3

Tries 0/99

8

Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. See figure below. Find x which maximizes the area of this window if the total perimeter is 10 feet.



A. $\frac{20}{\pi}$ ft

B. $\frac{10}{(\pi+2)^2}$ ft

C. $\frac{20}{\pi+4}$ ft

D. $\frac{10}{\pi}$ ft

E. $\frac{20}{(\pi+4)^2}$ ft

F. $\frac{20}{\pi+2}$ ft

Tries 0/99

9

Find the x-coordinate of the point on the graph of $y = \sqrt{x} + 2$ that is the closest to the point $(3, 2)$.

A. $\frac{9}{2}$

B. 0

C. $\frac{5}{2}$

D. $\frac{3}{2}$

E. $\frac{1}{2}$

F. $\frac{7}{2}$

Tries 0/99

10

SheSellsSeaShells is an ocean boutique offering shells and handmade shell crafts on Sanibel Island in Florida. Find the price SheSellsSeaShells should charge to maximize revenue if $p(x) = 160 - 2x$, where $p(x)$ is the price in dollars at which x shells will be sold per day.

A. \$120

B. \$20

C. \$80

D. \$60

E. \$40

F. \$100

Tries 0/99

11

Find the open interval where $f(x) = \frac{1}{2}x^4 + 2x^3$ is concave downward.

A. $(-3, \infty)$

B. $(-\infty, -3)$

C. $(-2, 0)$

D. $(-3, 0)$

E. $(-2, \infty)$

F. $(-3, -2)$

Tries 0/99

12

Let $f(x) = -x^3 + 12x$. The y values of the absolute minimum and the absolute maximum of $f(x)$ over the closed interval $[-3, 5]$ are respectively:

A. -65 and -16

B. -65 and -9

C. -65 and 16

D. -16 and 16

E. -16 and -9

F. -9 and 16

Tries 0/99

13

$\lim_{x\to\infty} f(x) = \infty$ is true for which of the following functions?

A. $f(x) = \dfrac{2x^2}{x^2 + x}$

B. $f(x) = \dfrac{2x^3 + x^2 - 2}{-3x^3 + 7}$

C. $f(x) = \dfrac{x - x^2}{-x + 5}$

D. $f(x) = \dfrac{x + 9}{x^2 + x + 6}$

E. $f(x) = \dfrac{2}{x} + 3$

F. $f(x) = \dfrac{x^3 + x^2 - 2}{-x + 5}$

Tries 0/99

14

Choose the correct statement regarding the asymptotes of $f(x)$.

$f(x) = \dfrac{x^2 - 2x + 6}{x + 1}$

A. Horizontal Asymptote: $y = 0$; Vertical Asymptote: $x = -1$; Slant Asymptote: None

B. Horizontal Asymptote: None; Vertical Asymptote: $x = -1$; Slant Asymptote: None

C. Horizontal Asymptote: $y = -1$; Vertical Asymptote: $x = 1$; Slant Asymptote: None

D. Horizontal Asymptote: $y = 0$; Vertical Asymptote: $x = 1$; Slant Asymptote: $y = x - 3$

E. Horizontal Asymptote: $y = -1$; Vertical Asymptote: $x = 1$; Slant Asymptote: $y = x - 3$

F. Horizontal Asymptote: None; Vertical Asymptote: $x = -1$; Slant Asymptote: $y = x - 3$

Tries 0/99

15

Find the point on the graph of $y = 5x + 2$ that is the closest to the point $(0,4)$.

A. $(\frac{5}{13}, \frac{51}{26})$

B. $(\frac{10}{13}, \frac{102}{13})$

C. $(\frac{5}{26}, \frac{51}{13})$

D. $(\frac{10}{13}, \frac{51}{13})$

E. $(\frac{5}{13}, \frac{102}{13})$

F. $(\frac{5}{13}, \frac{51}{13})$

Tries 0/99

16

$f(x)$ is a polynomial and

$$f'(2) = 0, \qquad f'(5) = 0$$

$$f''(3.5) = 0,\ f''(x) < 0 \text{ on } (-\infty, 3.5) \text{ and } f''(x) > 0 \text{ on } (3.5, \infty)$$

Which of the following statements are true?

I. $(2, f(2))$ is an inflection point of $f(x)$.

II. $(3.5, f(3.5))$ is an inflection point of $f(x)$.

III. $f(x)$ has a relative maximum at $x = 2$.

IV. $f(x)$ has a relative minimum at $x = 5$.

A. Only I and IV are true.

B. Only II and III are true.

C. Only I and III are true.

D. Only II and IV are true.

E. Only I, II and IV are true.

F. Only II, III and IV are true.

Tries 0/99

---

17 $\int \frac{\sin x - 2\cos x}{4} dx =$

A. $\frac{2\sin x - \cos x}{4} + C$

B. $\frac{-\sin x + 2\cos x}{4} + C$

C. $\frac{2\sin x + \cos x}{4} + C$

D. $\frac{\sin x + 2\cos x}{4} + C$

E. $\frac{-2\sin x + 2\cos x}{4} + C$

F. $\frac{-2\sin x - \cos x}{4} + C$

Tries 0/99

18

An evergreen nursery usually sells a certain shrub after 5 years of growth and shaping. The growth rate during those 5 years is approximated by

$$\frac{dh}{dt} = 1.4t + 8,$$

where t is the time in years and h is the height in centimeters. The seedlings are 14 centimeters tall when planted. How tall are the shrubs when they are sold?

A. 36 cm

B. 57.5 cm

C. 29 cm

D. 42 cm

E. 92.5 cm

F. 71.5 cm

Tries 0/99

19

A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2800 - 200p$ units. Each unit costs \$10 to make. What is the maximum profit that the company can make?

A. 980 dollars

B. 1000 dollars

C. 600 dollars

D. 880 dollars

E. 1200 dollars

F. 800 dollars

Tries 0/99

20

Find the absolute extrema of $f(x) = 2x^3 + 3x^2 - 36x$ on the closed interval $[0, 4]$.

A. absolute minimum: $(0, 0)$; absolute maximum: $(4, 32)$

B. absolute minimum: $(-3, 0)$; absolute maximum: $(2, 0)$

C. absolute minimum: $(2, -44)$; absolute maximum: $(0, 0)$

D. absolute minimum: $(-3, 0)$; absolute maximum: $(0, 0)$

E. absolute minimum: $(2, -44)$; absolute maximum: $(-3, 81)$

F. absolute minimum: $(2, -44)$; absolute maximum: $(4, 32)$

Tries 0/99

21

A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 160 m of wire at your disposal, what is the **largest area** you can enclose?

A. $6400\,\text{m}^2$

B. $4800\,\text{m}^2$

C. $1600\,\text{m}^2$

D. $3600\,\text{m}^2$

E. $4000\,\text{m}^2$

F. $3200\,\text{m}^2$

Tries 0/99

22

A rectangular box with square base and top is to be constructed using sturdy metal. The volume is to be $16\ \text{m}^3$. The material used for the sides costs \$4 per square meter, and the material used for the top and bottom costs \$1 per square meter. What is the least amount of money that can be spent to construct the box?

A. \$55

B. \$136

C. \$30

D. \$120

E. \$160

F. \$96

Tries 0/99

23

Choose the correct statement(s) about the function $f(x) = 2x^3 - 9x^2$.

[I.] $f(x)$ has a relative maximum at $x = 0$.

[II.] $f(x)$ has a relative minimum at $x = 3$.

[III.] $f(x)$ is concave downward on $(-\infty, \frac{3}{2})$.

A. I only

B. II only

C. I & III only

D. II & III only

E. All of the statements are true.

F. I & II only

Tries 0/99

24 Find the point of inflection of $h(x) = xe^{-2x}$.

A. $(-\frac{1}{2}, -\frac{e}{2})$

B. $(0,0)$

C. $(-1, -e^2)$

D. $(\frac{1}{2}, \frac{e}{2})$

E. $(\frac{1}{2}, \frac{1}{2e})$

F. $(1, \frac{1}{e^2})$

Tries 0/99

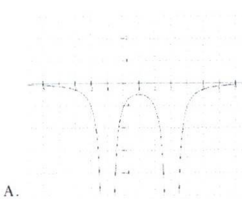
25 A function $f(x)$ satisfies the following conditions:

$f'(x) > 0$ on $(-\infty, -1)$

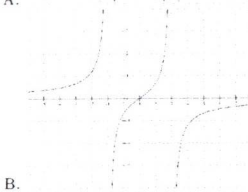
$f''(x) < 0$ on $(-1, 0)$

$f'(x) = 0$ at $x = 1$

Which of the following graphs is a possible graph of $f(x)$?



A.



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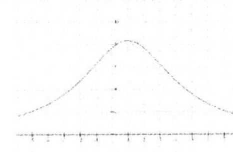

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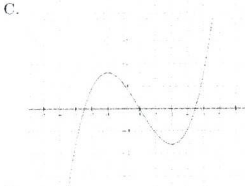

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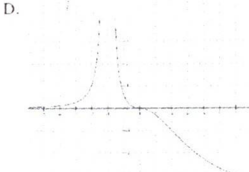


E.

Tries 0/99

26

Which of the following functions satisfies $\lim_{x\to-\infty} f(x) = -\infty$?

A. $f(x) = \dfrac{2x-5}{x^2+25}$

B. $f(x) = \dfrac{x^2-3x}{x-5x^2}$

C. $f(x) = \dfrac{x^2-10}{2x^3+x}$

D. $f(x) = \dfrac{x^3-27x}{7-4x^2}$

E. $f(x) = \dfrac{x^4-16}{6x+2}$

F. $f(x) = \dfrac{6}{x} + 3$

Tries 0/99

27

Which of the following describes all the asymptotes of the function $f(x) = \dfrac{-2x^2-5x+7}{x+3}$?

A. $x = 3, y = 0$

B. $x = -2, y = 0$

C. $x = 3, y = -2$

D. $x = -3, y = -2x + 1$

E. $x = -3, y = -2$

F. $x = -2, y = 2x + 1$

Tries 0/99

28

A box with a square base and open top is to be made from 300 square inches of material. What is the volume of the largest box that can be made.

A. 600 cubic inches

B. 560 cubic inches

C. 400 cubic inches

D. 500 cubic inches

E. 472 cubic inches

F. 532 cubic inches

Tries 0/99

29

A poster is to have an area of 200 square inches with 1 inch margins on the left and right sides, and 2 inch margins on the top and bottom. Varying the dimensions of the poster changes the area of the region inside the margins. What is the maximum area inside the margins?

A. 168 square inches

B. 148 square inches

C. 138 square inches

D. 128 square inches

E. 88 square inches

F. 108 square inches

Tries 0/99

30

Find the x-coordinate of the point on the line of $y = 2x + 1$ that is closest to the point (5,1).

A. 4

B. 3

C. 5

D. 1

E. 0

F. 2

Tries 0/99

31 $\int \frac{3x^2 - 4}{2\sqrt{x}} dx =$

A. $\frac{3}{7}\sqrt{x^7} - \frac{4}{3}\sqrt{x^3} + C$

B. $\frac{9}{4}\sqrt{x} + \frac{1}{\sqrt{x^3}} + C$

C. $\frac{3}{5}\sqrt{x^3} - \frac{4}{3}\sqrt{x} + C$

D. $\frac{3}{5}\sqrt{x^5} - 4\sqrt{x} + C$

E. $\frac{3}{4}\sqrt{x^3} - \frac{3}{\sqrt{x}} + C$

F. $\frac{9}{4}\sqrt{x^5} + \sqrt{x} + C$

Tries 0/99

Find the particular solution that satisfies the following differential equation and the initial conditions.

32 $$f''(x) = 3\cos(x), \quad f'(0) = 4, \quad f(0) = 7$$

A. $f(x) = 3\cos(x) + x + 7$

B. $f(x) = 3\cos(x) + 4x + 10$

C. $f(x) = -3\cos(x) + x + 7$

D. $f(x) = -3\cos(x) + 4x + 10$

E. $f(x) = -3\cos(x) + 4x + 7$

F. $f(x) = 3\cos(x) + 4x + 7$

Tries 0/99

---

33 Find the inflection point of $y = x^3 + 3x^2$.

A. $(-1, 0)$

B. $(0, 0)$

C. $(0, 2)$

D. $(-1, 2)$

E. $(-2, 4)$

F. $(-2, 0)$

Tries 0/99

---

34 A particle is moving on a straight line with an initial velocity of 10 ft/sec and an acceleration of

$$a(t) = \sqrt{t} + 2,$$

where t is time in seconds and $a(t)$ is in ft/sec^2. What is its velocity after 9 seconds?

A. 24 ft/sec

B. 135 ft/sec

C. 72 ft/sec

D. 46 ft/sec

E. 90 ft/sec

F. 140 ft/sec

Tries 0/99

35

Which of the following limits equals $-\infty$?

A. $\lim_{x\to-\infty} \frac{x^3+5x^2-7x}{-2x^2-5x+6}$

B. $\lim_{x\to-\infty} \frac{-x^3+8}{x^2+x-2}$

C. $\lim_{x\to-\infty} \frac{-2x^2+7x}{x^3+5x^2+1}$

D. $\lim_{x\to-\infty} \frac{x^4+8x}{x^3+1}$

E. $\lim_{x\to-\infty} \frac{x^2-4}{x^2+1}$

F. $\lim_{x\to-\infty} \frac{x^2+4x-5}{x^4-1}$

Tries 0/99

36

Choose the correct statement regarding the y values of the absolute maximum and the absolute minimum of $f(x) = x^3 - 3x + 10$ on the interval of $[0, 3]$.

A. The y values of the absolute maximum and the absolute minimum are 28 and 10 respectively.

B. The y values of the absolute maximum and the absolute minimum are 28 and 8 respectively.

C. The y values of the absolute maximum and the absolute minimum are 12 and 12 respectively.

D. The y values of the absolute maximum and the absolute minimum are 12 and 8 respectively.

E. The y values of the absolute maximum and the absolute minimum are 28 and 12 respectively.

F. The y values of the absolute maximum and the absolute minimum are 12 and 10 respectively.

Tries 0/99

37

Which of the following statements is true regarding the function $f(x) = \frac{2x^2-3x+4}{x-1}$?

A. $f(x)$ has a slant asymptote which is $y = x - 1$.

B. $f(x)$ has a slant asymptote which is $y = 2x - 1$.

C. $f(x)$ has a slant asymptote which is $y = x + 1$.

D. $f(x)$ has a horizontal asymptote which is $y = 2$.

E. $f(x)$ has a horizontal asymptote which is $y = 3$.

F. $f(x)$ has a horizontal asymptote which is $y = \frac{1}{2}$.

Tries 0/99

38

Find the x values at which the inflection points of $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{15}{2}x^2 + 7$ occur.

A. $x = 0$ and $x = 3$

B. $x = -3$ and $x = \frac{5}{3}$

C. $x = -3$ and $x = 0$

D. $x = -5$ and $x = -3$

E. $x = -5$ and $x = 3$

F. $x = 0$ and $x = \frac{5}{3}$

Tries 0/99

39

Find the largest open interval(s) where $f(x) = 4x^5 - 5x^4$ is concave upward.

A. $(-\infty, 0)$ and $(1, \infty)$

B. $(\frac{3}{4}, \infty)$

C. $(-\infty, \frac{3}{4})$ and $(1, \infty)$

D. $(-\infty, 0)$ and $(\frac{3}{4}, \infty)$

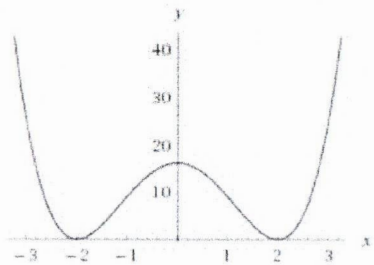
E. $(0, \infty)$

F. $(-\infty, \frac{3}{4})$

Tries 0/99

40

The following graph is of $f'(x)$. Choose the correct statement(s) about $f(x)$.



I. On $(-2, 2)$, $f(x)$ is increasing.

II. On $(-\infty, -2)$, $f(x)$ is concave up.

III. $f(x)$ has a relative maximum at $x = 0$.

A. I, II only

B. I only

C. I, III only

D. II, III only

E. II only

F. III only

Tries 0/99

41

Evaluate the indefinite integral $\int \sec x(\tan x - \sec x)\, dx$.

A. $\sec x + \tan x + C$

B. $\sec x - \tan x + C$

C. $-\sec x + \tan x + C$

D. $\sec x + \cot x + C$

E. $\csc x + \tan x + C$

F. $-\sec x - \tan x + C$

Tries 0/99

42

Solve the following initial value problem

$$y' = \frac{1}{x^2} + x, \quad y(2) = 1$$

A. $y = -\frac{2}{x^3} + 4$

B. $y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$

C. $y = -\frac{1}{x} + \frac{x^2}{2} + \frac{1}{2}$

D. $y = -\frac{2}{x^3} + \frac{x^2}{2} + \frac{7}{2}$

E. $y = -\frac{2}{x^3} + \frac{x^2}{2} - \frac{3}{4}$

F. $y = -\frac{1}{x} + \frac{x^2}{2} + \frac{5}{2}$

Tries 0/99

43 Solve the initial value problem $y'' = 2 + 4e^x$ with $y'(0) = 1$ and $y(0) = 4$.

A. $y = 8e^{2x}$

B. $y = x^2 + 4e^x - 3x$

C. $y = 8e^{2x} + 2x$

D. $y = x^2 + 4e^x - 4x + 3$

E. $y = x^2 + 4e^x - 4$

F. $y = x^2 + 4e^x - 3x - 4$

Tries 0/99

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44 A family wants to fence a rectangular play area alongside the wall of their house. The wall of their house bounds one side of the play area. If they want the play area to be exactly 2500 ft^2, what is the least amount of fencing needed? Round your answer to the nearest tenth place.

A. 70.7 ft

B. 141.4 ft

C. 93.3 ft

D. 212.1 ft

E. 106.1 ft

F. 186.6 ft

Tries 0/99

A box with a square base and an open top must have a volume of 4000 cm^3. If the cost of the material used is \$1 per cm^2, the smallest possible cost of the box is

45

A. \$500

B. \$1200

C. \$1500

D. \$1000

E. \$600

F. \$2000

Tries 0/99

$\boxed{9\ pt}$ Which of the following is NOT equal to $-\infty$?

1.A○ $\lim_{x\to 8^-} \frac{4}{(x-8)^5}$

B○ $\lim_{x\to 3^-} \frac{1}{x-3}$

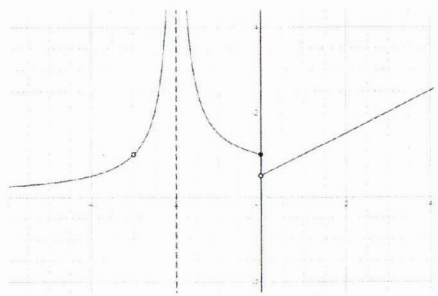
C○ $\lim_{x\to 2^+} \frac{-1}{\sqrt{x-2}}$

D○ $\lim_{x\to 8^+} \frac{4}{(x-8)^5}$

E○ $\lim_{x\to 4^-} \frac{-1}{(4-x)^2}$

F○ $\lim_{x\to 1^+} \frac{x}{1-x}$

$\boxed{9\ pt}$ Choose the correct statement(s) regarding $f(x)$ shown in the graph below.



(a) $f(0) = \frac{1}{2}$

(b) $\lim_{x\to -3} f(x)$ does not exist.

(c) f is discontinuous at $x = -3$, $x = -2$ and $x = 0$.

(d) $\lim_{x\to 0^-} f(x) = 1$

2.A○ a and c only

B○ All statements are true.

C○ d only

D○ b only

E○ c and d only

F○ b and d only

$\boxed{8\ pt}$ Compute the following limit:

$$\lim_{x\to 2} \frac{x^2-2x}{x^2-5x+6}$$

3.A○ 6

B○ -2

C○ -6

D○ 0

E○ -3

F○ 2

$\boxed{8\ pt}$ Given

$$f(x) = \begin{cases} x^2-3, & x<1 \\ -\frac{1}{4}x+1, & x\geq 1 \end{cases}$$

Find $\lim_{x\to 1^+} f(x)$.

4.A○ $\frac{3}{4}$

B○ Does not exist.

C○ 0

D○ -2

E○ 1

F○ 1.75

$\boxed{8\ pt}$ Given $f(x) = \frac{x^2-36}{x^2-7x+6}$. Which of the following are true?

I. There is a hole at $x = 6$.

II. There is a hole at $x = 1$.

III. There is a hole at $x = -6$.

IV. There is a vertical asymptote at $x = 6$.

V. There is a vertical asymptote at $x = 1$.

VI. There is a vertical asymptote at $x = -6$.

5.A○ II and III

B○ II and IV

C○ IV and V

D○ I and VI

E○ I and II

F○ I and V

$\boxed{8\ pt}$ Find the derivative of $y = \frac{x}{10} + 9x^4 - \frac{1}{4}\sqrt{x}$.

6.A○ $\frac{x}{10} + 9x^3 - \frac{\sqrt{x^3}}{8}$

B○ $\frac{1}{10} + 36x^3 - \frac{\sqrt{x^3}}{8}$

C○ $\frac{x}{10} + 9x^3 - \frac{1}{8\sqrt{x}}$

D○ $\frac{1}{10} + 36x^3 - \frac{1}{8\sqrt{x}}$

E○ $\frac{1}{10} + 9x^3 - \frac{1}{4\sqrt{x}}$

F○ $\frac{x}{10} + 36x^3 - \frac{1}{4\sqrt{x}}$

$\boxed{8\ pt}$ The derivative of a function is found by computing $f'(x) = \lim_{h\to 0} \frac{\frac{4}{7(x+h)^3} - \frac{4}{7x^3}}{h}$. Which of the following could be $f(x)$?

7.A◯ $f(x) = \frac{4}{7x^3} + \frac{4}{7x}$

B◯ $f(x) = \frac{4}{7x^3}$

C◯ $f(x) = \frac{4x^3}{7}$

D◯ $f(x) = \frac{4}{7x}$

E◯ $f(x) = \frac{4}{7x^3} - \frac{4}{7x}$

F◯ $f(x) = \frac{1}{x^3}$

$\boxed{8\ pt}$ $s(t) = -5t^2 + 4t + 7$ describes the position, in meters, of a moving particle on a straight line in terms of time t, in hours. At what time does the particle stop?

8.A◯ $t = \frac{2}{5}$ hours

B◯ $t = \frac{4}{5}$ hours

C◯ $t = \frac{1}{10}$ hours

D◯ $t = 1$ hours

E◯ $t = 2$ hours

F◯ $t = 4$ hours

$\boxed{8\ pt}$ The population of a city since 1990 can be modeled as

$$P(t) = t^2 + 1234t + 26000,$$

where $t = 0$ corresponds to the year 1990. In which year is the population increasing at the rate of 1306 people per year?

9.A◯ 2017

B◯ 1995

C◯ 2036

D◯ 3260

E◯ 2043

F◯ 2026

$\boxed{8\ pt}$ Find the equation of the tangent line to $f(x) = 6x + \cos x$ at $x = \pi$.

10.A◯ $y = 6x + 12\pi - 1$

B◯ $y = 6x - 1$

C◯ $y = 6x - 6\pi$

D◯ $y = 6x + 6\pi - 1$

E◯ $y = 6x + 1$

F◯ $y = 6x$

$\boxed{9\ pt}$ Given $g(t) = \dfrac{2\sqrt{t} - 5}{t - 1}$, find $g'(4)$.

11.A◯ $-\frac{1}{3}$

B◯ $\frac{45}{2}$

C◯ $\frac{5}{\sqrt{2}-2}$

D◯ $\frac{5}{18}$

E◯ $\frac{5}{6}$

F◯ 0

$\boxed{9\ pt}$ Find the derivative of $f(x) = (e^x + 5)\sec x$.

12.A◯ $e^x \sec^2 x + e^x \sec x + 5\sec^2 x$

B◯ $e^x \sec x \tan x + e^x \sec x + 5 \sec x \tan x$

C◯ $-e^x \sec x \tan x + e^x \sec x - 5 \sec x \tan x$

D◯ $(e^x + 5)\sec^2 x$

E◯ $e^x \sec x \tan x$

F◯ $e^x \sec^2 x$

$\boxed{9\ pt}$ Given $y = \left(\frac{1}{8}x^2 + x - \frac{1}{x}\right)^4$, find $y'(2)$.

1.A○ 40

B○ 32

C○ 64

D○ 28

E○ 10

F○ 56

$\boxed{9\ pt}$ Find the derivative of $f(x) = \tan(e^{3x})$.

2.A○ $\sec^2(e^{3x})$

B○ $\tan(e^{3x})$

C○ $3e^{3x}\sec(e^{3x})\tan(e^{3x})$

D○ $3\tan(e^{3x})$

E○ $\sec(e^{3x})\tan(e^{3x})$

F○ $3e^{3x}\sec^2(e^{3x})$

$\boxed{8\ pt}$ Find the derivative of $f(x) = \ln(\cos x)$.

3.A○ $\tan x$

B○ $\sec x$

C○ $-\sec x$

D○ $\cot x$

E○ $-\tan x$

F○ $-\cot x$

$\boxed{8\ pt}$ A particle travels along a straight line. Its position in feet is given by $s(t) = t^3 - 3t^2 + 12$, where $t \geq 0$ is in seconds. Find the acceleration of the particle when the velocity is 9 ft/sec.

4.A○ 48 ft/sec^2

B○ 3 ft/sec^2

C○ -18 ft/sec^2

D○ -27 ft/sec^2

E○ 489 ft/sec^2

F○ 12 ft/sec^2

$\boxed{8\ pt}$ On which interval(s) is $y = \frac{1}{4}x^4 - x^3 + x^2$ decreasing?

5.A○ $(-\infty, \infty)$

B○ $(0, 1)$

C○ $(0, 1)$ and $(2, \infty)$

D○ $(-\infty, 2)$

E○ $(0, 2)$

F○ $(-\infty, 0)$ and $(1, 2)$

$\boxed{8\ pt}$ At what x value does $f(x) = \frac{x^3}{3} - x + 100000$ achieve its relative minimum?

6.A○ -100

B○ 100000

C○ -1

D○ 1

E○ -10

F○ 0

$\boxed{8\ pt}$ Which of the following is a critical number of $f(x) = \frac{x^3}{3} + x^2 - 8x + 5$?

7.A○ -8

B○ -2

C○ 3

D○ 5

E○ 0

F○ 2

$\boxed{8\ pt}$ Given $y^2 + 2xy^2 - 3x + 5 = 0$. Use implicit differentiation to find $\frac{dy}{dx}$.

8.A○ $\dfrac{-2y + 3x - 5}{4xy}$

B○ $\dfrac{-2y^2 - 2y + 3}{4xy}$

C○ $\dfrac{1}{2y}$

D○ $\dfrac{-2y^2 + 3}{2y + 4xy}$

E○ $\dfrac{2y^2 - 2}{2y + 4xy}$

F○ $\dfrac{-4y + 3}{2y}$

$\boxed{8\ pt}$ Use implicit differentiation to find $\frac{dy}{dx}$ at the point $(0, -1)$ when $e^{xy} = 3x^2 - y^3$.

9.A○ $\frac{3}{2}$

B○ 3

C○ $\frac{3}{5}$

D○ $\frac{5}{3}$

E○ $\frac{1}{3}$

F○ $\frac{2}{3}$

$\boxed{8\ pt}$ The radius of a circle is increasing at a rate of 3 mm/sec. Find the rate at which the perimeter/circumference of the circle changes.

10.A○ 3 mm/sec

B○ 6 mm/sec

C○ 3π mm/sec

D○ 6π mm/sec

E○ $\frac{2\pi}{3}$ mm/sec

F○ 2π mm/sec

$\boxed{9\ pt}$ Water flows into a right cylindrical shaped swimming pool with a circular base at a rate of 4 m^3/min. The radius of the base is 3 m. How fast is the water level rising inside the swimming pool? The volume of a right cylinder with a circular base is $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

11.A○ $\frac{3}{8\pi}$ m/min

B○ $\frac{3}{16\pi}$ m/min

C○ $\frac{2}{3\pi}$ m/min

D○ $\frac{4}{3\pi}$ m/min

E○ $\frac{2}{9\pi}$ m/min

F○ $\frac{4}{9\pi}$ m/min

$\boxed{9\ pt}$ A 20 ft long ladder leans against a building. Suppose that the bottom of the ladder slides away horizontally at a rate of 4 ft/sec. How fast is the ladder sliding down the building when the top of the ladder is 12 ft from the ground?

12.A◯ $\frac{1216}{3}$ ft/sec

B◯ $\frac{16}{3}$ ft/sec

C◯ $\frac{473}{3}$ ft/sec

D◯ $\frac{26}{3}$ ft/sec

E◯ $\frac{50}{3}$ ft/sec

F◯ $\frac{34}{3}$ ft/sec

$\boxed{9\ pt}$ Which of the following functions satisfies $\lim_{x\to-\infty} f(x) = \infty$?

1.A◯ $f(x) = \dfrac{x-5}{x^2+25}$

B◯ $f(x) = \dfrac{x^3-x}{1+x^2}$

C◯ $f(x) = \dfrac{x^4-16}{6x+2}$

D◯ $f(x) = \dfrac{x^2+7}{x^3+3}$

E◯ $f(x) = \dfrac{10-x^2}{2x^3+x}$

F◯ $f(x) = \dfrac{x^5+2x}{x+3}$

$\boxed{9\ pt}$ Find the open interval where $y = 3x^3 - x^4$ is concave up.

2.A◯ $(0, 3)$

B◯ $(-\infty, 0)$ and $(2.25, \infty)$

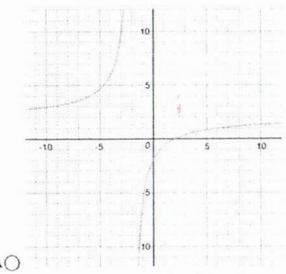
C◯ $(0, 2.25)$

D◯ $(-\infty, 0)$ and $(1.5, \infty)$

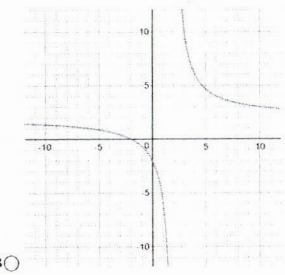
E◯ $(0, 1.5)$

F◯ $(-\infty, 0)$ and $(3, \infty)$

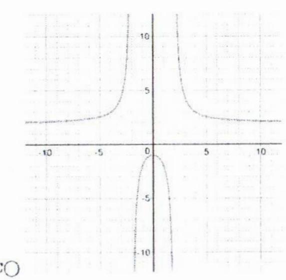
$\boxed{8\ pt}$ Which of the following is the graph of $y = \dfrac{2x^2+4}{x^2-4}$?
Note: There are only four choices for this question. The letters (A, B, C and D) are at the bottom left corner of each corresponding graph.



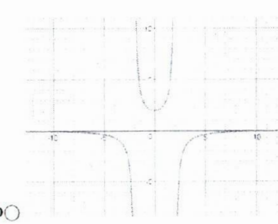
3.A◯



B◯



C◯



D◯

$\boxed{8\ pt}$ The critical numbers of a function $f(x)$ are $x = -2$ and $x = 1$. If the second derivative of the function is $f''(x) = 12x + 6$, then which of the following statements are true?

I. The relative minimum of $f(x)$ occurs at $x = 1$.

II. The relative minimum of $f(x)$ occurs at $x = -2$.

III. The relative maximum of $f(x)$ occurs at $x = 1$.

IV. The relative maximum of $f(x)$ occurs at $x = -2$.

V. The inflection point of $f(x)$ occurs at $x = \frac{1}{2}$.

VI. The inflection point of $f(x)$ occurs at $x = -\frac{1}{2}$.

4.A◯ Only I, II and VI are true

B◯ Only VI is true

C◯ Only I, IV and VI are true

D◯ Only II, III and V are true

E◯ Only V is true.

F◯ Only II, III and VI are true

$\boxed{8\ pt}$ Find the absolute extrema of $f(x) = \frac{1}{3}x^3 - 9x + 2$ on the closed interval $[0, 6]$.

5.A◯ Absolute minimum: $(3, -16)$; Absolute maximum: $(6, 20)$

B◯ Absolute minimum: $(0, 2)$; Absolute maximum: $(6, 20)$

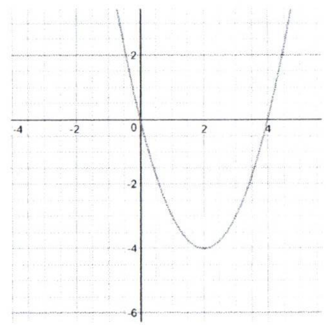
C◯ Absolute minimum: $(3, -16)$; Absolute maximum: $(-3, 20)$ and $(6, 20)$

D◯ Absolute minimum: $(3, -16)$; Absolute maximum: $(-3, 20)$

E◯ Absolute minimum: $(3, -16)$; Absolute maximum: None

F◯ Absolute minimum: $(0, 2)$; Absolute maximum: $(-3, 20)$

$\boxed{8\ pt}$ The graph of the derivative, $f'(x)$, is given below. What is the x-coordinate of the relative minimum of $f(x)$?



6.A◯ 3

B◯ 1

C◯ 5

D◯ 0

E◯ 4

F◯ 2

$\boxed{8\ pt}$ Evaluate $\int \left(6x^2 + \frac{\sqrt[3]{x^2}}{6}\right) dx$

7.A◯ $6x^3 + \frac{\sqrt[3]{x^5}}{10} + C$

B◯ $6x^3 + \frac{1}{9\sqrt[3]{x}} + C$

C◯ $2x^2 + \frac{\sqrt{x^3}}{6} + C$

D◯ $2x^3 + \frac{\sqrt[3]{x^5}}{10} + C$

E◯ $12x + \frac{1}{9\sqrt[3]{x}} + C$

F◯ $2x^3 + \frac{\sqrt{x^3}}{6} + C$

$\boxed{8\ pt}$ Solve the following initial value problem

$$f'(x) = \frac{1}{2}\sin x + \cos x, \qquad f(0) = 1$$

8.A◯ $f(x) = -\frac{1}{2}\cos x + \sin x + \frac{3}{2}$

B◯ $f(x) = -\frac{1}{2}\cos x - \sin x + \frac{3}{2}$

C◯ $f(x) = -\frac{1}{2}\cos x - \sin x + \frac{1}{2}$

D◯ $f(x) = \frac{1}{2}\cos x + \sin x + \frac{1}{2}$

E◯ $f(x) = -\frac{1}{2}\cos x + \sin x + \frac{1}{2}$

F◯ $f(x) = \frac{1}{2}\cos x - \sin x + \frac{1}{2}$

$\boxed{8\ pt}$ Find $y(1)$ given that $y'' = 4e^x$, $y'(0) = -4$, and $y(0) = 0$.

9.A◯ $4e - 8$

B◯ $4e - 4$

C◯ $4e - 12$

D◯ -4

E◯ -12

F◯ -8

$\boxed{8\ pt}$ You have 80 feet of fence to create a rectangular dog run, which will be bounded on one side by the wall of your house. What is the area of the largest dog run that you can create?

10.A◯ 80 ft^2

B◯ 1000 ft^2

C◯ 972 ft^2

D◯ $8\sqrt{10}$ ft^2

E◯ 842 ft^2

F◯ 800 ft^2

$\boxed{9\ pt}$ A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2800 - 200p$ units. Each unit costs \$10 to make. What is the maximum profit that the company can make?

11.A◯ 1000 dollars

B◯ 980 dollars

C◯ 800 dollars

D◯ 600 dollars

E◯ 880 dollars

F◯ 1200 dollars

9 pt Anita wants to construct a kitchen island in the shape of a rectangular box with a square base. The wood for the base and sides costs **6** dollars per square foot, and the butcher block for the top costs 18 dollars per square foot. What is the largest volume of the island that Anita can create for 240 dollars? Round your answer to the nearest hundredth.

12.A◯ 8.65 ft^3

B◯ 12.17 ft^3

C◯ 10.64 ft^3

D◯ 4.41 ft^3

E◯ 18.43 ft^3

F◯ 20.79 ft^3