practice for final exam

April 24, 2019

1. Recall that
$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$
Give the first three nonzero terms of the Taylor series for $\int_0^x t\cos(t^3)\ dt$.

A. $x - \frac{1}{14}x^7 + \frac{1}{312}x^{13}$

B. $x - \frac{1}{12}x^7 + \frac{1}{288}x^{13}$

C. $x^2 - \frac{1}{14}x^8 + \frac{1}{312}x^{14}$

D. $x - \frac{1}{2}x^7 + \frac{1}{24}x^{13}$

E. $\frac{1}{2}x^2 - \frac{1}{16}x^8 + \frac{1}{336}x^{14}$

2. Evaluate $\int_{-1}^{0} x^2\sqrt{x+1}\ dx$.

A. 16/15 B. 20/21 C. 16/105 D. -8/21 E. -8/5

3. Write an integral in polar coordinates to find the area enclosed by the curves $y = 0$, $x = 4$, and $y = \sqrt{3}x$.

A. $\int_0^{\pi/3} 4\sec(\theta)\ d\theta$

B. $\int_0^{\pi/3} 8\sec^2(\theta)\ d\theta$

C. $\int_0^{\pi/3} 4\tan(\theta)\ d\theta$

D. $\int_0^{\pi/3} 8\tan^2(\theta)\ d\theta$

E. $\int_0^{4} \sqrt{3}x\ dx$

4. Evaluate $\displaystyle\int_1^\infty \frac{\ln x}{x^4}\,dx$

A. 1/16 B. 1/9 C. -1/9 D. 1/12 E. -1/12

5. After performing an appropriate trig substitution, the integral $\displaystyle\int \frac{x^8}{(x^2+4)^{5/2}}\,dx$ becomes

A. $\displaystyle\int \frac{8\tan^8\theta}{\sec^5\theta}\,d\theta$

B. $\displaystyle\int \frac{8\sec^8\theta}{\tan^5\theta}\,d\theta$

C. $\displaystyle\int \frac{16\sec^8\theta}{\tan^3\theta}\,d\theta$

D. $\displaystyle\int \frac{16\tan^8\theta}{\sec^3\theta}\,d\theta$

E. $\displaystyle\int \frac{32\tan^5\theta}{\sec^8\theta}\,d\theta$

6. Find the first two nonzero terms in the Maclaurin series for $\dfrac{\sqrt[3]{8-x}-2}{x}$.

A. $-\dfrac{1}{12}-\dfrac{x}{288}$

B. $-\dfrac{1}{8}+\dfrac{x}{64}$

C. $-\dfrac{1}{8}-\dfrac{x}{64}$

D. $-\dfrac{1}{2}+\dfrac{x}{288}$

E. $\dfrac{1}{8}-\dfrac{x}{288}$

7. Which of these series converge?

I. $\displaystyle\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$ II. $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

(a) I and II. (b) Only I. (c) Only II. (d) Neither of these.

ANSWERS

1. E
2. C
3. B
4. B
5. D
6. A
7. B