Quiz 2 — MA261 — June 20, 2017

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1. (6 points) Find the length of the curve $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 1$.

   $\mathbf{r}'(t) = 2t\mathbf{j} + 3t^2\mathbf{k}$, $|r'(t)| = t\sqrt{4+9t^2}$.
   $L = \int_0^1 t\sqrt{4+9t^2}dt = \frac{1}{27}(13^{3/2} - 8)$.

2. (6 points) At what points does the helix $\mathbf{r}(t) = \langle \sin t, \cos t, t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$? Give the coordinates, not just the t-values.

   Substitute $\mathbf{r}(t)$ into the equation of the sphere:

   $$\begin{aligned} \sin^2 t + \cos^2 t + t^2 &= 5 \\ t^2 &= 4 \\ t &= \pm 2 \end{aligned}$$

   Now put the t-values found back into $\mathbf{r}(t)$ to get $\langle \sin 2, \cos 2, 2 \rangle$ and $\langle \sin(-2), \cos(-2), -2 \rangle$.

3. (8 points) Let $\mathbf{r}(t) = te^t\mathbf{i} - 2\mathbf{j} + \sin(t)\mathbf{k}$.

   (a) Find $\mathbf{r}'(t)$.

   (b) Find a vector equation for the line $\mathbf{u}(t)$ tangent to the curve at the point where $t = 0$.

   (a) $\mathbf{r}'(t) = (1+t)e^t\mathbf{i} + \cos(t)\mathbf{k}$.

   (b) $\mathbf{r}(0) = -2\mathbf{j}$, $\mathbf{r}'(0) = \mathbf{i} + \mathbf{k}$. $\mathbf{u}(t) = (\mathbf{i} + \mathbf{k})t - 2\mathbf{j}$.