Quiz 4 — MA261 — July 7, 2017

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1. (4 points) Find and classify the critical point of the function $f(x, y) = y^2 - 4y + 2x - x^2$ using the second derivatives test.

   > $f_x = 2 - 2x$, so $x = 1$. $f_y = 2y - 4$, so $y = 2$. $f_{xx} = -2$ and $f_{yy} = 2$ and $f_{xy} = 0$, so $D = -4$ and the critical point $(1, 2)$ is a saddle point.

2. (8 points) Find the (x, y) coördinates of the extreme value of $f(x, y) = e^{xy}$ subject to the constraint $x^3 + y^3 = 16$ using the method of Lagrange multipliers.

   > $f_x = ye^{xy}$, $f_y = xe^{xy}$, $g_x = 3x^2$, $g_y = 3y^2$. Then $ye^{xy} = \lambda 3x^2$ and $xe^{xy} = \lambda 3y^2$. Solving for λ and substituting gives the solution $y^3 = x^3$, or $y = x$. Then $x^3 + y^3 = 16$ tells us $x = y = 2$.

3. (8 points) Set up **but do not evaluate** an iterated integral to compute $\iint_D y^2 \, dA$ where D is the triangular region with vertices $(0, 1), (1, 2), (4, 1)$.

   > $$\iint_D y^2 \, dA = \int_1^2 \int_{y-1}^{7-3y} y^2 \, dx dy$$