Quiz 6 — MA261 — July 14, 2017
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1. Find the volume of the region bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy coördinate plane by changing to cylindrical coördinates.

$$\int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r \; dz dr d\theta = \int_0^{2\pi} d\theta \int_0^2 (4-r^2) r \; dr$$
$$= 2\pi \left(2r^2 - \frac{r^4}{4} \right) \Big|_0^2$$
$$= 8\pi$$

2. Evaluate $\iiint_R (x^2 + y^2) \; dV$, where R is the region between the spheres of radius 1 and 2 centered at the origin.

Hint: $\int_0^{\pi} \sin^3 u \; du = 4/3$

$$\iiint_R (x^2 + y^2) \; dV = \int_0^{2\pi} \int_0^{\pi} \int_1^2 (\rho^2 \sin^2 \phi)\rho^2 \sin \phi \; d\rho d\phi d\theta$$
$$= \int_0^{2\pi} d\theta \int_0^{\pi} \sin^3 \phi \; d\phi \int_1^2 \rho^4 \; d\rho$$
$$= (2\pi)(4/3)(31/5)$$
$$= 248\pi/15$$