Quiz 7 — MA261 — July 20, 2017

Christina Jamroz, Alden Bradford

1. (4 points) Find the gradient vector field of $f(x,y,z) = xy^2z$.

$$\nabla f = \langle f_x, f_y, f_z \rangle = \langle y^2z, 2xyz, xy^2 \rangle$$

2. (6 points) Let $\mathbf{F}(x,y) = \langle 2y, x \rangle$. Integrate $\mathbf{F} \cdot d\mathbf{r}$ along the path $y = x^2$ from $(0,0)$ to $(1,1)$.

Using $y = x^2$ we find $dy = 2xdx$ and so

$$\begin{aligned}\int 2y\ dx + x\ dy &= \int_0^1 2x^2\ dx + x2x\ dx \\ &= \int_0^1 (4x^2)\ dx \\ &= 4/3\end{aligned}$$

3. (10 points) For each function $\mathbf{F}$ below, determine whether $\mathbf{F}$ is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$.

(a) $\mathbf{F} = \langle 2xy, x^2 + y^2 \rangle$ (b) $\mathbf{F} = \langle xe^y, e^y \rangle$

(a) $\frac{\partial}{\partial y} 2xy = 2x = \frac{\partial}{\partial x}\left(x^2 + y^2\right)$ so the field is conservative.

$$\begin{aligned}f(x,y) &= \int 2xy\ dx &&= x^2y + g(y) \\ f(x,y) &= \int (x^2 + y^2)\ dy &&= x^2y + \frac{y^3}{3} + h(x) \\ f(x,y) &= x^2y + \frac{y^3}{3} + C\end{aligned}$$

(b) $\frac{\partial}{\partial y} xe^y = xe^y$ while $\frac{\partial}{\partial x} e^y = 0$. The function is not conservative.