Quiz 8 — MA261 — July 25, 2017

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1. (8 points) Use Green's theorem to evaluate $\int_C y^3\,dx - x^3\,dy$, where C is the positively-oriented circle $x^2+y^2=4$.

Here $P=y^3$, $Q=-x^3$, so $\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=-3x^2-3y^2=-3r^2$. Do the integral in polar coordinates:

$$\begin{aligned}\int_0^{2\pi}\int_0^2(-3r^2)r\,dr\,d\theta &= -6\pi\int_0^2 r^3\,dr\\ &= -6\pi\frac{r^4}{4}\Big|_0^2\\ &= -24\pi\end{aligned}$$

2. (12 points) Find (a) the divergence and (b) the curl of $\mathbf{F}(x,y,z)=xy^2z^3\mathbf{i}+x^3yz^2\mathbf{j}+x^2y^3z\mathbf{k}$.

$\nabla\cdot\mathbf{F}=y^2z^3+x^3z^2+x^2y^3$

$\nabla\times\mathbf{F}=$

$(3x^2y^2z-2x^3yz)\mathbf{i}+(3xy^2z^2-2xy^3z)\mathbf{j}+(3x^2yz^2-2xyz^3)\mathbf{k}$